



Province of the
EASTERN CAPE
EDUCATION

**NASIONALE
SENIOR SERTIFIKAAT**

GRAAD 12

SEPTEMBER 2010

WISKUNDE – VRAESTEL 1

MEMORANDUM

Hierdie memorandum bestaan uit 13 bladsye.

Konsekwente akkuraatheid sal as 'n algemene reël geld.

VRAAG 1

1.1 1.1.1 $x(2x - 5) = 0$
 $x = 0$ of $x = \frac{5}{2}$ ✓✓ antwoorde (2)

1.1.2 $(3 - x)(2x - 1) = 1$
 $6x - 3 - 2x^2 + x = 1$
 $2x^2 - 7x + 4 = 0$ ✓ vereenvoudig
 $x = \frac{-b \pm \sqrt{b^2 - 4ac}}{2a}$ ✓ formule
 $= \frac{7 \pm \sqrt{49 - 32}}{2(2)}$ ✓ instelling in formule
 $= \frac{7 \pm \sqrt{17}}{4}$
 $= 2,78$ of $0,72$ ✓✓ antwoorde
 (-1 vir verkeerde afronding) (5)

1.1.3 $x^2 - 2x \leq 15$
 $x^2 - 2x - 15 \leq 0$
 $(x + 3)(x - 5) \leq 0$ ✓ standaard vorm
 ✓ faktore en korrekte ongelykheid

$$\begin{array}{ccccccc} & -3 & & & 5 & & \\ + & 0 & & - & 0 & & + \\ & \bullet & & & \bullet & & \\ -3 \leq x \leq 5 & \text{ of } & x \in [-3 ; 5] \end{array}$$
 ✓✓ notasie en kritiese waardes (4)

1.2 OPSIE 1:

$y = -4x + 5 \dots\dots\dots(1)$
 Vervang (1) in $y = x^2 - 2x - 3$
 $-4x + 5 = x^2 - 2x - 3$
 $x^2 + 2x - 8 = 0$ ✓ instelling / gelykstel
 $(x - 2)(x + 4) = 0$ ✓ standaard vorm
 $x = 2$ of $x = -4$ ✓ faktore
 ✓ x waardes
 Vervang in (1):
 $y = -4(2) + 5$ of $y = -4(-4) + 5$
 $= -3$ $= 21$ ✓ y waardes (5)

OPSIE 2:

$$x = \frac{-y+5}{4} \dots\dots\dots (1)$$

Stel. (1) in $y = x^2 - 2x - 3$

$$y = \left(\frac{-y+5}{4}\right)^2 - 2\left(\frac{-y+5}{4}\right) - 3$$

✓ instelling

$$y = \frac{y^2 - 10y + 25}{16} + \frac{2y - 10}{4} - 3$$

$$16y = y^2 - 10y + 25 + 8y - 40 - 48$$

$$y^2 - 18y - 63 = 0$$

✓ standaard vorm

$$(y - 21)(y + 3) = 0$$

✓ faktore

$$y = 21 \quad \text{of} \quad y = -3$$

✓ y waardes

Vervang in (1):

$$x = \frac{-21+5}{4} \quad \text{of} \quad x = \frac{3+5}{4}$$

$$x = -4$$

$$x = 2$$

✓ x waardes

$$\begin{aligned} 1.3 \quad & \frac{6^{2010} \times 10^{2011}}{4^{2010} \times 15^{2009}} \\ &= \frac{2^{2010} \times 3^{2010} \times 2^{2011} \times 5^{2011}}{2^{4020} \times 3^{2009} \times 5^{2009}} \\ &= 2^{4021-4020} \times 3^{2010-2009} \times 5^{2011-2009} \\ &= 2^1 \times 3^1 \times 5^2 \\ &= 6 \times 25 \\ &= 150 \end{aligned}$$

✓ magte van priemgetalle

✓ eksp. wette

✓ vereenvoudiging

✓ antwoord

(4)
[20]**VRAAG 2**

Opsie 1:

$$2.1 \quad 11; 7; 3; -1; -5; \dots$$

$$\begin{aligned} T_n &= a + (n-1)d \\ &= 11 + (n-1)(-4) \\ &= -4n + 15 \end{aligned}$$

✓ formule

✓ instelling

✓ antwoord

(slegs antwoord – vol punte)

(3)

Opsie 2:

$$T_n = pn + q$$

✓ formule

$$T_1 = -4n + q$$

✓ p = -4

$$11 = -4(1) + q$$

$$q = 15$$

✓ q = 15

$$T_n = -4n + 15$$

$$2.2 \quad -293 = -4n + 15$$

✓ instelling

$$4n = 293 + 15$$

$$n = 77$$

✓ antwoord

∴ -293 is 'n term van die ry.

(2)

2.3 $S_1 = 11$; $S_2 = 18$; $S_3 = 21$;
 $S_4 = 20$
 $\therefore 11 ; 18 ; 21 ; 20 ; \dots$

✓ antwoord (1)

2.4 $11 \searrow 7 \swarrow 18 \searrow 3 \swarrow 21 \searrow -1 \swarrow 20 \searrow -5 \swarrow 15 ; \dots$

✓ 1^{ste} verskil

$\therefore 2a = -4$
 $a = -2$

✓ a-waarde

$3a + b = 7$ OF $-2 + b + c = 11$
 $3(-2) + b = 7$ $-8 + 2b + c = 18$
 $b = 13$

✓ instelling

✓ b-waarde

$a + b + c = 11$
 $-2 + 13 + c = 11$
 $c = 0$

✓ c-waarde

$\therefore T_n = -2n^2 + 13n$

(5)

OF

$S_n = \frac{n}{2}[2a + (n-1)d]$

✓ formule

$= \frac{n}{2}[2(11) + (n-1)(-4)]$

✓✓ instelling

$= \frac{n}{2}[22 - 4n + 4]$

✓ vereenvoudiging

$= \frac{n}{2}[26 - 4n]$

$T_n = -2n^2 + 13n$

✓ antwoord

2.5 a ; $a + d$; $a + 2d$; $a + 3d$; ...

$S_1 = a$
 $S_2 = 2a + d$
 $S_3 = 3a + 3d$
 $S_4 = 4a + 6d$
 \therefore Nuwe ry:

} ✓ som

$a \searrow a+d \swarrow a+2d \searrow a+3d \swarrow \dots$

✓ ry

✓ 2^{de} verskil

\therefore konstante 2^{de} verskil van d.
 \therefore altyd kwadratiese.

✓ konstante tweede verskil

(4)
[15]

-

VRAAG 3

$$3.1 \quad -4 - 8 - 16 - 32$$

$$T_k = a \cdot r^{k-1} \\ = -4 \cdot 2^{k-1}$$

✓ formule
✓ instelling

$$\sum_{k=1}^4 -2^{k+1} \quad \text{of} \quad \sum_{k=0}^3 -2^{k+2} \quad \text{of} \quad \sum_{k=1}^4 (-4 \cdot 2^{k-1})$$

✓ antwoord

(3)

$$3.2 \quad 30 ; 24 ; 19,2 ; 15,36 ; \dots$$

$$3.2.1 \quad r = \frac{24}{30} \\ = \frac{4}{5} \quad (\text{of } 0,8)$$

✓ antwoord

(1)

$$3.2.2 \quad S_8 = \frac{a(r^n - 1)}{r - 1} \\ = \frac{30[(\frac{4}{5})^8 - 1]}{\frac{4}{5} - 1} \\ = 124,83$$

✓ formule

✓ instelling
✓ antwoord

(3)

$$3.2.3 \quad S_\infty = \frac{a}{1 - r} \\ = \frac{30}{1 - \frac{4}{5}} \\ = 150$$

✓ formule

✓ instelling
✓ antwoord

(3)

OPSIE 1:

$$3.2.4 \quad T_n = a \cdot r^{n-1} \\ 30 \cdot (\frac{4}{5})^{n-1} \geq 3,6$$

✓ instelling

$$(\frac{4}{5})^{n-1} \geq \frac{3}{25}$$

✓ vereenvoudiging

$$(n-1) \log(\frac{4}{5}) \geq \log(\frac{3}{25})$$

$$n-1 \leq \frac{\log(\frac{3}{25})}{\log(\frac{4}{5})}$$

✓ logs

$$n-1 \leq 9,5017\dots$$

$$n \leq 10,5017$$

∴ Ekonomies vir 10 herhalings

✓ vereenvoudiging
✓ antwoord

(5)

OPSIE 2

$$30. \left(\frac{4}{5}\right)^{n-1} = 3,6$$

✓ instelling

$$\left(\frac{4}{5}\right)^{n-1} = \frac{3}{25}$$

✓ vereenvoudiging

$$(n-1) \log\left(\frac{4}{5}\right) = \log\left(\frac{3}{25}\right)$$

$$n-1 = \frac{\log\left(\frac{3}{25}\right)}{\log\left(\frac{4}{5}\right)}$$

✓ logs

$$n-1 = 9,5017\dots$$

✓ vereenvoudiging

$$n = 10,5017\dots$$

∴ Ekonomies vir 10 herhalings

✓ antwoord

OPSIE 3:

$$30. (0,8)^{n-1} = 3,6$$

✓ instelling

$$(0,8)^{n-1} = \frac{3}{25}$$

✓ vereenvoudiging

$$(n-1) \log(0,8) = \log(0,12)$$

$$n-1 = \log_{0,8} 0,12$$

✓ logs

$$n-1 = 9,5017\dots$$

✓ vereenvoudiging

$$n = 10,5017$$

∴ Ekonomies vir 10 herhalings

✓ antwoord

[15]

VRAAG 4

$$4.1 \quad \begin{aligned} x &= -2 \\ y &= 1 \end{aligned}$$

✓ antwoord

✓ antwoord

(2)

$$4.2 \quad y\text{-afsnit: } y = \frac{-3}{0+2} + 1$$

$$y = \frac{-3}{2} + 1$$

$$y = -\frac{1}{2}$$

✓ antwoord

$$x\text{-afsnit: } 0 = \frac{-3}{x+2} + 1$$

$$\frac{3}{x+2} = 1$$

✓ vereenvoudiging

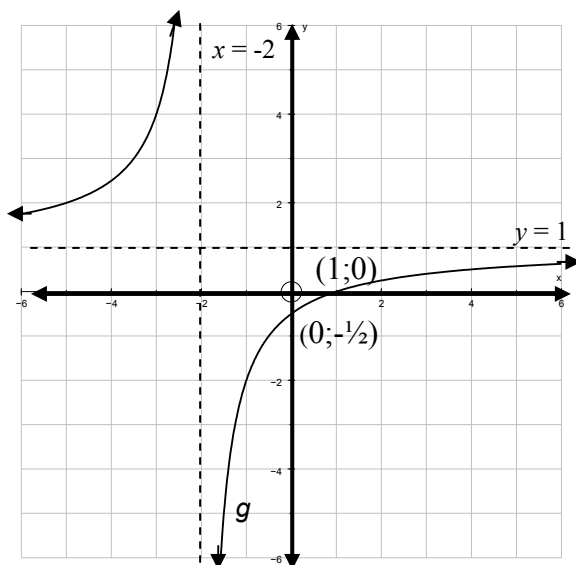
$$3 = x + 2$$

$$x = 1$$

✓ antwoord

(3)

4.3



- ✓ asymptote
- ✓ afsnitte
- ✓ vorm

(3)

4.3 $x = -4$

(3)

[9]

VRAAG 5

5.1 $y = \log_m x$
 $2 = \log_m 4$ (4 ; 2)
 $m^2 = 4$
 $m = 2$

✓ instelling

✓ antwoord

(2)

5.2 Definisieversameling: $x > 0$

✓ antwoord

(1)

5.3 $y = \log_2 x$
 $g^{-1}: x = \log_2 y$
 $y = 2^x$

✓ ruil x en y

✓ antwoord

(2)

OF: $x = 2^y$
 $g^{-1}: y = 2^x$

✓ magsvorm

✓ antwoord

5.4 $h: y = 2^{x+2}$

✓ antwoord

(1)

5.5 E(-1 ; 0) deur simmetrie

$$y = a(x + 1)(x - 5)$$

$$= a(x^2 - 4x - 5)$$

$$2 = a(16 - 16 - 5) \quad (4 ; 2)$$

$$a = \frac{-2}{5}$$

$$y = \frac{-2}{5}(x^2 - 4x - 5)$$

$$\therefore y = \frac{-2}{5}x^2 + \frac{8}{5}x + 2$$

$$a = \frac{-2}{5} \quad ; \quad b = \frac{8}{5} \quad \text{en} \quad c = 2$$

✓ punt E

✓ wortels

✓ a waarde

✓ vereenvoudiging

(4)

OF

$$y = a(x - 2)^2 + q$$

$$(5;0): \quad 0 = a(5-2)^2 + q$$

$$0 = 9a + q$$

(1) ✓ vergelyking 1

$$(4;2): \quad 2 = a(4-2)^2 + q$$

$$2 = 4a + q$$

(2) ✓ vergelyking 2

$$(1) - (2) \quad -2 = 5a$$

$$a = \frac{-2}{5}$$

$$q = \frac{18}{5}$$

✓ a waarde

(2)

$$y = \frac{-2}{5}(x - 2)^2 + \frac{18}{5}$$

$$\therefore y = \frac{-2}{5}x^2 + \frac{8}{5}x + 2$$

✓ vereenvoudiging

$$a = \frac{-2}{5} \quad ; \quad b = \frac{8}{5} \quad \text{en} \quad c = 2$$

5.6 OPSIE 1:

$$y = \frac{-2}{5}(x^2 - 4x - 5)$$

$$= \frac{-2}{5}(x^2 - 4x + 4 - 4 - 5)$$

$$= \frac{-2}{5}[(x - 2)^2 - 9]$$

$$= \frac{-2}{5}(x - 2)^2 + \frac{18}{5}$$

✓ kwadraatsvoltooing

✓ perfekte vierkant

✓ antwoord

(3)

OPSIE 2:

$$a = \frac{-2}{5} \quad \text{en} \quad p = 2$$

$$y = \frac{-2}{5} (x - 2)^2 + q$$

✓ vorm

$$0 = \frac{-2}{5} (5 - 2)^2 + q \quad (5 ; 0)$$

✓ instelling

$$q = \frac{2}{5} (3)^2$$

$$q = \frac{18}{5}$$

$$\therefore y = \frac{-2}{5} (x - 2)^2 + \frac{18}{5}$$

✓ antwoord

- 5.7 Maksimum waarde van $f(x) = 3,6$
 As grafiek 4 eenhede af geskuif word
 sal dit nie meer die x -as sny nie.

✓ maksimum

✓ 4 af

✓ antwoord

(3)

OF

$$f(x) - 4 = 0$$

$$f(x) = 4$$

As die grafiek van $y = 4$ getrek word,
 sal dit nie vir f sny nie.

✓ $f(x) = 4$

✓✓ verduideliking

- 5.8 $1 < x < 5$ of $x \in (1 ; 5)$

✓✓ antwoord

(2)

[18]

VRAAG 6

6.1 $y = x^2$

$$x = y^2$$

$$y = -\sqrt{x}$$

✓ omruil van x en y

✓ antwoord

(2)

6.2 $x \geq 0$

✓ antwoord

(1)

6.3 $y \leq 0$

✓✓ antwoord

(1)

- 6.4 die inverse het 'n een-tot-een afbeelding.
 OF
 elke element van die
 definisieversameling word met slegs een
 van die waardeversameling verbind.

✓✓ antwoord

(2)

[6]

VRAAG 7

- 7.1 7.1.1 $A = P(1+i)^n$ ✓ formule
- $15000 = 10000 (1.01)^n$ ✓ instelling
- $1.5 = (1.01)^n$
- $\log 1.5 = n \log 1.01$ ✓ log vorm
- $n = 40,75$ maande ✓ antwoord (4)
- 7.1.2 $(1+i) = (1 + \frac{0,12}{12})^{12}$ ✓ instelling
- $i = 1,1268 - 1$ ✓ vereenvoudiging
- $i = 0,1268$ $r = 12,68\%$ ✓ antwoord (3)
- 7.2 7.2.1 $P = \frac{x[1 - (1+i)^{-n}]}{i}$ ✓ formule
- $= \frac{500[1 - (1 + \frac{0,08}{12})^{-36}]}{\frac{0,08}{12}}$ ✓ instelling
- $= R15\ 955,90$ ✓ antwoord (3)
- 7.2.2 $\text{Saldo} = \frac{500[1 - (1 + \frac{0,08}{12})^{-30}]}{\frac{0,08}{12}}$ ✓ i waarde
- ✓ n waarde
- $= R13\ 554,42$ ✓ antwoord (3)
- 7.2.3 $13554,42 = \frac{x[1 - (1 + \frac{0,12}{12})^{-30}]}{\frac{0,12}{12}}$ ✓ p waarde
- ✓ n waarde
- $135,5442 = x(0,258077\dots)$
- $x = R525,21$ ✓ antwoord (3)

[16]

VRAAG 8

8.1 $5h$ nader 10

OF $5h \rightarrow 10$

✓ antwoord

(5h = 10 is NIE aanvaarbaar NIE) (1)

8.2 $f'(x) = \lim_{h \rightarrow 0} \frac{f(x+h) - f(x)}{h}$

✓ formule

$$= \lim_{h \rightarrow 0} \frac{\frac{5}{x+h} - \frac{5}{x}}{h}$$

✓ instelling

$$= \lim_{h \rightarrow 0} \frac{5x - 5(x+h)}{x(x+h)} \times \frac{1}{h}$$

✓ vereenvoudiging

$$= \lim_{h \rightarrow 0} \frac{5x - 5x - 5h}{x(x+h)} \times \frac{1}{h}$$

$$= \lim_{h \rightarrow 0} \frac{-5}{x(x+h)}$$

✓ vereenvoudiging

$$= \frac{-5}{x^2}$$

✓ antwoord

(slegs antwoord – geen punte) (5)

8.3

$$D_x[8x^4 - 6\sqrt{x} + \frac{5}{x}]$$

$$D_x[8x^4 - 6x^{\frac{1}{2}} + 5x^{-1}]$$

✓ magsvorm

$$= 32x^3 - 3x^{-\frac{1}{2}} - 5x^{-2}$$

✓✓✓ afgeleides (4)

[10]**VRAAG 9**

9.1 $x^3 - 3x^2 - 9x - 5 = 0$

$(x+1)(x^2 - 4x - 5) = 0$

$(x+1)(x+1)(x-5) = 0$

$x = -1 \text{ of } x = 5$

$D(5; 0)$

$f'(x) = 3x^2 - 6x - 9 = 0$

$f'(5) = 3(5)^2 - 6(5) - 9$

$\therefore m = 36$

✓ kwadratiese faktor

✓ faktore

✓ x waardes

✓ afgeleide

✓ stel $x = 5$

✓ antwoord (6)

9.2 $f'(x) = 3x^2 - 6x - 9 = 0$

$x^2 - 2x - 3 = 0$

$(x-3)(x+1) = 0$

$x = 3 \text{ of } x = -1$

✓ afgeleide = 0

✓ faktore

✓ x waardes

$y = (3)^3 - 3(3)^2 - 9(3) - 5$

$= -32$

✓ instelling

$\therefore y = -32$

✓ antwoord (5)

- 9.3 $-1 < x < 3$ of $x \in (-1 ; 3)$ ✓✓ antwoord (2)
- 9.4 $B(0 ; -5)$ en $D(5 ; 0)$ ✓ koördinate van B.
 Gem. grad. = $\frac{-5 - 0}{0 - 5}$ ✓ instelling
 $= 1$ ✓ antwoord (3)
- 9.5 $f''(x) = 6x - 6$ ✓ $f''(x)$
 $6x - 6 = 0$ ✓ $f''(x) = 0$
 $x = 1$
 $\therefore x < 1$ ✓ antwoord (3)

[19]

VRAAG 10

- 10.1 $SR = x + 30$ ✓ antwoord
 $EH = \frac{5400}{x}$ ✓ $\frac{5400}{x}$
 $PS = \frac{5400}{x} + 20$ ✓ antwoord
 $AreaPQRS = (x + 30)(\frac{5400}{x} + 20)$ ✓ area
 $Area = 5400 + 20x + \frac{162000}{x} + 600$ (4)
 $Area PQRS = 6000 + 20x + 162000x^{-1}$
- 10.2 $\frac{dA}{dx} = 20 - 162000x^{-2}$ ✓ afgeleide
 $20 - 162000x^{-2} = 0$ ✓ afgeleide = 0
 $\frac{162000}{x^2} = 20$
 $162000 = 20x^2$ ✓ vereenvoudiging
 $8100 = x^2$
 $90 = x$ ✓ x-waarde
 $\therefore HG = 90 \text{ mm}$
 so $SR = 120 \text{ mm}$ ✓ antwoord (5)

[9]

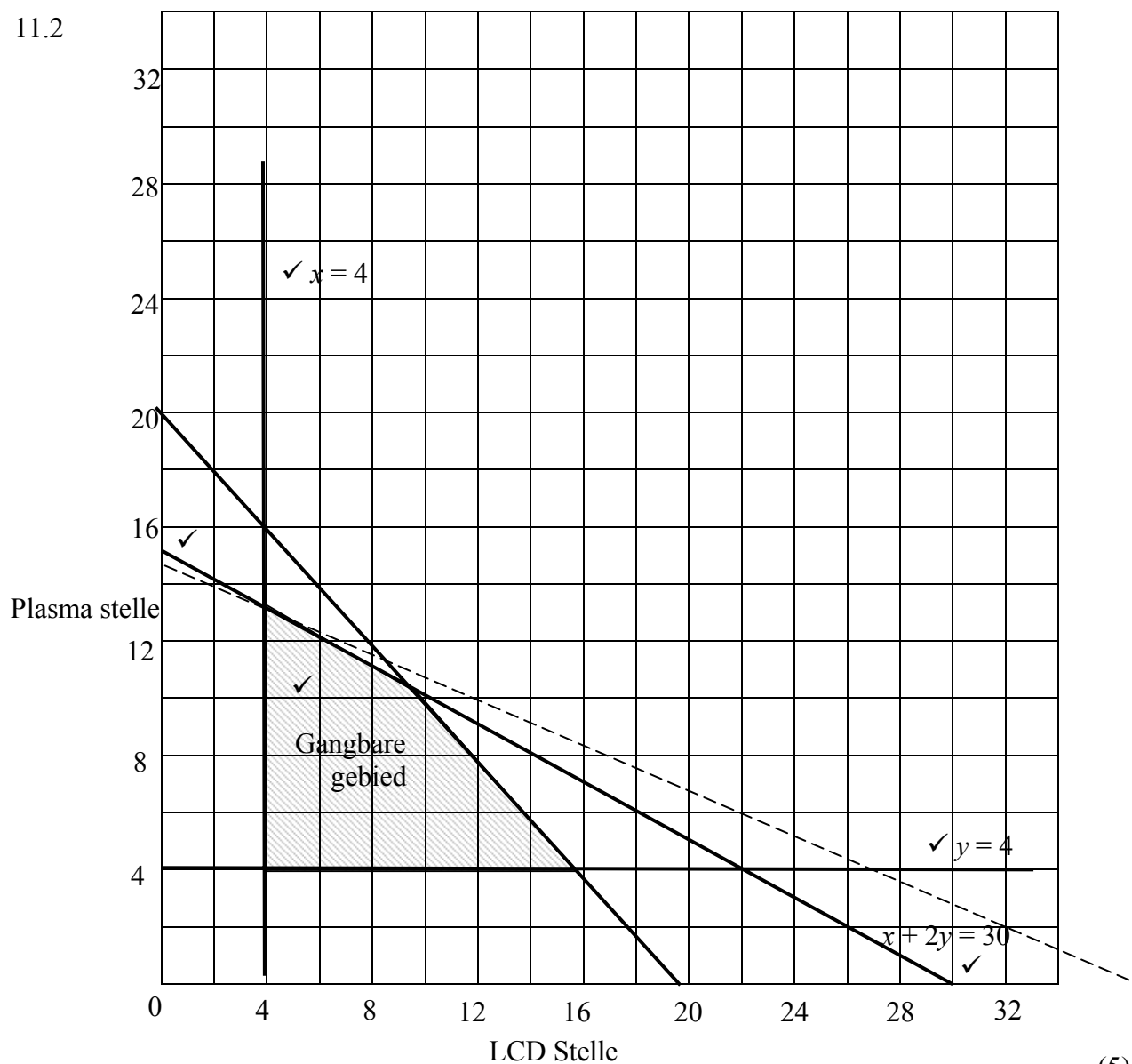
VRAAG 11

11.1 $x + 2y \leq 30$ OF $3000x + 6000y \leq 90000$
 $y \geq 4$

✓ ongelykheid
 ✓ ongelykheid

(2)

11.2



(5)

11.3 11.3.1 $P = 400x + 1000y$

✓ antwoord

(1)

11.3.2 $y = -\frac{2}{5}x + \frac{P}{1000}$

✓ gradiënt van soeklyn
 ✓ soeklyn op diagram

(2)

11.3.3 Maks wins by (4 ; 13)
 $P = 400(4) + 1000(13)$
 $= R14\ 600$

✓✓ x en y waardes

✓ antwoord

(3)

[13]

TOTAAL: 150