Province of the

EASTERN CAPE

EDUCATION

NATIONAL SENIOR CERTIFICATE

GRADE 12

SEPTEMBER 2010

MATHEMATICS – PAPER 1

MEMORANDUM

This memorandum consists of 13 pages.

Consistent accuracy applies as a general rule.

QUESTION 1

1.1 1.1.1 $x(2x - 5) = 0$

$x = 0$ or $x = \frac{5}{2}$ ✓✓ answers (2)

1.1.2 $(3 - x)(2x - 1) = 1$

$6x - 3 - 2x^2 + x = 1$

$2x^2 - 7x + 4 = 0$ ✓ simplification

$x = \frac{-b \pm \sqrt{b^2 - 4ac}}{2a}$ ✓ formula

$= \frac{7 \pm \sqrt{49 - 32}}{2(2)}$ ✓ substitution into formula

$= \frac{7 \pm \sqrt{17}}{4}$

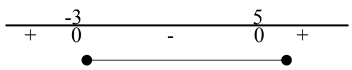
$= 2{,}78$ or $0{,}72$ ✓✓ answers

(-1 for rounding off incorrectly) (5)

1.1.3 $x^2 - 2x \leq 15$

$x^2 - 2x - 15 \leq 0$ ✓ standard form

$(x + 3)(x - 5) \leq 0$ ✓ factors with correct inequality



$-3 \leq x \leq 5$ or $x \in [-3 ; 5]$ ✓✓notation and critical values (4)

1.2 OPTION 1

$y = -4x + 5$……….(1)

Substitute (1) into $y = x^2 - 2x - 3$

$-4x + 5 = x^2 - 2x - 3$ ✓ substitution/equating

$x^2 + 2x - 8 = 0$ ✓ standard form

$(x - 2)(x + 4) = 0$ ✓ factors

$x = 2$ or $x = -4$ ✓ x values

Substitute into (1):

$y = -4(2) + 5$ or $y = -4(-4) + 5$

$= -3$ $= 21$ ✓ y values (5)

OPTION 2

$x=\frac{-y+5}{4}$ ………… (1)

Subst. (1) into $y = x^2 - 2x - 3$ ✓ substitution

$y=(\frac{-y+5}{4})^2 - 2(\frac{-y+5}{4}) - 3$

$y = \frac{y^2 - 10y + 25}{16} + \frac{2y - 10}{4} - 3$

$16y = y^2 - 10y + 25 + 8y - 40 - 48$

$y^2 - 18y - 63 = 0$ ✓ standard form

$(y - 21)(y + 3) = 0$ ✓ factors

$y = 21$ or $y = -3$ ✓ y values

Subst. into (1):

$x = \frac{-21+5}{4}$ or $x = \frac{3+5}{4}$

$x = -4$ $x = 2$ ✓ x values

1.3 $\frac{6^{2010} \times 10^{2011}}{4^{2010} \times 15^{2009}}$

$= \frac{2^{2010} \times 3^{2010} \times 2^{2011} \times 5^{2011}}{2^{4020} \times 3^{2009} \times 5^{2009}}$ ✓ powers of prime numbers

$= 2^{4021 - 4020} \text{ x } 3^{2010 - 2009} \text{ x } 5^{2011 - 2009}$ ✓ exp laws

$= 2^1 \text{ x } 3^1 \text{ x } 5^2$ ✓ simplification

$= 6 \text{ x } 25$

$= 150$ ✓ answer (4)

[20]

QUESTION 2

Option 1:

2.1 11 ; 7 ; 3 ; -1 ; -5 ; …

$T_n = a + (n - 1)d$ ✓ formula

$= 11 + (n - 1)(-4)$ ✓ substitution

$= -4n + 15$ ✓ answer

(answer only – full marks) (3)

Option 2:

$T_n = pn + q$ ✓ formula

$= -4n + q$ ✓ $p = -4$

$T_1 = 11 = -4(1) + q$

$q = 15$ ✓ $q = 15$

$T_n = -4n + 15$

2.2 $-293 = -4n + 15$ ✓ substitution

$4n = 293 + 15$

$n = 77$ ✓ answer

$\therefore$ -293 is a term of the sequence. (2)

2.3 $S_1 = 11$; $S_2 = 18$; $S_3 = 21$;
$S_4 = 20$
$\therefore$ 11 ; 18 ; 21 ; 20 ; … ✓ answer (1)

2.4 11 ; 18 ; 21 ; 20 ; 15 ; …..
7 3 -1 -5
-4 -4 -4 ✓ 1^{st} difference

$\therefore$ $2a = -4$
$a = -2$ ✓ a-value

$3a + b = 7$ OR $-2 + b + c = 11$
$3(-2) + b = 7$ $-8 + 2b + c = 18$ ✓ substitution
$b = 13$ ✓ b-value

$a + b + c = 11$
$-2 + 13 + c = 11$
$c = 0$ ✓ c-value
(5)

$\therefore T_n = -2n^2 + 13n$

OR

$S_n = \frac{n}{2}[2a + (n-1)d]$ ✓ formula

$= \frac{n}{2}[2(11) + (n-1)(-4)]$ ✓✓ substitution

$= \frac{n}{2}[22 - 4n + 4]$ ✓ simplification

$= \frac{n}{2}[26 - 4n]$

$T_n = -2n^2 + 13n$ ✓ answer

2.5 a ; a + d ; a + 2d ; a + 3d ; …

$S_1 = a$
$S_2 = 2a + d$
$S_3 = 3a + 3d$
$S_4 = 4a + 6d$
✓ sum
$\therefore$ New sequence:

a ; 2a + d ; 3a + 3d ; 4a + 6d ;….. ✓ sequence
a+d a+2d a+3d
d d ✓ 2^{nd} difference

$\therefore$ constant 2^{nd} difference of d. ✓ constant second difference
$\therefore$ always quadratic. (4)
[15]

QUESTION 3

3.1 $-4-8-16-32$

$T_k = a \cdot r^{k-1}$ ✓ formula

$= -4.2^{k-1}$ ✓ substitution

$\sum_{k=1}^{4} -2^{k+1}$ or $\sum_{k=0}^{3} -2^{k+2}$ or $\sum_{k=1}^{4} (-4.2^{k-1})$ ✓ answer (3)

3.2 30 ; 24 ; 19,2 ; 15,36 ; …

3.2.1 $r = \frac{24}{30}$

$= \frac{4}{5}$ (or 0,8) ✓ answer (1)

3.2.2 $S_8 = \frac{a(r^n - 1)}{r-1}$ ✓ formula

$= \frac{30[(\frac{4}{5})^8 - 1]}{\frac{4}{5} - 1}$ ✓ substitution

$= 124.83$ ✓ answer (3)

3.2.3 $S_\infty = \frac{a}{1-r}$ ✓ formula

$= \frac{30}{1 - \frac{4}{5}}$ ✓ substitution

$= 150$ ✓ answer (3)

OPTION 1:

3.2.4 $T_n = a \cdot r^{n-1}$

$30.\ (\frac{4}{5})^{n-1} \geq 3{,}6$ ✓ substitution

$(\frac{4}{5})^{n-1} \geq \frac{3}{25}$ ✓ simplification

$(n-1)\log(\frac{4}{5}) \geq \log(\frac{3}{25})$

$n - 1 \leq \frac{\log(\frac{3}{25})}{\log(\frac{4}{5})}$ ✓ logs

$n - 1 \leq 9{,}5017\ldots$

$n \leq 10{,}5017$ ✓ simplification

$\therefore$ Economical for 10 passes ✓ answer (5)

OPTION 2

$30.\ (\frac{4}{5})^{n-1} = 3,6$ ✓ substitution

$(\frac{4}{5})^{n-1} = \frac{3}{25}$ ✓ simplification

$(n-1)\log(\frac{4}{5}) = \log(\frac{3}{25})$

$n-1 = \dfrac{\log(\frac{3}{25})}{\log(\frac{4}{5})}$ ✓ logs

$n-1 = 9,5017\ldots$ ✓ simplification

$n = 10,5017\ldots$

$\therefore$ Economical for 10 passes ✓ answer

OPTION 3:

$30.\ (0,8)^{n-1} = 3,6$ ✓ substitution

$(0,8)^{n-1} = \frac{3}{25}$ ✓ simplification

$(n-1)\log(0,8) = \log(0,12)$

$n-1 = \log_{0,8}0,12$ ✓ logs

$n-1 = 9,5017\ldots$ ✓ simplification

$n = 10,5017$

$\therefore$ Economical for 10 passes ✓ answer **[15]**

QUESTION 4

4.1 $x = -2$ ✓ answer

$y = 1$ ✓ answer (2)

4.2 y-int: $y = \frac{-3}{0+2} + 1$

$y = \frac{-3}{2} + 1$

$y = -\frac{1}{2}$ ✓ answer

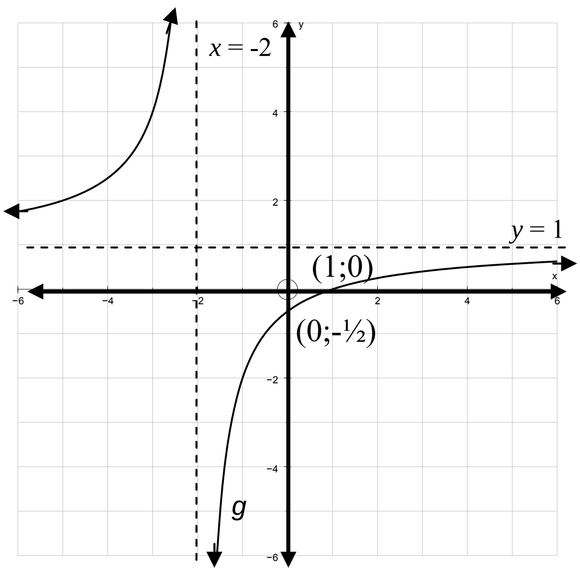
x int: $0 = \frac{-3}{x+2} + 1$

$\frac{3}{x+2} = 1$ ✓ simplification

$3 = x + 2$

$x = 1$ ✓ answer (3)

4.3



✓ asymptotes ✓ intercepts ✓shape	(3)

4.4	$x = -4$	✓ answer	(1) **[9]**

QUESTION 5

5.1	$y = \log_m x$ $2 = \log_m 4$ (4 ; 2) $m^2 = 4$ $m = 2$	✓ substitution ✓ answer	(2)
5.2	Domain: $x > 0$	✓ answer	(1)
5.3	$y = \log_2 x$ $g^{-1}: \quad x = \log_2 y$ $y = 2^x$	✓ interchange x and y ✓ answer	(2)
	OR: $x = 2^y$ $g^{-1}: \quad y = 2^x$	✓ exponential form ✓ interchange x and y	
5.4	$h: \quad y = 2^{x+2}$	✓ answer	(1)

5.5 E(-1 ; 0) by symmetry

$y = a(x + 1)(x - 5)$ ✓ point E

$= a(x^2 - 4x - 5)$ ✓ roots

$2 = a(16 - 16 - 5)$ (4 ; 2)

$a = \frac{-2}{5}$

$y = \frac{-2}{5}(x^2 - 4x - 5)$ ✓ a value

$\therefore\ y = \frac{-2}{5}x^2 + \frac{8}{5}x + 2$

$a = \frac{-2}{5}$; $b = \frac{8}{5}$ and $c = 2$ ✓ simplification

(4)

OR

$y = a(x-2)^2 + q$

(5;0): $0 = a(5 - 2)^2 + q$

$0 = 9a + q$ ………………(1) ✓ equation 1

(4;2): $2 = a(4 - 2)^2 + q$

$2 = 4a + q$……………….(2) ✓ equation 2

(1) − (2) $-2 = 5a$

$a = \frac{-2}{5}$ ✓ a value

$q = \frac{18}{5}$

$y = \frac{-2}{5}(x - 2)^2 + \frac{18}{5}$

$\therefore\ y = \frac{-2}{5}x^2 + \frac{8}{5}x + 2$ ✓ simplification

$a = \frac{-2}{5}$; $b = \frac{8}{5}$ and $c = 2$

5.6 OPTION 1:

$y = \frac{-2}{5}(x^2 - 4x - 5)$

$= \frac{-2}{5}(x^2 - 4x + 4 - 4 - 5)$ ✓ completing the square

$= \frac{-2}{5}[(x - 2)^2 - 9]$ ✓ perfect square

$= \frac{-2}{5}(x - 2)^2 + \frac{18}{5}$ ✓ answer (3)

OPTION 2:

$a = \frac{-2}{5}$ and $p = 2$

$y = \frac{-2}{5}(x-2)^2 + q$ ✓ form

$0 = \frac{-2}{5}(5-2)^2 + q$ (5 ; 0) ✓ substitution

$q = \frac{2}{5}(3)^2$

$q = \frac{18}{5}$

$\therefore\ y = \frac{-2}{5}(x-2)^2 + \frac{18}{5}$ ✓ answer

5.7	Maximum value of $f(x) = 3{,}6$ If graph is moved down 4 units it will no longer cut the x-axis.	✓ maximum ✓ 4 down ✓ answer	(3)
	OR		
	$f(x) - 4 = 0$ $f(x) = 4$ If you draw the graph of $y = 4$, it will not intersect *f*.	✓ $f(x) = 4$ ✓✓ explanation	
5.8	$1 < x < 5$ or $x \in (1\ ;\ 5)$	✓✓ answer	(2) **[18]**

QUESTION 6

6.1	$y = x^2$ $x = y^2$ $y = -\sqrt{x}$	✓interchanging *x* and *y* ✓answer	(2)
6.2	$x \geq 0$	✓answer	(1)
6.3	$y \leq 0$	✓answer	(1)
6.4	the inverse is a one-to-one mapping OR every element of the domain is associated with only one of the range.	✓✓answer	(2)

[6]

QUESTION 7

7.1 7.1.1

$A = P(1+i)^n$ ✓ formula

$15000 = 10000\ (1.01)^n$ ✓ substitution

$1.5 = (1.01)^n$

$\log 1.5 = n \log 1.01$ ✓ log form

$n = 40{,}75$ months ✓ answer (4)

7.1.2

$(1+i) = (1+\frac{0{,}12}{12})^{12}$ ✓ substitution

$i = 1{,}1268 - 1$ ✓ simplification

$i = 0{,}1268 \qquad r = 12{,}68\%$ ✓ answer (3)

7.2 7.2.1

$$P = \frac{x[1-(1+i)^{-n}]}{i}$$ ✓ formula

$$= \frac{500[1-(1+\frac{0{,}08}{12})^{-36}}{\frac{0{,}08}{12}}$$ ✓ substitution

$= R15\ 955{,}90$ ✓ answer (3)

7.2.2

$$\text{Balance} = \frac{500[1-(1+\frac{0{,}08}{12})^{-30}]}{\frac{0{,}08}{12}}$$ ✓ i value ✓ n value

$= R13\ 554{,}42$ ✓ answer (3)

7.2.3

$$13554{,}42 = \frac{x[1-(1+\frac{0{,}12}{12})^{-30}]}{\frac{0{,}12}{12}}$$ ✓ p value ✓ n value

$135{,}5442 = x(0{,}258077\ldots.)$

$x = R525{,}21$ ✓ answer (3)

[16]

QUESTION 8

8.1	$5h$ approaches 10 OR $5h \to 10$	✓ answer ($5h = 10$ NOT acceptable)	(1)
8.2	$f'(x) = \lim_{h\to 0} \frac{f(x+h) - f(x)}{h}$	✓ formula	
	$= \lim_{h\to 0} \frac{\frac{5}{x+h} - \frac{5}{x}}{h}$	✓ substitution	
	$= \lim_{h\to 0} \frac{5x - 5(x+h)}{x(x+h)} \times \frac{1}{h}$	✓ simplification	
	$= \lim_{h\to 0} \frac{5x - 5x - 5h}{x(x+h)} \times \frac{1}{h}$		
	$= \lim_{h\to 0} \frac{-5}{x(x+h)}$	✓ simplification	
	$= \frac{-5}{x^2}$	✓ answer (answer only – no marks)	(5)
8.3	$D_x[8x^4 - 6\sqrt{x} + \frac{5}{x}]$		
	$D_x[8x^4 - 6x^{\frac{1}{2}} + 5x^{-1}]$	✓ power form	
	$= 32x^3 - 3x^{-\frac{1}{2}} - 5x^{-2}$	✓✓✓ derivatives	(4)
			[10]

QUESTION 9

9.1	$x^3 - 3x^2 - 9x - 5 = 0$		
	$(x + 1)(x^2 - 4x - 5) = 0$	✓quadratic factor	
	$(x + 1)(x + 1)(x - 5) = 0$	✓ factors	
	$x = -1$ or $x = 5$	✓ x values	
	D (5 ; 0)		
	$f'(x) = 3x^2 - 6x - 9 = 0$	✓ derivative	
	$f'(5) = 3(5)^2 - 6(5) - 9$	✓ subst $x = 5$	
	$\therefore$ m = 36	✓ answer	(6)
9.2	$f'(x) = 3x^2 - 6x - 9 = 0$	✓ derivative = 0	
	$x^2 - 2x - 3 = 0$		
	$(x - 3)(x + 1) = 0$	✓ factors	
	$x = 3$ or $x = -1$	✓ x values	
	$y = (3)^3 - 3(3)^2 - 9(3) - 5$	✓ substitution	
	$= -32$		
	$\therefore y = -32$	✓ answer	(5)

9.3	$-1 < x < 3$ or $x \in (-1\,;\,3)$	✓✓ answer	(2)
9.4	B(0 ; -5) and D(5 ; 0)	✓ coordinates of B.	
	Ave grad $= \dfrac{-5-0}{0-5}$	✓ substitution	
	$= 1$	✓ answer	(3)
9.5	$f''(x) = 6x - 6$	✓ $f''(x)$	
	$6x - 6 = 0$	✓ $f''(x) = 0$	
	$x = 1$		
	$\therefore\ x < 1$	✓ answer	(3)
			[19]

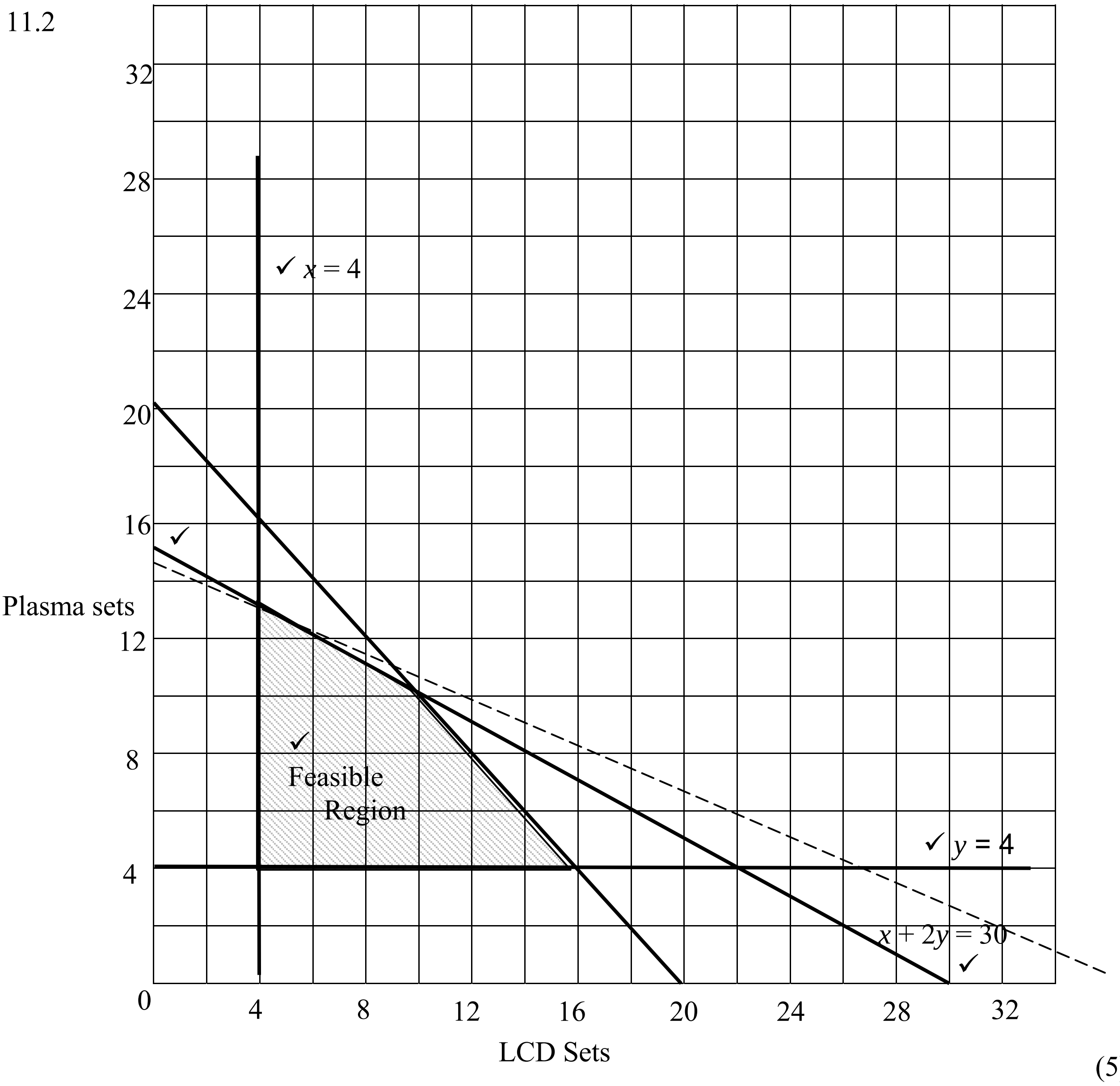
QUESTION 10

10.1	SR = $x + 30$	✓ answer	
	EH $= \dfrac{5400}{x}$	✓ $\dfrac{5400}{x}$	
	PS $= \dfrac{5400}{x} + 20$	✓ answer	
	$AreaPQRS = (x+30)(\dfrac{5400}{x} + 20)$	✓ area	
	$Area = 5400 + 20x + \dfrac{162000}{x} + 600$		(4)
	Area PQRS = $6000 + 20x + 162000\,x^{-1}$		
10.2	$\dfrac{dA}{dx} = 20 - 162000x^{-2}$	✓ derivative	
	$20 - 162000\,x^{-2} = 0$	✓ derivative = 0	
	$\dfrac{162000}{x^2} = 20$		
	$162000 = 20\,x^2$	✓ simplification	
	$8100 = x^2$		
	$90 = x$	✓ x value	
	$\therefore$ HG = 90 mm		
	so SR = 120 mm	✓ answer	(5)
			[9]

QUESTION 11

11.1 $x + 2y \leq 30$ OR $3000x + 6000y \leq 90000$ ✓ inequality
$y \geq 4$ ✓ inequality (2)

11.2



(5)

11.3 11.3.1 P = 400x + 1000y ✓ answer (1)

11.3.2 $y = -\frac{2}{5}x + \frac{P}{1000}$ ✓ gradient of search line
✓ search line on diagram (2)

11.3.3 Max profit at (4 ; 13) ✓✓ x and y values
P = 400(4) + 1 000(13)
= R14 600 ✓ answer (3)

[13]

TOTAL: 150