Province of the
EASTERN CAPE
EDUCATION

NATIONAL SENIOR CERTIFICATE

GRADE 12

SEPTEMBER 2010

MATHEMATICS – THIRD PAPER

MARKS: 100

TIME: 2 hours

This question paper consists of 9 pages, a formula sheet and two diagram sheets.

INSTRUCTIONS AND INFORMATION

Read the following instructions carefully before answering the questions.

1. This question paper consists of 10 questions. Answer ALL the questions.

2. Clearly show ALL calculations, diagrams, graphs, et cetera, which you have used in determining your answers.

3 An approved scientific calculator (non-programmable and non-graphical) may be used, unless stated otherwise.

4. If necessary, answers should be rounded off to TWO decimal places, unless stated otherwise.

5. Number the answers correctly according to the numbering system used in this question paper.

6. Diagrams are NOT necessarily drawn to scale.

7. It is in your own interest to write legibly and to present work neatly.

8. TWO diagram sheets for answering QUESTION 6.3 and QUESTIONS 7 to 10 are attached at the end of this question paper. Write your NAME/EXAMINATION NUMBER in the spaces provided and hand them in together with your ANSWER BOOK.

QUESTION 1

Consider the following sequence: 2 ; x ; y ; 6 ; ... with recursive formula $T_{n+1} = T_n + T_{n-1}, n \geq 2\ and\ n \in N$.

1.1 Determine the values of x and y. (4)

1.2 Calculate the sum of the first five terms. (1)

[5]

QUESTION 2

A survey of 300 learners at a local high school showed that 55% of the learners indicated that soccer was their favourite sport. The school has a total enrolment of 1500 learners.

2.1 What percentage is the sample size of the school population? (1)

2.2 How many learners in the school would you expect to say soccer is their favourite sport if ALL the learners in the school were asked to respond? (1)

2.3 A sample is useful only if it is unbiased and representative of the entire group. Name TWO factors that should have been considered to ensure that a good sample was used. (2)

2.4 There are 80 high schools, with an average learner enrolment of 800 learners per school in and around Port Elizabeth. Do you think that the survey conducted is representative of all the schools in Port Elizabeth? Motivate your answer. (2)

[6]

QUESTION 3

The mean weight (mass) of 700 players that took part in a soccer tournament was 75 kg and the standard deviation was 7,5 kg. Assume that the weights (masses) are normally distributed:

3.1 Determine how many players weighed between 67,5 kg and 82,5 kg. (2)

3.2 What percentage of the players weighed between 60 kg and 90 kg? (2)

3.3 It was observed that players with a weight (mass) of less than 60 kg were more likely to get injured. How many of the players fell in this category? (2)

[6]

QUESTION 4

4.1 A group of 200 Soccer World Cup fans were interviewed about, Algeria, Bafana-Bafana and Cameroon. They were asked whether these teams would progress to the second round of the tournament. An analysis of the data indicated the following:

- 28 indicated that Algeria and Bafana-Bafana will progress.
- 42 indicated that Bafana-Bafana will progress, but not Algeria or Cameroon.
- 64 indicated that only Cameroon will progress, but not Algeria or Bafana-Bafana.
- 14 indicated that only Algeria and Cameroon will progress.
- 98 indicated that Algeria or Bafana-Bafana will progress, but not Cameroon.
- 122 indicated that Bafana-Bafana or Cameroon will progress, but not Algeria.
- 5 indicated that not one of the teams will progress.
- Some indicated that all three teams will progress.

4.1.1 Draw a Venn-diagram to represent the information. (6)

4.1.2 Determine how many fans said that all three teams will progress to the second round. (2)

4.1.3 How many fans felt that Bafana-Bafana will progress to the second round, irrespective of what happens to Algeria or Cameroon? (2)

4.1.4 What is the probability, based on the fans predictions, that only one team will progress to the second round? (2)

4.2 The probability that goals are scored by strikers is $\frac{2}{3}$, and the probability that goals are scored by defenders is $\frac{3}{5}$. What is the probability that ...

4.2.1 both strikers and defenders will score? (1)

4.2.2 only strikers will score? (1)

4.2.3 at least one of the two (striker/defender) will score? (3)

[17]

QUESTION 5

Your school has started the process of RCL (Representative Council of Learners) elections. 50 learners in total were shortlisted for the voting list. Of the 50 learners, 20 learners have previously served on the RCL, 15 learners are in the FET (Grades 10 – 12) phase and 10 learners previously served on the RCL and are in the FET phase.

5.1 Are previous experience on the RCL and being in the FET phase independent events? (Show ALL calculations to support your answer.) (4)

5.2 The executive committee of the RCL consist of eight members. Each of the eight members can either be a boy or a girl. How many different combinations of boys and girls are possible? (2)

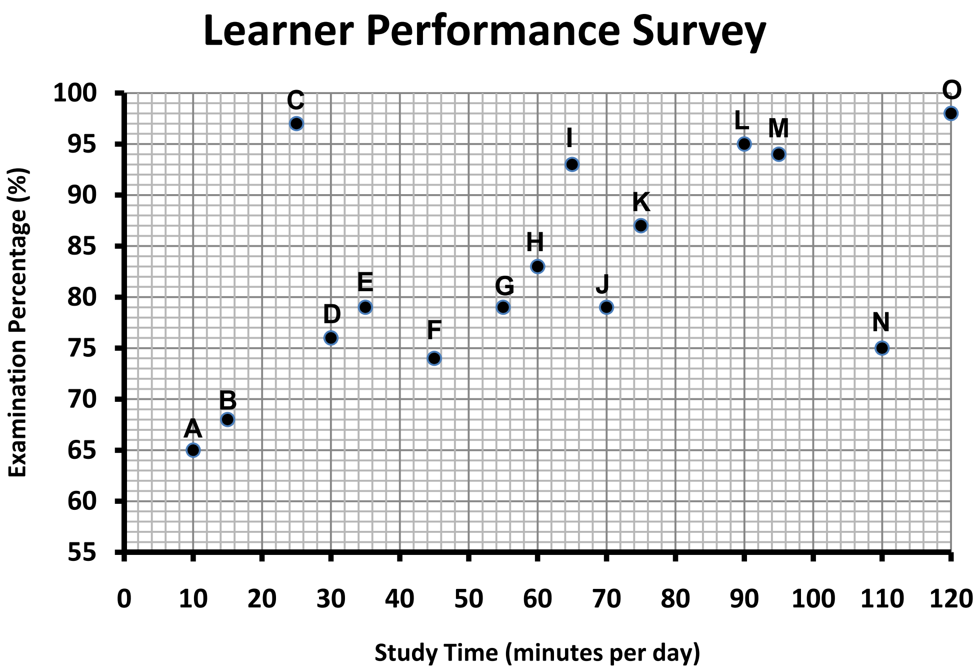
5.3 Three of the eight executive committee members must represent the learners on the SGB (School Governing Body). In how many different ways can these three representatives be selected? (3)

5.4 The three positions of Chairperson, Secretary and Treasurer must be filled from the eight executive committee members. How many different possibilities are there? (2)

[11]

QUESTION 6

Learners in a Mathematics class were given an assignment to conduct a survey and interpreting the results. John decided to conduct a survey to determine whether there is a relationship between amount of time spent studying for an examination and the percentage obtained for the examination. Data was obtained from 15 learners (A, B, C, …, O). He asked the learners how many minutes they spent studying for the examinations per day and the percentage they obtained for the examination. In order to determine if there is a relationship, the study time needs to be compared with the percentage obtained. The data is displayed in the scatter plot diagram below:



6.1 Which two points would you describe as outliers? (2)

6.2 Which learner devoted a lot of time to studying, but obtained a fairly low examination percentage? (Write down the letter of the learner, e.g. Learner Z) (1)

6.3 Complete the table provided on the diagram sheet and determine the correlation coefficient. (2)

6.4 What kind of relationship is suggested by the correlation coefficient? (1)

6.5 Determine the equation of the least squares line. (4)

6.6 Determine the correlation coefficient by ignoring the outliers. (2)

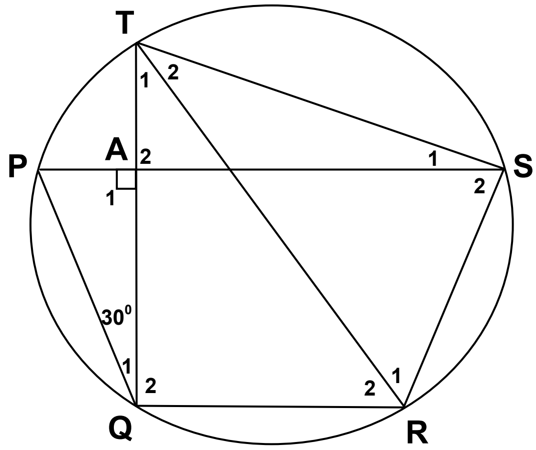
6.7 What kind of relationship is suggested if we ignore the outliers? (2)

6.8 Estimate the examination percentage that might be obtained if a learner studies for 50 minutes per day. (1)

[15]

QUESTION 7

7.1 The accompanying figure shows cyclic quadrilateral PQRS. QA $\perp$ PS and QA produced meets the circle in T. $Q\hat{P}S = P\hat{S}R$ and $P\hat{Q}A = 30^0$.

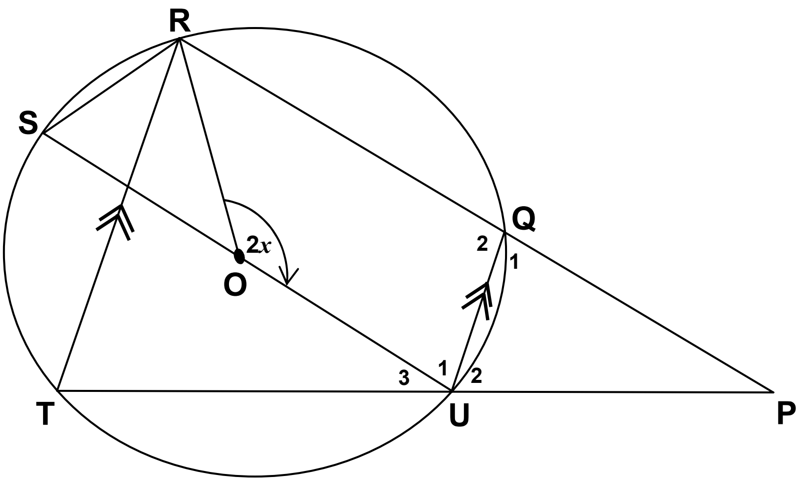


7.1.1 Give, with reasons, THREE angles each equal to 60^0. (3)

7.1.2 Calculate $Q\hat{R}S$ and show that PS || QR. (3)

7.1.3 Show that TR is a diameter of the circle. (3)

7.2 In the figure below, O is the centre of the circle, RT || QU and $R\hat{O}U = 2x$.



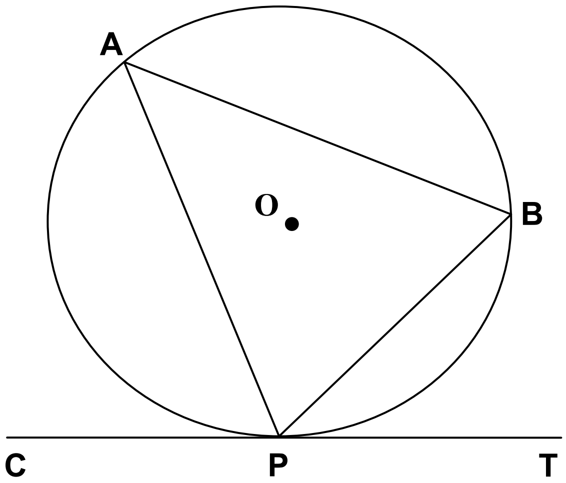
7.2.1 Write down, stating a reason, the size of $\hat{T}$ in terms of x. (1)

7.2.2 Prove that ROUP is a cyclic quadrilateral. (4)

[14]

QUESTION 8

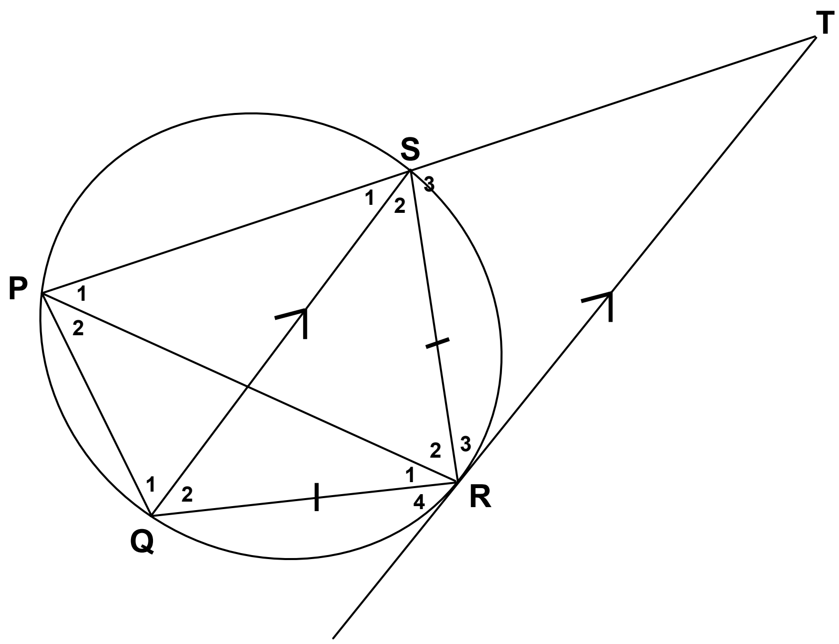
In the diagram below CPT is a tangent to circle ABP, with centre O, at P.
Prove the THEOREM which states that the angle between the tangent and the chord is equal to the angle at the circumference in the alternate circle segment.
(i.e. $P\hat{A}B = B\hat{P}T$)



[6]

QUESTION 9

The accompanying figure shows cyclic quadrilateral PQRS with QR = RS.
TR is a tangent to the circle at R, TR || QS, TR meets PS produced in T.



Prove, giving reasons, that:

9.1 $\hat{R}_1 = \hat{T}$. (2)

9.2 PR bisects $Q\hat{P}S$. (3)

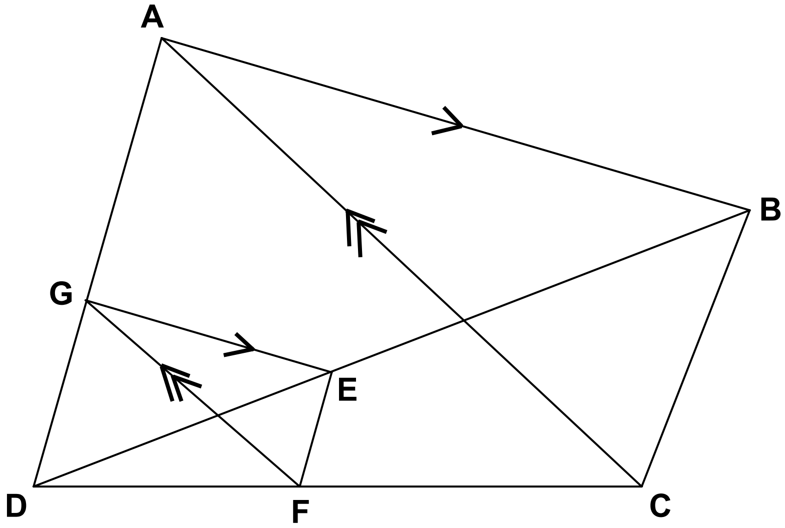
9.3 $\Delta\, RST \;|||\; \Delta\, PQR$. (3)

9.4 $QR^2 = PQ \times ST$. (2)

[10]

QUESTION 10

In quadrilateral ABCD, G and F are points on AD and DC respectively such that GE || AB with E on BD and GF || AC.



10.1 Write down, with reasons, TWO ratios each equal to $\frac{AG}{GD}$ (2)

10.2 Hence, or otherwise prove that EF || BC. (2)

10.3 If $\frac{DE}{BE} = \frac{3}{5}$ and BC = 16 units, calculate the length of EF. (2)

10.4 If it is further given that GE = 9 units, determine the following ratio:

$$\frac{Area\ of\ \Delta GEF}{Area\ of\ \Delta ABC}$$ (4)

[10]

TOTAL: 100

INFORMATION SHEET: MATHEMATICS

$$x = \frac{-b \pm \sqrt{b^2 - 4ac}}{2a}$$

$A = P(1 + ni)$ $\quad$ $A = P(1 - ni)$ $\quad$ $A = P(1 - i)^n$ $\quad$ $A = P(1 + i)^n$

$\sum_{i=1}^{n} 1 = n$ $\quad$ $\sum_{i=1}^{n} i = \frac{n(n+1)}{2}$ $\quad$ $T_n = a + (n-1)d$ $\quad$ $S_n = \frac{n}{2}(2a + (n-1)d)$

$T_n = ar^{n-1}$ $\quad$ $S_n = \frac{a(r^n - 1)}{r - 1}$; $\quad r \neq 1$ $\quad$ $S_\infty = \frac{a}{1-r}$; $-1 < r < 1$

$F = \frac{x[(1+i)^n - 1]}{i}$ $\quad$ $P = \frac{x[1 - (1+i)^{-n}]}{i}$

$$f'(x) = \lim_{h \to 0} \frac{f(x+h) - f(x)}{h}$$

$d = \sqrt{(x_2 - x_1)^2 + (y_2 - y_1)^2}$ $\quad$ $\mathrm{M}\left(\frac{x_1 + x_2}{2} ; \frac{y_1 + y_2}{2}\right)$

$y = mx + c$ $\quad$ $y - y_1 = m(x - x_1)$ $\quad$ $m = \frac{y_2 - y_1}{x_2 - x_1}$ $\quad$ $m = \tan\theta$

$$(x - a)^2 + (y - b)^2 = r^2$$

In ΔABC:

$\frac{a}{\sin A} = \frac{b}{\sin B} = \frac{c}{\sin C}$ $\quad$ $a^2 = b^2 + c^2 - 2bc.\cos A$ $\quad$ $area\, \Delta ABC = \frac{1}{2} ab.\sin C$

$\sin(\alpha + \beta) = \sin\alpha.\cos\beta + \cos\alpha.\sin\beta$ $\quad$ $\sin(\alpha - \beta) = \sin\alpha.\cos\beta - \cos\alpha.\sin\beta$

$\cos(\alpha + \beta) = \cos\alpha.\cos\beta - \sin\alpha.\sin\beta$ $\quad$ $\cos(\alpha - \beta) = \cos\alpha.\cos\beta + \sin\alpha.\sin\beta$

$\cos 2\alpha = \begin{cases} \cos^2\alpha - \sin^2\alpha \\ 1 - 2\sin^2\alpha \\ 2\cos^2\alpha - 1 \end{cases}$ $\quad$ $\sin 2\alpha = 2\sin\alpha.\cos\alpha$

$(x; y) \to (x\cos\theta + y\sin\theta\,; y\cos\theta - x\sin\theta)$ $\quad$ $(x; y) \to (x\cos\theta - y\sin\theta\,; y\cos\theta + x\sin\theta)$

$\bar{x} = \frac{\sum fx}{n}$ $\quad$ $\partial^2 = \frac{\sum_{i=1}^{n}(x_i - \bar{x})^2}{n}$

$P(A) = \frac{n(A)}{n(S)}$ $\quad$ $P\,(A \text{ or } B) = P\,(A) + P\,(B) - P\,(A \text{ and } B)$

$\hat{y} = a + bx$ $\quad$ $b = \frac{\sum(x - \bar{x})(y - \bar{y})}{\sum(x - \bar{x})^2}$

NAME/EXAMINATION NUMBER:

DIAGRAM SHEET 1

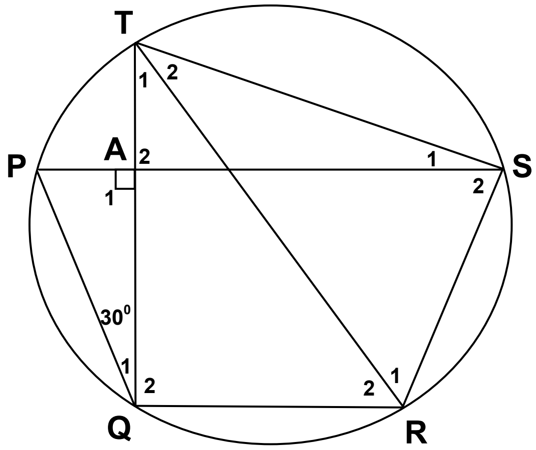
QUESTION 6

6.3

	A	B	C	D	E	F	G	H	I	J	K	L	M	N	O
Time															
Percentage															

QUESTION 7

7.1



7.2

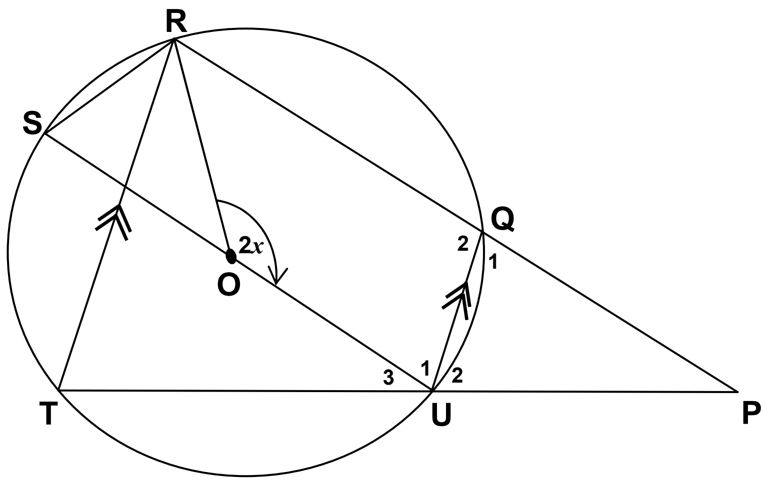


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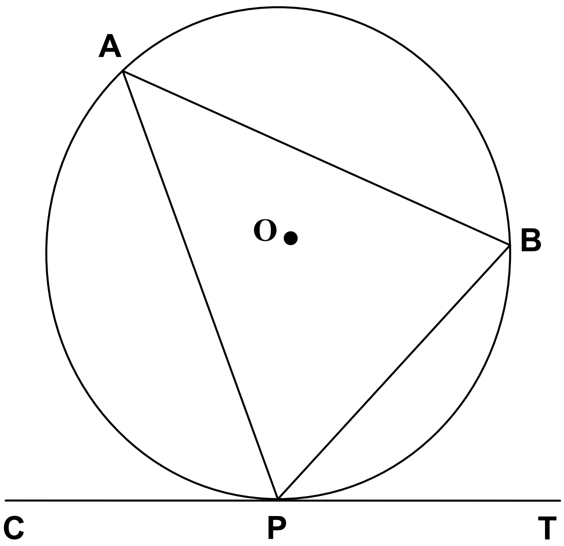
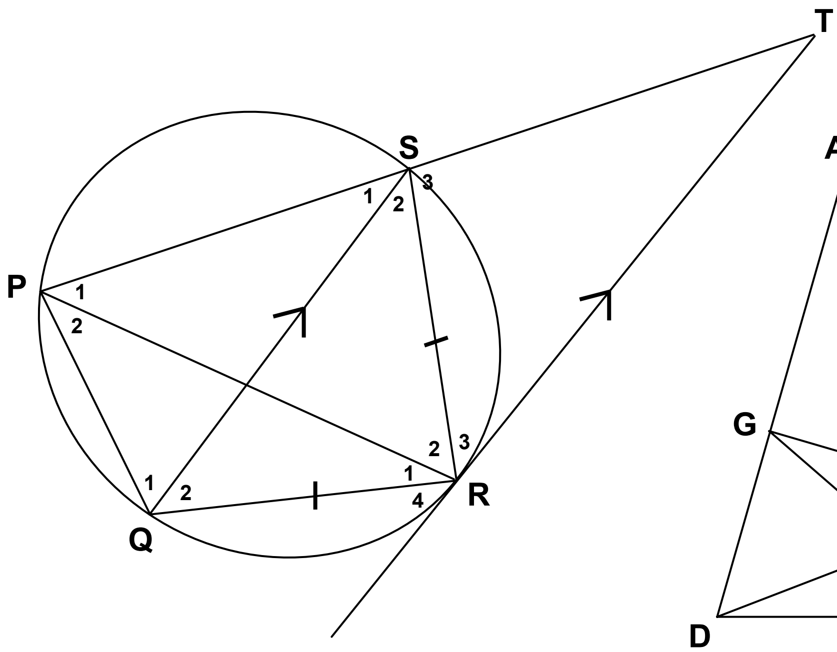
DIAGRAM SHEET 2

QUESTION 8



QUESTION 9



QUESTION 10