



Province of the  
**EASTERN CAPE**  
EDUCATION

**NATIONAL  
SENIOR CERTIFICATE**

**GRADE 12**

**SEPTEMBER 2010**

**MATHEMATICS – SECOND PAPER  
WISKUNDE – TWEEDE VRAESTEL  
MEMORANDUM**

**MARKS/PUNTE: 150**

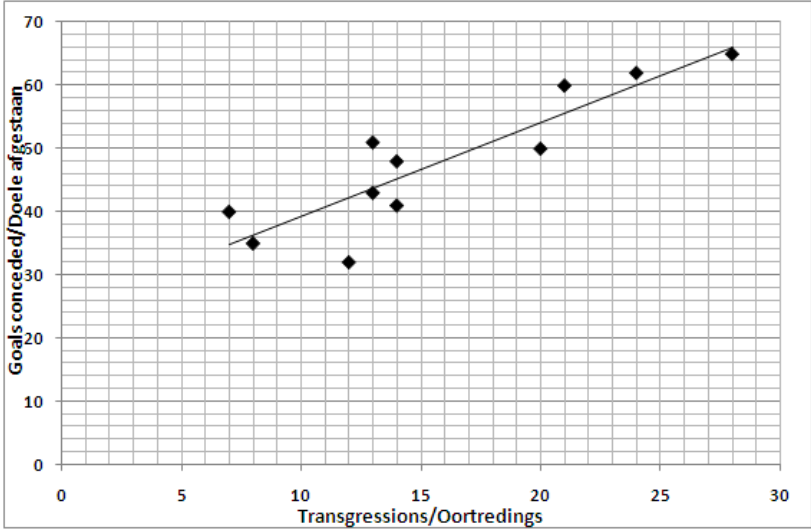
**TIME/TYD: 3 hours/uur**

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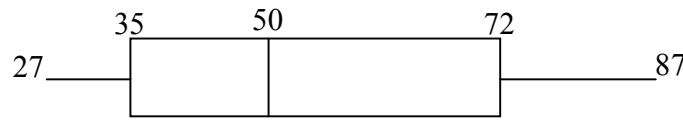
This memorandum consists of 10 pages.  
*Hierdie memorandum bestaan uit 10 bladsye.*

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**QUESTION/VRAAG 1**

1.1		✓ labels ✓✓✓ plotting of at least 6 points correctly  (4)
1.2	See graph/Sien grafiek	✓✓ line (2)
1.3	16	✓✓ answer (2)
		<b>[8]</b>

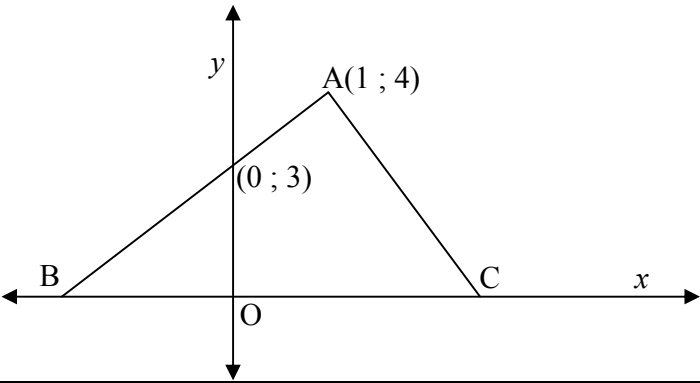
**QUESTION/VRAAG 2**

2.1	$87 - 27 = 60$ $\therefore$ Highest mark = 87 Answer only: FULL MARKS	✓ range ✓ answer  (2)
2.2	27 35 50 72 87 (Numbers must be specified if not in order)	✓ min. & max. ✓ median ✓ 1st & 2 <sup>nd</sup> quartiles  (3)
2.3		✓ box with med. ✓ min/max. ✓ quartiles  (3)
2.4	Positively skewed or skewed to the right/Positief skeef/na regs	✓✓ answer (2)
2.5	9 learners/leerders	✓✓ answer (2)
		<b>[12]</b>

## QUESTION/VRAAG 3

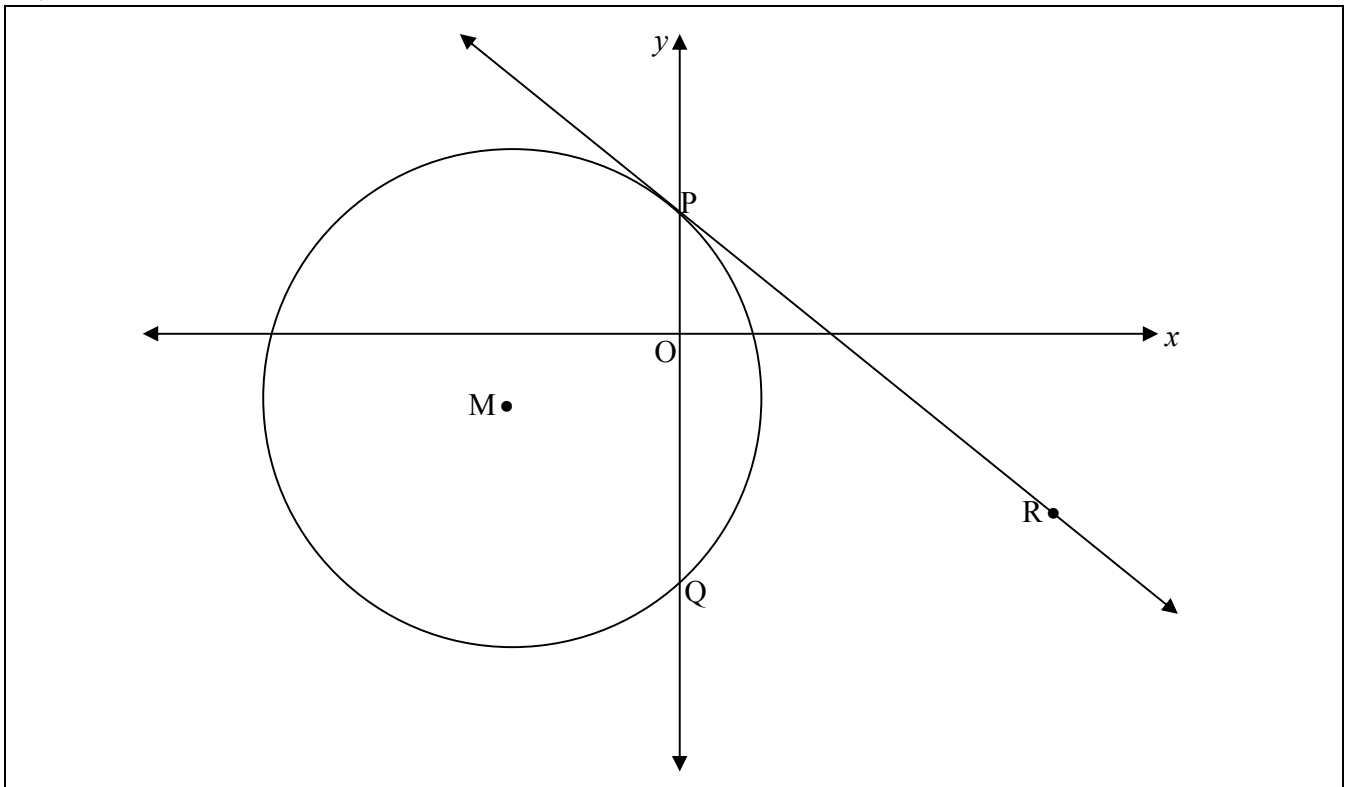
3.1	$\bar{x} = \frac{88}{8} = 11$	$\sqrt{\frac{88}{8}}$ $\sqrt{11}$ (2)
3.2	Standard deviation = 2,35	$\sqrt{\sqrt{\sqrt{\text{answer}}}}$ (3)
3.3	$11 + 2,35 = 13,35$ $11 - 2,35 = 8,65 \therefore$ values from 9 to 13 $\therefore$ 5 players scored within one stand. deviation (12, 10, 9, 9, 11)	$\sqrt{\text{interval}}$ $\sqrt{\text{answer}}$ (2) <b>[7]</b>

## QUESTION/VRAAG 4

		
4.1	$AB_m = \frac{y_2 - y_1}{x_2 - x_1} = \frac{4 - 3}{1 - 0} = 1$	$\sqrt{\text{subst. in gra. Form.}}$ $\sqrt{\text{grad. AB}}$ (2)
4.2	$AC_m = -1$ (perp. gradients/loodregte grad.) $\therefore$ Equ. of AC: $y = -1x + c$ $\therefore 4 = -1 \cdot 1 + c$ $\therefore c = 5 \therefore y = -x + 5$ <b>OR/OF</b> $AC_m = -1$ (perp. gradients/loodregte grad.) $y - y_1 = m(x - x_1)$ $y - 4 = -1(x - 1)$ $y - 4 = -x + 1$ $\therefore y = -x + 5$	$\sqrt{\text{grad. AC}}$ $\sqrt{\text{equ. str. Line}}$ $\sqrt{\text{subst. (1 ; 4)}}$ $\sqrt{\text{value of c}}$  $\sqrt{\text{grad. AC}}$ $\sqrt{\text{equ. str. Line}}$ $\sqrt{\text{subst. (1 ; 4)}}$ $\sqrt{\text{value of c}}$  (4)

4.3	$\tan B = 1 \quad \therefore \hat{B} = 45^\circ$ $\therefore \hat{ACB} = 45^\circ$ [triangle/driehoek] $\therefore AB = AC$ [2 $\angle$ 's =]	$\sqrt{\tan B = 1}$ $\sqrt{\hat{B} = 45^\circ}$ $\sqrt{\hat{ACB} = 45^\circ}$ (3)
4.4	$B(x; 0) \quad \therefore \frac{4-0}{1-x} = 1$ $\therefore \frac{4}{1-x} = 1 \quad \therefore 4 = 1 - x$ $\therefore x = -3$	$\sqrt{AB_m = 1}$ $\sqrt{4 = 1 - x}$ or substitution (2)
4.5	C(5; 0)	$\sqrt{x\text{-coordinate}}$ $\sqrt{y\text{-coordinate}}$ (2)
4.6	BC = 8 units/eenhede, perp. height/loodregte hoogte = 4, $\therefore \text{Area} = \frac{1}{2}bh = \frac{1}{2}(8).(4) = 16$ square units/vk. eenh.  <b>OR/OF</b> $\text{Area} = \frac{1}{2}bh = \frac{1}{2}AB.AC = \frac{1}{2}\sqrt{32}.\sqrt{32}$ $\frac{1}{2}.32 = 16$ square units/vk. eenh.	$\sqrt{\text{value of BC}}$ $\sqrt{\text{perp. height}}$ $\sqrt{\text{subst. in area form.}}$ $\sqrt{\text{answer}}$ (4)  $\sqrt{\text{length of AB}}$ $\sqrt{\text{length of AC}}$ $\sqrt{\text{subst. in area form.}}$ $\sqrt{\text{answer}}$  <b>[17]</b>

## QUESTION/VRAAG 5



5.1.1	$(x + 3)^2 + (y + 2)^2 = 9 + 12 + 4$ $\therefore (x + 3)^2 + (y + 2)^2 = 25$ $\therefore M(-3; -2)$	$\checkmark$ compl. Both squares $\checkmark 25$ $\checkmark\checkmark$ each coordinate (4)
5.1.2	$x = 0$ at P $\therefore (3)^2 + (y + 2)^2 = 25$ $(y + 2)^2 = 16$ $y + 2 = \pm 4$ $\therefore y = -6$ or $2 \therefore P(0; 2)$	$\checkmark$ subst. $x = 0$ $\checkmark y + 2 = \pm 4$ $\checkmark$ values of $y$ $\checkmark$ coord. of P (4)
5.1.3	Equation of tangent at P: $MP_m = \frac{2 - (-2)}{0 - (-3)} = \frac{4}{3}$ $\therefore PR_m = -\frac{3}{4}$ $y = -\frac{3}{4}x + 2$	$\checkmark$ grad. of MP $\checkmark$ grad. of PR $\checkmark$ equation of PR (3)
5.1.4	$PR_m = -\frac{3}{4}$ $\therefore$ Inclination of PR is $143,13^\circ$ $\therefore \hat{RPO} = 53,13^\circ$	$\checkmark PR_m = -\frac{3}{4}$ $\checkmark\checkmark 143,13^\circ$ $\checkmark 53,13^\circ$ (4)

5.2	$MP^2 + PR^2 = MR^2$ , $MP = 5$ units (radius of circle), $R(x ; y)$ , $P(0;2)$ $\therefore 5^2 + x^2 + (2 - y)^2 = (\sqrt{125})^2$ $25 + x^2 + (2 - (-\frac{3}{4}x + 2))^2 = 125$ $x^2 + \frac{9}{16}x^2 = 100$ $\frac{25}{16}x^2 = 100$ $x^2 = 64 \therefore x = 8 \text{ or } -8$ $y = -4 \text{ (} x > 0 \text{)}$ $\therefore R(8 ; -4)$	✓ Pyth. equation or dist. form. ✓ radius = 5 ✓ subst. in dist. form. ✓ subst. $y$ ✓ simpl. Bracket ✓ simplific. ✓ value of $x$ ✓ value of $y$
		(8) <b>[23]</b>

**QUESTION/VRAAG 6**

6.1.1	$P'(2 ; 2)$	✓✓ each coordinate (2)
6.1.2	A rotation of $180^\circ$	✓ rotation ✓ $180^\circ$ (2)
6.1.3	$(x ; y) \longrightarrow (-x ; -y)$	✓✓ each coordinate (2)
6.2.1	$A'(-5 ; 2)$ , $B'(-2 ; 2)$ and $C'(-2 ; -2)$	✓✓✓ each point's coordinates (3)
6.2.2	Length of $AB = 3$ units $\therefore$ length of $A'B' = 9$ units $\therefore$ perimeter of $\triangle ABC = 3 + 4 + 5 = 12$ units perimeter of $\triangle A'B'C' = 9 + 12 + 15 = 36$ units $\therefore$ Ratio: $12 : 36$ or $1 : 3$	✓ perim. of $\triangle ABC$ ✓ perim. of $\triangle A'B'C'$ ✓ ratio (3)
6.2.3	$9x^2$	✓✓ answer (2) <b>[14]</b>

**QUESTION/VRAAG 7**

7.1	$T(-3 ; 5) \longrightarrow T'(y ; -x) \longrightarrow T'(5 ; 3)$ $\therefore x = 3$ and $y = 5$	✓✓ coordinates for $T'$ ✓✓ values for $x$ and $y$ . (4)
7.2	$T''(x\cos\alpha - y\sin\alpha ; x\sin\alpha + y\cos\alpha)$ $T''(3\cos 120^\circ - 5\sin 120^\circ ; 3\sin 120^\circ + 5\cos 120^\circ)$ $T''(3(-\frac{1}{2}) - 5(\frac{\sqrt{3}}{2}) ; 3(\frac{\sqrt{3}}{2}) + 5(-\frac{1}{2}))$ $T''(\frac{-3 - 5\sqrt{3}}{2} ; \frac{3\sqrt{3} - 5}{2})$	✓✓ formula-each coordinate ✓ substitution ✓✓ $\sin 120^\circ$ & $\cos 120^\circ$ ✓ simplification (6) <b>[10]</b>

## QUESTION/VRAAG 8

8.1	$\sin A \cos B + \cos A \sin B = \sin(A + B)$ $= \sin 90^\circ$ $= 1$	$\checkmark$ identity $\checkmark$ substit./vervang $\checkmark$ answer. (3)
8.2	$\cos 2x = \cos(x + x)$ $= \cos x \cos x - \sin x \sin x$ $= \cos^2 x - \sin^2 x$ $= \cos^2 x - (1 - \cos^2 x)$ $= \cos^2 x - 1 + \cos^2 x$ $= 2\cos^2 x - 1$	$\checkmark$ expansion of $\cos 2x$ $\checkmark \sin^2 x = 1 - \cos^2 x$ (2)
8.3	$2\cos^2 22,5^\circ - 1 = \cos 2(22,5^\circ)$ $2\cos^2 22,5^\circ - 1 = \cos 45^\circ$ $2\cos^2 22,5^\circ - 1 = \frac{1}{\sqrt{2}}$ $2\cos^2 22,5^\circ = 1 + \frac{1}{\sqrt{2}}$ $2\cos^2 22,5^\circ = \frac{\sqrt{2} + 1}{\sqrt{2}}$ $\cos^2 22,5^\circ = \frac{\sqrt{2} + 1}{2\sqrt{2}}$ $\cos^2 22,5^\circ = \frac{\sqrt{2} + 1}{2\sqrt{2}} \cdot \frac{\sqrt{2}}{\sqrt{2}}$ $\cos^2 22,5^\circ = \frac{2 + \sqrt{2}}{2.2}$ $\cos 22,5^\circ = \sqrt{\frac{2 + \sqrt{2}}{4}} = \frac{\sqrt{2 + \sqrt{2}}}{2}$	$\checkmark$ substitution of $22,5^\circ$ $\checkmark$ ratio for $\cos 45^\circ$ $\checkmark$ finding $\cos^2 22,5^\circ$ $\checkmark$ rationalizing numerator (4)
8.4	$\tan^2 \alpha = \cos 2\alpha + 2\sin^2 \alpha$ $\tan^2 \alpha = 1 - 2\sin^2 \alpha + 2\sin^2 \alpha$ $\tan^2 \alpha = 1 \therefore \tan \alpha = \pm 1$ $\therefore \alpha = 45^\circ \text{ or } 225^\circ \text{ or } \alpha = 135^\circ \text{ or } 315^\circ$ $\therefore \alpha = 45^\circ + k.180^\circ, k \in \mathbb{Z}$ $\alpha = 135^\circ + k.180^\circ, k \in \mathbb{Z}$	$\checkmark \cos 2\alpha$ identity $\checkmark \tan \alpha = \pm 1$ $\checkmark$ values for 1 $\checkmark$ values for -1 $\checkmark$ gen. solution $\checkmark$ gen. solution $k \in \mathbb{Z}$ not included, 5/6 (6)

[15]

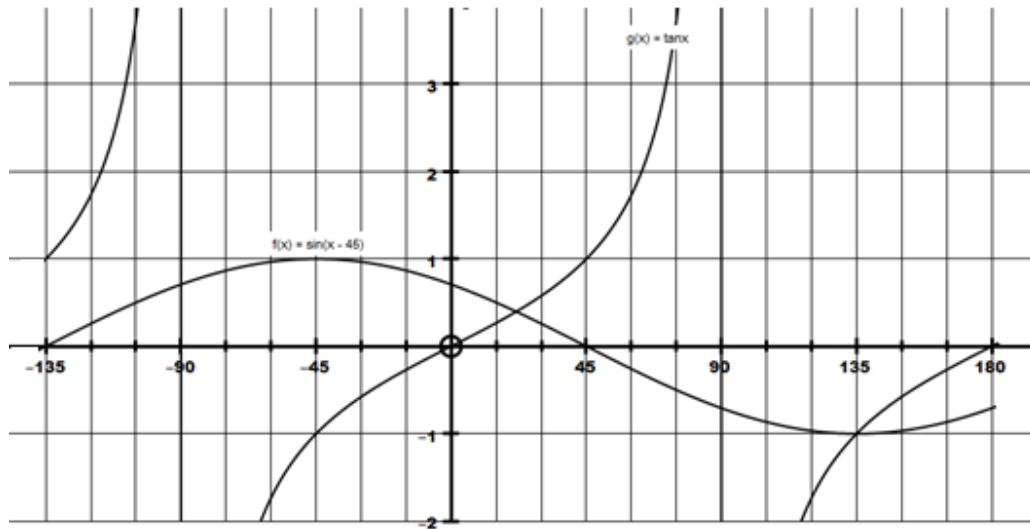
## QUESTION/VRAAG 9

9.1	$\frac{-\cos x \cdot \tan x}{\sin x} = \frac{-\cos x \cdot \frac{\sin x}{\cos x}}{\sin x} = -1$	$\sqrt{\cos(x - 540^\circ) = -\cos x}$ $\sqrt{\tan(x + 180^\circ) = \tan x}$ $\sqrt{\cos(90^\circ - x) = \sin x}$ $\sqrt{\tan x = \frac{\sin x}{\cos x}}$ $\sqrt{\text{answer}}$ (5)
9.2.1	$\frac{1 - (1 - 2\sin^2 x)}{2\sin x \cos x} = \frac{1 - 1 + 2\sin^2 x}{2\sin x \cos x} = \frac{2\sin^2 x}{2\sin x \cos x} = \frac{\sin x}{\cos x} = \tan x$	$\sqrt{\text{subst. of ident.}}$ $\sqrt{\text{subst. of ident.}}$ $\sqrt{\text{simplification.}}$ $\sqrt{\text{simplification.}}$ (4)
9.2.2	$\tan 15^\circ = \frac{1 - \cos 2 \cdot 15^\circ}{\sin 2 \cdot 15^\circ} = \frac{1 - \cos 30^\circ}{\sin 30^\circ} = \frac{1 - \frac{\sqrt{3}}{2}}{\frac{1}{2}} = 2 - \sqrt{3}$	$\sqrt{\text{subst. in ident.}}$ $\sqrt{\text{value of } \cos 30^\circ}$ $\sqrt{\text{value of } \sin 30^\circ}$ $\sqrt{\text{answer}}$ (4).
9.3	$\begin{aligned} (\sin 75^\circ + \cos 75^\circ)^2 &= \sin^2 75^\circ + 2\sin 75^\circ \cos 75^\circ + \cos^2 75^\circ \\ &= \sin^2 75^\circ + \cos^2 75^\circ + 2\sin 75^\circ \cos 75^\circ \\ &= 1 + \sin 150^\circ \\ &= 1 + \sin 30^\circ \\ &= 1 + \frac{1}{2} = \frac{3}{2} \end{aligned}$	$\sqrt{\text{removing of } ( )}$ $\sqrt{\text{use of ident.}}$ $\sqrt{\text{use of ident.}}$ $\sqrt{\text{value for } \sin 150^\circ \text{ or } \sin 30^\circ}$ $\sqrt{\text{answer}}$ (5) <b>[18]</b>



## QUESTION/VRAAG 10

10.1



**f:** ✓ turning points (both) ✓ x intercepts (all)

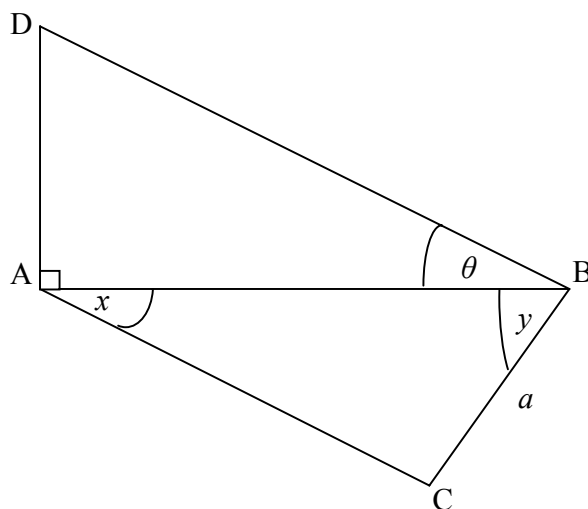
**g:** ✓ asymptotes (both) ✓  $(-45^\circ; -1)$ ;  $(45^\circ; 1)$  and  $(135^\circ; -1)$  ✓ x intercepts (all)

(5)

10.2.1	22,5° or 135° (any one of the two)	✓ answer (1)
10.2.2	$-135^\circ < x < -90^\circ$ or $0^\circ < x < 45^\circ$ or $90^\circ < x < 180^\circ$	✓ end values ✓ notation (2)
10.3	$f(x) = \sin x + 2$	✓ $\sin x$ ✓ $+ 2$ (2)
10.4	360°	✓ answer (1)

**[11]**

## QUESTION/VRAAG 11



11.1	$\frac{\sin B}{AC} = \frac{\sin A}{BC} \quad \therefore \sin y = \frac{2a \cdot \sin x}{a}$ $\frac{\sin y}{2a} = \frac{\sin x}{a} \quad \therefore \sin y = 2 \sin x$	$\sqrt{\text{subst. in sine rule}}$ $\sqrt{\sin y = \frac{2a \cdot \sin x}{a}}$ <p>(2)</p>
11.2	$\frac{AD}{AB} = \tan \theta \quad \therefore AD = AB \cdot \tan \theta$ $AB^2 = AC^2 + BC^2 - 2 \cdot AC \cdot BC \cdot \cos C$ $AB^2 = (2a)^2 + a^2 - 2 \cdot 2a \cdot a \cdot \cos[180^\circ - (x + y)]$ $AB^2 = 4a^2 + a^2 - 4a^2 \cdot [-\cos(x + y)]$ $AB^2 = 5a^2 + 4a^2 \cdot \cos(x + y)$ $\therefore AB^2 = a^2(5 + 4\cos(x + y))$ $\therefore AB = a\sqrt{5 + 4\cos(x + y)}$ $\therefore AD = \tan \theta \cdot a\sqrt{5 + 4\cos(x + y)}$ $\therefore AD = a \tan \theta \sqrt{5 + 4\cos(x + y)}$	$\sqrt{\frac{DA}{AB} = \tan \theta}$ $\sqrt{DA = AB \cdot \tan \theta}$ $\sqrt{\text{cosine rule}}$ $\sqrt{C = 180^\circ - (x + y)}$ $\sqrt{\text{subst. in cos rule}}$ $\sqrt{-\cos(x + y)}$ $\sqrt{\text{finding AB}}$ $\sqrt{\text{subst. AB in } AB \cdot \tan \theta}$ <p>(8)</p>
11.3	$\sin y = 2 \sin x \quad \therefore \sin y = 2 \sin 29^\circ$ $\therefore \sin y = 0,96961924 \dots$ $\therefore y = 75,84^\circ$ $\therefore AD = 27,3 \cdot \tan 36,4^\circ \cdot \sqrt{5 + 4 \cos(29^\circ + 75,84^\circ)}$ $\therefore AD = 20,12729587 \dots \cdot \sqrt{5 + 4 \cos(104,84^\circ)} = 40,13 \text{ m}$	$\sqrt{\sin y = 2 \sin 29^\circ}$ $\sqrt{\text{value of } y}$ $\sqrt{\text{subst. in AD}}$ $\sqrt{\sqrt{\text{answer for AD}}}$ <p>(5)</p> <p>[15]</p>

TOTAL/TOTAAL: 150