Province of the
EASTERN CAPE
EDUCATION

MATHEMATICS P1

MEMORANDUM

COMMON TEST

JUNE 2014

NATIONAL SENIOR CERTIFICATE

GRADE 12

MARKS: 125

TIME : 2½ hours

This memorandum consists of 8 pages.

QUESTION 1

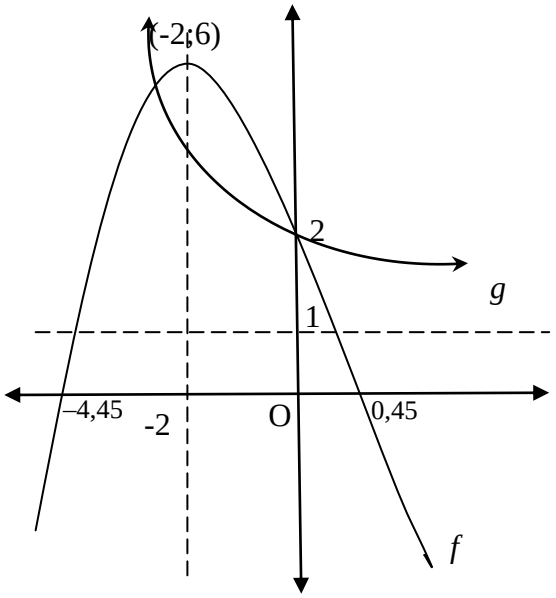
1.1.1	$(x+6)(x-1)=0$ $x=-6 \quad or \quad x=1$	✓ factors ✓✓ Answers	(3)
1.1.2	$x=\frac{-b\pm\sqrt{b^2-4ac}}{2a}$ $=\frac{-(4)\pm\sqrt{(4)^2-4(-3)(2)}}{2(-3)}$ $=-0,39 \quad or \quad 1,72$	✓ substitution into quadratic formula ✓✓ answers	(3)
1.1.3	$x^2\geq 0 \quad for\ all\ x\in R$ $\therefore x+2<0$ $\Rightarrow x<-2$	✓ $x^2\geq 0 \quad for\ all\ x\in R$ ✓ $x+2<0$ ✓ answer	(3)
1.1.4	$2^x(2^3-3.2^{-1})=104$ $2^x\left(8-\frac{3}{2}\right)=104$ $2^x=16$ $2^x=2^4$ $\therefore x=4$	✓ removal of common factor ✓ simplifying bracket ✓ 16 ✓ writing 16 as base 2 ✓ answer	(5)
1.2	$\sqrt{36\times 2}x-\sqrt{49\times 2}x+2\sqrt{144\times 2}x$ $6\sqrt{2}x-7\sqrt{2}x+24\sqrt{2}x$ $=23\sqrt{2}x$	✓ perfect squares ✓ for writing all as mixed surds ✓ answers	(3)
1.3	$y=2x-2$ $2x-2=(x-2)(x-1)$ $2x-2=x^2-3x+2$ $x^2-5x+4=0$ $(x-1)(x-4)=0$ $x=1 \quad or \quad x=4$ $y=0 \quad or \quad y=6$ **OR** $x-1=\frac{y}{2}$ $y=\left(\frac{y}{2}-1\right)\left(\frac{y}{2}\right)$ $y=\frac{y^2}{4}-\frac{y}{2}$ $4y=y^2-2y$ $y^2-6y=0$ $y(y-6)=0$ $y=0 \quad or \quad y=6$ $x=1 \quad or \quad x=4$	✓ for y as subject ✓ substitution of y ✓ form of equation ✓factors ✓x values ✓y values ✓ for y as subject ✓ substitution of y ✓ form of equation ✓factors ✓y values ✓x values	(6) (6)
			[23]

QUESTION 2

2.1.1	11 ; 14	✓ answer (both terms)	(1)
2.1.2	$T_n = 3n-1$ *Now if* $3n-1=n^2$ $n^2-3n+1=0$ $n=\frac{3\pm\sqrt{5}}{2}$ $\Rightarrow n$ is irrational But n must be natural $\therefore$ No term is a perfect square ($b^2-4ac \neq$ a perfect square) or $b^2-4ac=(-3)^2-4(1)(1)$ $=5$ $\Delta \neq$ *a perfect square*	✓✓ for nth term ✓ for equating nth term to n^2 ✓ for value of n ✓for concluding n is irrational and for deducing n is natural	(5)
2.2.1	$T_n = 2n-1$ *for the first difference sequence* $T_6 = 2(6)-1=11$ $\therefore T_6$ *of original seq.* $= 35-11=24$ $T_5 = 2(5)-1=9$ $\therefore T_5$ *of original seq.* $= 24-9=15$	✓ 6th term of 1st difference seq. ✓ 6th term of quadratic seq. ✓ 5th term of 1st difference seq. ✓ 5th term of quadratic sequence	(4)
2.2.2	$36a+6b+c=24 \rightarrow (1)$ $25a+5b+c=15 \rightarrow (2)$ $49a+7b+c=35 \rightarrow (3)$ $(1)-(2):$ $11a+b=9 \rightarrow (4)$ $(3)-(1)$ $13a+b=11 \rightarrow (5)$ $(5)-(4)$ $2a=2$ $\therefore a=1$ $b=-2$ $T_n=n^2-2n$ **OR** $2a=2$ $\therefore a=1$ $c=0$ $T_n=an^2+bn+c$ $35=1(7)^2+b(7)+0$ $-14=7b$ $b=-2$ $T_n=n^2-2n$	✓ $11a+b=9$ ✓ $13a+b=11$ ✓ a value ✓ b value ✓ answer ✓ a value ✓ c value ✓ $T_7=35$ ✓ b value ✓ answer	(5) (5)

2.3	$S_n = a + ar + ar^2 + ... + ar^{n-1} \quad \rightarrow \quad (1)$ $rS_n = ar + ar^2 + ... + ar^n \quad \rightarrow \quad (2)$ $(2) - (1):$ $rS_n - S_n = ar^n - a$ $S_n(r-1) = a(r^n - 1)$ $\therefore S_n = \frac{a(r^n - 1)}{r-1}$	✓ for equation (1) ✓ for equation (2) ✓ subtraction on LHS and RHS ✓ factorising	(4)
2.4	$S_n = \frac{a(r^n - 1)}{r-1}$ $531440 = \frac{2(3^n - 1)}{3-1}$ $3^n = 531441$ $n = \log_3 531441$ $n = 12$	✓✓ substitution ✓ for 3^n as subject ✓ writing in log form ✓ answer	(5)
			[24]

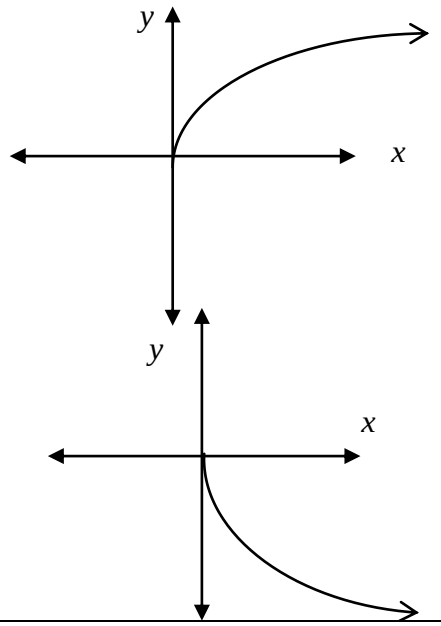
QUESTION 3

3.1		f: ✓ turning point ✓ *y* - intercept ✓✓ *x* – intercepts: 1 +ve and 1 –ve ✓ shape g: ✓ Horizontal asymptote ✓ *y* - intercept ✓ shape	(8)
3.2.1	$t = 6$	✓✓answer	(2)
3.2.2	$t = 2$	✓✓answer	(2)
3.3	$y = 2$	✓✓answer	(2)
			[14]

QUESTION 4

4.1	$p=-3$ $q=4$ $y=\frac{a}{x-3}+4$ *Now subst.* $A(4;6)$: $6=\frac{a}{4-3}+4$ $\therefore a=2$	✓ p value ✓ q value ✓ subst. p, q and point A ✓a value	(4)
4.2	$y\in R;\quad y\neq 2$	✓✓ answer	(2)
4.3	$4=3+c$ $\therefore c=1$ **OR** $y=(x+p)+q$ $y=(x-3)+4$ $y=x+1$ $\therefore c=1$	✓✓subst. of point A ✓ answer ✓substitution of p and q values ✓equation of line of symmetry ✓answer	(3) (3)
			[9]

QUESTION 5

5.1	$y=5^x$	✓✓ answer	(2)
5.2.1	$y=\pm\sqrt{x}$	✓✓ answer	(2)
5.2.2	h is a many-to-one-function **OR** For each x-value, there is more than 1 y-value **OR** If you draw a vertical line parallel to the y-axis, it will cut h twice.	✓✓ answer ✓✓ answer ✓✓ answer	(2) (2) (2)
5.2.3	$x\leq 0$ $x\geq 0$	✓ answer ✓ answer	(2)
5.2.4		✓shape ✓intercept ✓shape ✓intercept	(4)

5.2.5	$\sqrt{x} \le 2$ *So consider* $\sqrt{x} = 2$ $x = 4$ $\therefore 0 \le x \le 4$	✓✓answer[end points + inequality]	(2)
			[14]

QUESTION 6

6.1	$f'(x) = \lim_{h\to 0} \frac{f(x+h) - f(x)}{h}$ $= \lim_{h\to 0} \frac{(x+h)^3 - (x)^3}{h}$ $= \lim_{h\to 0} \frac{(x^3 + 3x^2h + 3xh^2 + h^3) - x^3}{h}$ $= \lim_{h\to 0} \frac{h(3x^2 + 3xh + h^2)}{h}$ $= 3x^2$ OR $f'(x) = \lim_{h\to 0} \frac{f(x+h) - f(x)}{h}$ $= \lim_{h\to 0} \frac{(x+h)^3 - (x)^3}{h}$ $= \lim_{h\to 0} \frac{(x+h-x)(x^2 + 2xh + h^2 + xh + x^2 + x^2)}{h}$ $= \lim_{h\to 0} (3x^2 + 3xh + h^2)$ $= 3x^2$	✓ formula ✓ substitution ✓ simplifying ✓ factorising ✓ answer ✓ formula ✓ substitution ✓ factorising ✓ simplifying ✓ answer	(5) (5)
6.2.1	$y = x^2 - x$ $\frac{dy}{dx} = 2x - 1$	✓✓ for simplifying ✓✓both answers	(4)
6.2.2	$h(x) = x^{\frac{1}{6}} - 3x^{\frac{1}{2}}$ $h'(x) = \frac{1}{6}x^{\frac{-5}{6}} - \frac{3}{2}x^{-\frac{1}{2}}$	✓✓rewriting both terms in exponential form ✓✓ both answers	(4)
			[13]

QUESTION 7

7.1	A(2 ; 0)	✓✓for x and y coordinates	(2)
7.2	$y = a(x-x_1)(x-x_2)(x-x_3)$ $-2 = a(0+1)(0+1)(0-2)$ $-2 = -2a$ $\therefore a = 1$ $y = (x+1)(x+1)(x-2)$ $= (x^2+2x+1)(x-2)$ $= x^3 - 3x - 2$ $\therefore c = 3$ **OR** $f'(x) = 3ax^2 - c$ $f'(-1) = 3a(-1)^2 - c = 0$ $3a - c = 0 \quad \rightarrow \quad (1)$ *Also* $f(-1) = a(-1)^3 - c(-1) - 2$ $-a + c - 2 = 0$ $-a + c = 2 \quad \rightarrow \quad (2)$ $(1)+(2)$: $2a = 2$ $\therefore a = 1$ *subst.in*(2): $c = 3$ **OR** $f(x) = a(x+1)^2(x-2)$ $= a(x-2)(x^2+2x+1)$ $= a(x^3+2x^2+x-2x^2-4x-2)$ $= a(x^3-3x-2)$ $\therefore a = 1$ and $c = 3$	✓✓substituting x intercepts and y intercept into formula ✓ for $-2 = -2a$ ✓simplifying any two brackets ✓cubic function ✓ for derivative ✓ subst. $x = -1$ into derivative ✓for subst. $x = -1$ into original equation f ✓for $2a = 2$ ✓ c value ✓for subst. x-intercepts into equation ✓for squaring binomial ✓for expanding ✓for collecting like terms ✓for answers	(5) (5)
7.3	$\frac{dy}{dx} = 3x^2 - 3$ $3x^2 - 3 = 0$ $3(x+1)(x-1) = 0$ $\therefore x = -1 \quad or \quad x = 1$ $\Rightarrow y = 0 \quad or \quad y = -4$ $B(1;-4)$ Also consider second derivative method	✓ derivative and equal to 0 ✓ x – values ✓ y – values ✓ coordinates of B	(4)
7.4	$x = 0$	✓✓ answer	(2)
7.5	$k > 0 \quad or \quad k < -4$	✓✓✓ answers and accuracy mark for the word OR	(3)
7.6	$-1 < x < 1$	✓✓ end points and inequality	(2)
			[18]

QUESTION 8

8.1	$V = l \times b \times h$ $1 = 2x \times x \times h$ $h = \frac{1}{2x^2}$	✓substituting into volume formula ✓h value in terms of x	(2)
8.2	$C = 2x^2 \times R200 + 6xh \times R120$ $= 400x^2 + 720x\left(\frac{1}{2x^2}\right)$ $= 400x^2 + 360x^{-1}$	✓ cost of base ✓cost of sides ✓substitution of h value	(3)
8.3	$C'(x) = 800x - 360x^{-2}$ $800x = \frac{360}{x^2}$ $x^3 = \frac{360}{800}$ $x = 0{,}77\ m$ *Minimum Cost* $= 400(0{,}77)^2 + 360(0{,}77)^{-1}$ $= R704{,}69$ *or* $R704{,}68$	✓derivative = 0 ✓for making x^3 the subject ✓x value ✓substitution ✓answer	(5)
			[10]

TOTAL MARKS: 125