Province of the
EASTERN CAPE
EDUCATION

# NATIONAL SENIOR CERTIFICATE/ *NASIONALE SENIOR SERTIFIKAAT*

## GRADE/*GRAAD* 12

## JUNE/*JUNIE* 2018

# MATHEMATICS P2/*WISKUNDE V2* MARKING GUIDELINE/*NASIENRIGLYN*

**MARKS /*PUNTE*: 150**

This marking guideline consists of 20 pages./
*Hierdie nasienriglyn bestaan uit 20 bladsye.*

**NOTE:**

- If a candidate answers a question **TWICE**, only mark the **FIRST** attempt.
- If a candidate has crossed out an attempt of a question and not redone the question, mark the crossed out version.
- Consistent accuracy applies in **ALL** aspects of the marking guideline.
- Assuming answers/ values in order to solve a problem is **NOT** acceptable.

***NEEM KENNIS:***

- *As 'n kandidaat 'n vraag **TWEEKEER** beantwoord, merk slegs die **EERSTE** poging*
- *As 'n kandidaat 'n poging om 'n vraag te beantwoord deurgehaal het en nie weer probeer het nie, merk die deurgehaalde poging.*
- *Volgehoue akkuraatheid is van toepassing op **ALLE** aspekte van die nasienriglyn.*
- *Aanvaarding van antwoorde/waardes om sodoende probleme op te los is **NIE** aanvaarbaar **NIE**.*

| **GEOMETRY/*MEETKUNDE*** | |
|---|---|
| **S** | **A mark for a correct statement.**<br>**(A statement mark is independent of a reason.)** |
| | ***'n Punt vir korrekte stelling/bewering.***<br>***('n Stelling/bewering punt is onafhanklik van die rede.)*** |
| **R** | **A mark for a correct reason.**<br>**(A reason mark may only be awarded if the statement is correct.)** |
| | ***'n Punt vir korrekte rede.***<br>***('n Rede punt mag slegs toegeken word indien die stelling/bewering korrek is.)*** |
| **S/R** | **Award a mark if statement and reason are both correct.** |
| | ***Ken 'n punt toe indien die stelling/bewering sowel as die rede korrek is.*** |

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**QUESTION/*VRAAG 1***

| | | |
|---|---|---|
| 1.1 | $\bar{x} = \frac{\sum x}{n}$<br>$= \frac{619}{15}$<br>$= 41{,}27$<br>**Answer only full marks/** ***Slegs antwoord volpunte*** | ✓619<br>✓41,27 (2) |
| 1.2 | $\sigma = 10{,}63$ | ✓✓Answer / *antwoord* (2) |
| 1.3 | $41{,}27 - 10{,}63 \le x \le 41{,}27 + 10{,}63$<br>$30{,}64 \le x \le 51{,}90$<br>$\therefore 8$ learners/leerders | ✓ both c.v$^s$ / *beide k.w$^s$*<br>✓ notation / *notasie*<br>✓ 8 learners/*leerders* (3) |
| 1.4 | $Q_1 = 29$<br>$Q_3 = 49$<br>$\therefore \text{Semi} - \text{IQR/IKW} = \frac{49 - 29}{2}$<br>$= 10$ | ✓ $Q_1$<br>✓ $Q_3$<br>✓ answer/*antwoord* (3) |
| | | **[10]** |

**QUESTION/*VRAAG 2***

| | | |
|---|---|---|
| 2.1 | $56 + 2y = 64$<br>$2y = 8$<br>$y = 4$<br>**Answer only 1 mark** ***Slegs antwoord 1 punt*** | ✓ correct equation *korrekte vergelyking*<br>✓ *y*-value/*waarde* (2) |

2.2

| Time (in minutes) *Tyd (in minute)* | Frequency / *Frekwensie* | Cumulative frequency/ *Kumulatiewe frekwensie* |
|---|---|---|
| $5 \le t < 10$ | 3 | 3 |
| $10 \le t < 15$ | 5 | 8 |
| $15 \le t < 20$ | 4 | 12 |
| $20 \le t < 25$ | 16 | 28 |
| $25 \le t < 30$ | 15 | 43 |
| $30 \le t < 35$ | 17 | 60 |
| $35 \le t < 40$ | 4 | 64 |

✓ 8, 12 and/*en 28*

✓ 43, 60 and/*en 64* (2)

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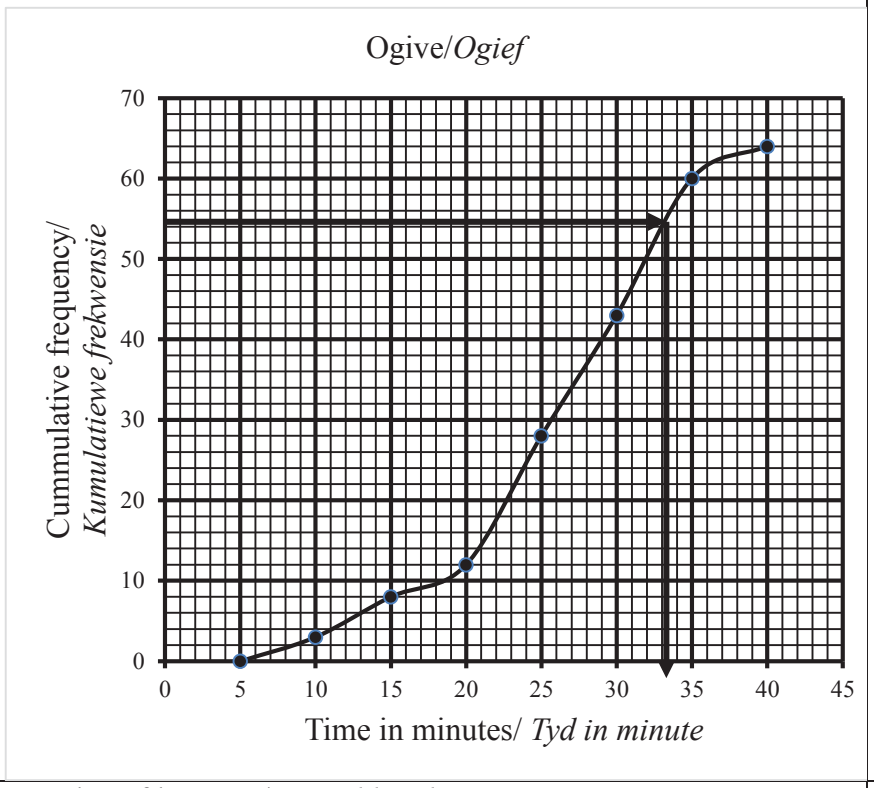
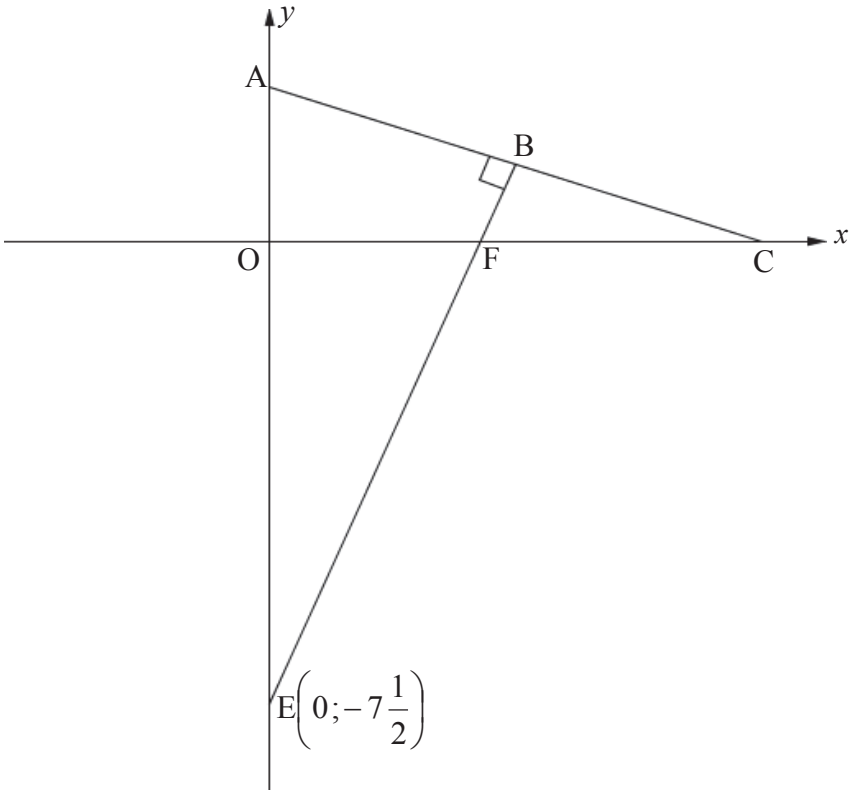
| | | |
|---|---|---|
| 2.3 |  | ✓ grounding/*anker*<br>✓ plotting against the upper limit/ *afsteek teen die boonste limiet*<br>✓ shape / *vorm*<br>(3) |
| 2.4 | Number of learners / *Aantal leerders* $\approx 64 - 54 \approx 10$<br>Accept/ *Aanvaar* $[9 - 11]$ | ✓ method/*metode*<br>✓ 54<br>✓ answer/*antwoord*<br>(3) |
| | | **[10]** |

**QUESTION/*VRAAG 3***




| | | |
|---|---|---|
| 3.1 | $x+py=p$<br>$y=-\frac{x}{p}+1$<br>$\therefore \text{A}(0;1)$ | ✓ *y*-subject of formula<br>*y-onderwerp van formule*<br>✓ coordinates of A<br>*koördinate van A*<br>(2) |
| 3.2 | $\text{OA}=1$<br>$\therefore \text{OC}=4(1)=4$<br>$\therefore C(4;0)$<br>$m_{\text{AC}}=-\frac{1}{p}=\frac{1-0}{0-4}$<br>$p=4$ | ✓ OC<br>✓ $-\frac{1}{p}=\frac{1-0}{0-4}$<br>✓ simplification/*vereenvoudiging*<br>✓ *p*-value/*waarde* (4) |
| 3.3 | $m_{\text{EB}}=4$ EB ⊥ AC<br>$y=4x-7\frac{1}{2}$ | ✓ $m_{EB}=4$<br>✓ equation/*vergelyking*<br>(2) |

| | | |
|---|---|---|
| 3.4 | $-\frac{x}{4}+1=4x-7\frac{1}{2}$<br>$-x+4=16x-30$<br>$17x=34$<br>$x=2$<br>$y=\frac{1}{2}$ | ✓ equating / *gelykstel*<br>✓ simplification / *vereenvoudiging*<br>✓ $x$-value/*waarde*<br>✓ $y$-value/*waarde* (4) |
| 3.5 | $4x-7\frac{1}{2}=0$<br>$4x=\frac{15}{2}$<br>$x=\frac{15}{8}$<br>$\mathrm{F}\left(\frac{15}{8};0\right)$ | ✓ $y=0$<br>✓ $x$-value/*waarde* (2) |
| 3.6 | $\mathrm{BF}=\sqrt{\left(\frac{15}{8}-2\right)^2+\left(0-\frac{1}{2}\right)^2}$<br>$=\frac{\sqrt{17}}{8}$<br>$\therefore \text{Area FOAB}=\text{Area }\Delta\text{AOC}-\text{Area }\Delta\text{BFC}$<br>$=\frac{1}{2}\times 4\times 1-\frac{1}{2}\left(\frac{\sqrt{17}}{2}\right)\frac{\sqrt{17}}{8}$<br>$=1{,}47$<br>**OR/*OF***<br>$\mathrm{BF}=\sqrt{\left(\frac{15}{8}-2\right)^2+\left(0-\frac{1}{2}\right)^2}$<br>$=\frac{\sqrt{17}}{8}$<br>$\text{Area FOAB}=\text{Area }\Delta\text{AOF}+\text{Area }\Delta\text{ABF}$<br>$=\frac{1}{2}\times 1\times\frac{15}{8}+\frac{1}{2}\times\frac{\sqrt{17}}{2}\times\frac{\sqrt{17}}{8}$<br>$=1{,}47$ | ✓ substitution / *vervanging*<br>✓ BF<br>✓ BC<br>✓ substitution / *vervanging*<br>✓answer / *antwoord*<br>✓ substitution / *vervanging*<br>✓ BF<br>✓ OF<br>✓ substitution / *vervanging*<br>✓answer / *antwoord* (5) |
| 3.7 | $r=\frac{17}{16}$ | ✓ answer / *antwoord* (1) |
| 3.8 | $\left(x-\frac{47}{16}\right)^2+y^2=\frac{289}{256}$ | ✓ $r^2$<br>✓ equation / *vergelyking* (2) |
| | | **[22]** |

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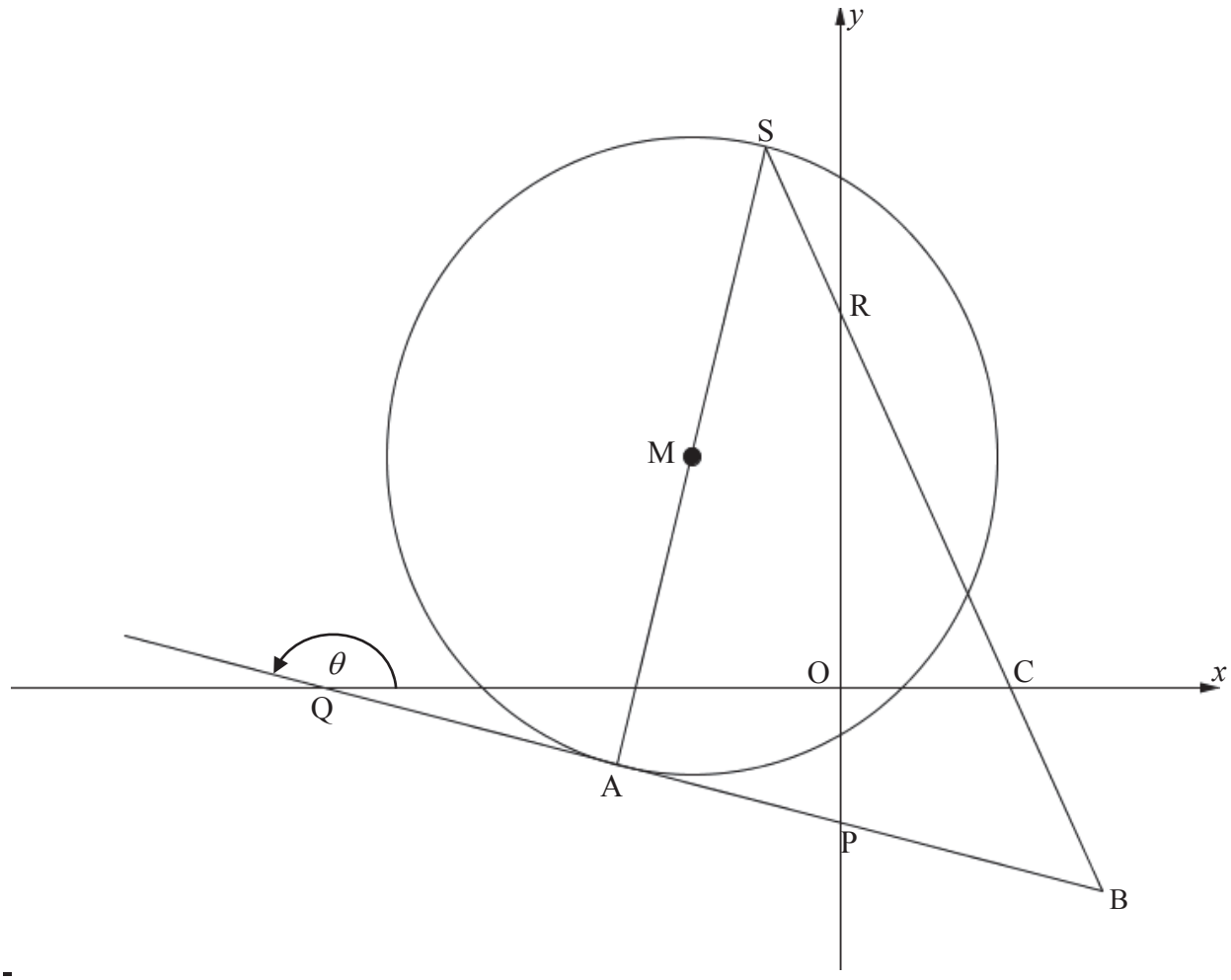
## QUESTION/*VRAAG 4*

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| | | |
|---|---|---|
| 4.1 | $x^2+6x+9+y^2-8y+16=-5+9+16$<br>$(x+3)^2+(y-4)^2=26$<br>$\therefore M(-3;4)$ | ✓ completing the square<br>*kwadraatsvoltooiing*<br>✓ *x*-value/*waarde*<br>✓ *y*-value/*waarde* (3) |
| 4.2 | $r=\sqrt{26}$ | ✓ answer/*antwoord* (1) |
| 4.3 | $m_{AS}=5$ [SA ⊥ QB]<br>$y-4=5(x+3)$<br>$y=5x+19$ | ✓ $m_{AS}=5$<br>✓ subst. /*verv.* $m_{AS}$ & $(-3;4)$<br>✓ equation/*vergelyking* (3) |

| | | | |
|---|---|---|---|
| 4.4 | $x^2+6x+9+(5x+19-4)^2=26$<br>$x^2+6x+9+25x^2+150x+225-26=0$<br>$26x^2+156x+208=0$<br>$x^2+6x+8=0$<br>$(x+4)(x+2)=0$<br>$x=-4 \text{ or } x\neq -2$<br>$\therefore y=5(-4)+19$<br>$=-1$<br>$Q(-4;-1)$ | ✓ substitution/*vervanging*<br>✓ simplification / *vereenvoudiging*<br>✓ standard form / *standaardvorm*<br>✓ factors / *faktore*<br>✓ choosing correct *x-value* *korrekte keuse van x-waarde*<br>✓ *y*-value/*waarde* | (6) |
| 4.5 | $-1=-\frac{1}{5}(-4)+k$<br>$\therefore k=-\frac{9}{5}$ | ✓ substitute/*vervang* $(-4;-1)$<br>✓ answer / *antwoord* | (2) |
| 4.6 | $\tan\theta=-\frac{1}{5}$<br>$\therefore \theta=169^0$<br>$O\hat{Q}P=11^0$ [∠s on a str line] / [∠*e op 'n reguitlyn*]<br>$\therefore O\hat{Q}P=O\hat{R}C$<br>$\therefore$ RCPQ is a cyclic quad [converse ∠s same seg]<br>*RCPQ is 'n koordevierhoek* [*omgekeerde ∠e in dieselfde seg*] | ✓ $\tan\theta=-\frac{1}{5}$<br>✓ size of $\theta$ / *grootte van* $\theta$<br>✓ size of $O\hat{Q}P$ / *grootte van* $O\hat{Q}P$<br>✓ R | (4) |
| | | | **[19]** |

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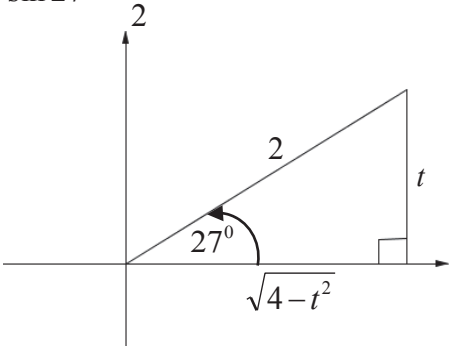
## QUESTION/*VRAAG 5*

| | | |
|---|---|---|
| 5.1.1 | $2\sin 27^0 = t$<br>$\sin 27^0 = \frac{t}{2}$<br>  <br><br>$x = \sqrt{4-t^2}$<br>$\sin 54^0 = 2\sin 27^0 \cos 27^0$<br>$= 2 \times \frac{t}{2} \times \frac{\sqrt{4-t^2}}{2}$<br>$= \frac{t\sqrt{4-t^2}}{2}$ | ✓ $\sin 27^0 = \frac{t}{2}$<br><br>✓ $x = \sqrt{4-t^2}$<br>✓ $2\sin 27^0 \cos 27^0$<br>✓ substitution/*vervanging*<br>(4) |
| 5.1.2 | $\tan 513^0.\cos 27^0 = (-\tan 27^0)\cos 27^0$<br>$= -\frac{\sin 27^0}{\cos 27^0}.\cos 27^0$<br>$= -\frac{t}{2}$ | ✓ $\left(-\tan 27^0\right)$<br>✓ $-\frac{\sin 27^0}{\cos 27^0}$<br>✓ $-\frac{t}{2}$<br>(3) |
| 5.1.3 | $\cos 87^0 = \cos\left(60^0 + 27^0\right)$<br>$= \cos 60^0.\cos 27^0 - \sin 60^0.\sin 27^0$<br>$= \frac{1}{2}.\frac{\sqrt{4-t^2}}{2} - \frac{\sqrt{3}}{2}.\frac{t}{2}$<br>$= \frac{\sqrt{4-t^2} - t\sqrt{3}}{4}$ | ✓ $60^0 + 27^0$<br>✓ expansion / *uitbreiding*<br>✓ subst./*verv.* $\frac{\sqrt{4-t^2}}{2}$ & $\frac{t}{2}$<br>✓ subst. / *verv.* $\frac{1}{2}$ & $\frac{\sqrt{3}}{2}$<br>(4) |

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| | | |
|---|---|---|
| 5.2 | $\frac{\sin(-2\alpha)\cos(90^0+\alpha)}{\sin(-\alpha+360^0).\cos(-\alpha-180^0)}$<br>$=\frac{(-\sin 2\alpha)(-\sin\alpha)}{(\sin\alpha)(-\cos\alpha)}$<br>$=-\frac{2\sin\alpha\cos\alpha}{\cos\alpha}$<br>$=-2\sin\alpha$ | ✓ $(-\sin 2\alpha)$<br>✓ $(-\sin\alpha)$<br>✓ $\sin\alpha$<br>✓ $(-\cos\alpha)$<br>✓ $2\sin\alpha\cos\alpha$<br>✓ $-2\sin\alpha$<br>(6) |
| 5.3 | $9\sin^2 x-4\cos^2 x=0$<br>$(3\sin x-2\cos x)(3\sin x+2\cos x)=0$<br>$\therefore 3\sin x=\pm 2\cos x$<br>$\tan x=\pm\frac{2}{3}$<br>$x=33,69^0+180^0.k$ or/of $x=146,31^0+180^0.k \quad k\in Z$ | ✓ factors/*faktore*<br>✓ both equations<br>*beide vergelykings*<br>✓ $\tan x=\pm\frac{2}{3}$<br>✓ both / *beide*<br>$33,69^0$ & $146,31^0$ / $-33,69^0$<br>✓ $180^0.k$ & $k\in Z$<br>(5) |
| | | **[22]** |

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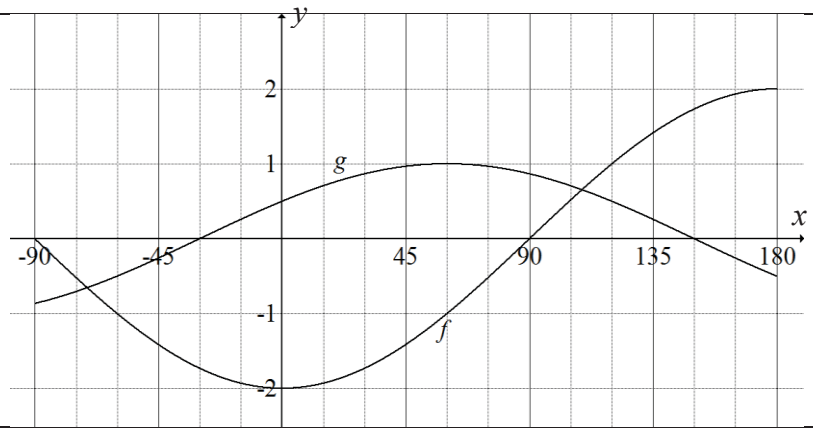
## QUESTION/*VRAAG 6*

| | | | |
|---|---|---|---|
| 6.1 | $\text{Amplitude} = 2$ | ✓ answer/*antwoord* | (1) |
| 6.2 | $1 \le y \le 5$ | ✓ min & max/*maks* ✓ notation/*notasie* | (2) |
| | | | |
| 6.3 |  | ✓ both *x*-intercepts *beide x-afsnitte* $-30^0$ & $150^0$<br>✓ max TP & *y*-intercept<br>✓ shape / *vorm* | (3) |
| 6.4 | $-30^0 < x < 0^0$ | ✓ both c.v.$^s$ / *beide k.w$^s$* ✓ notation/*notasie* | (2) |
| 6.5 | $h(x) = \sin(x + 90^0) - 2$<br>$= \cos x - 2$ | ✓✓ $\sin(x + 90^0) - 2$<br>✓ $\cos x$ | (3) |
| | | | **[11]** |

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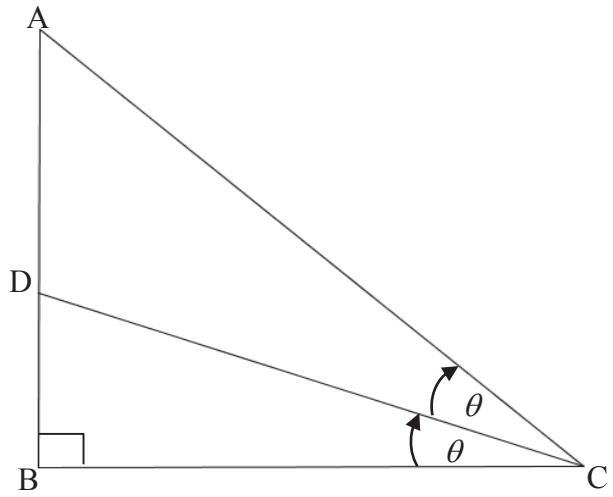
**QUESTION/*VRAAG 7***




| | | | |
|---|---|---|---|
| 7.1 | $\hat{A} = 90^0 - 2\theta$ | ✓ answer / *antwoord* | (1) |
| 7.2 | $\sin\theta = \frac{DB}{DC}$ | ✓ answer / *antwoord* | (1) |
| 7.3 | $DC = \frac{DB}{\sin\theta}$<br>*and/en* $AD = 2DB = 2DB$<br>$\frac{DC}{\sin(90^0 - 2\theta)} = \frac{AD}{\sin\theta}$ *in* $\Delta ADC$<br>$\frac{\frac{DB}{\sin\theta}}{\cos 2\theta} = \frac{2DB}{\sin\theta}$<br>$\frac{DB}{\sin\theta.\cos 2\theta} = \frac{2DB}{\sin\theta}$<br>$DB.\sin\theta = 2DB.\sin\theta.\cos 2\theta$<br>$2\cos 2\theta = 1$<br>$2\cos 2\theta - 1 = 0$ | ✓ AD = 2DB<br>✓ sine rule/*sinusreël in* ΔADC<br>✓ subst. in sine rule / *verv. in sinusreël*<br>✓ cos2θ<br>✓ equation / *vergelyking* | (5) |
| | | | **[7]** |

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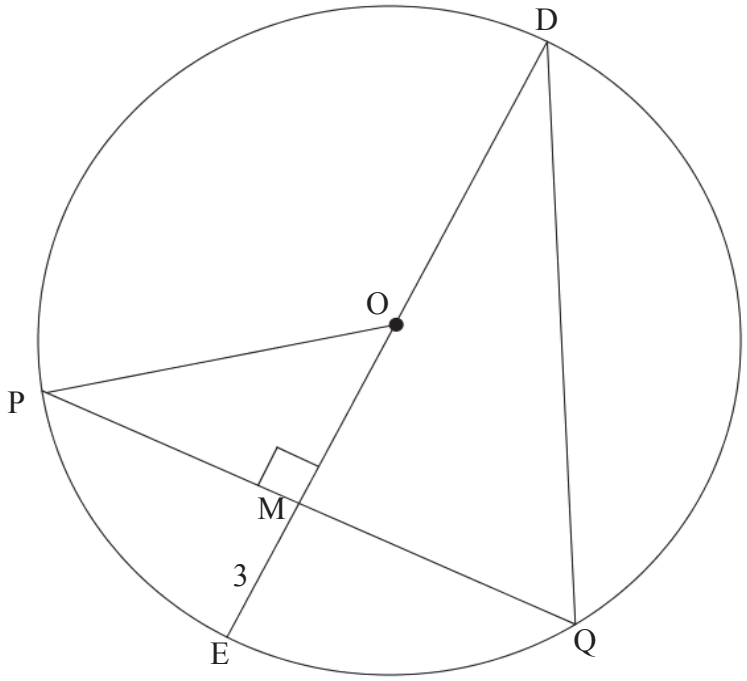
## QUESTION/*VRAAG 8*

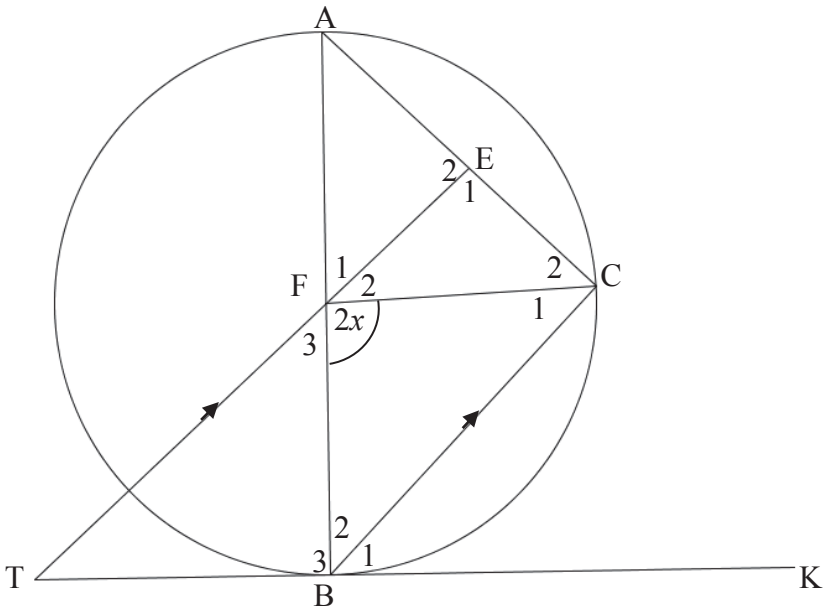
| | | | |
|---|---|---|---|
| 8.1 | **Perpendicular** to the chord / ***Loodreg*** *op die koord* | ✓ answer/*antwoord* | (1) |
| 8.2 | | | |




| | | | |
|---|---|---|---|
| 8.2.1 | $OM = \frac{2x+3}{2} - 3$<br>OR<br>$OM = \frac{2x-3}{2}$ | ✓ answer/*antwoord*<br>OR/*OF*<br>✓ answer/*antwoord* | (1) |
| 8.2.2 | $OQ^2 = OM^2 + QM^2$<br>$\left(\frac{2x+3}{2}\right)^2 = \left(\frac{2x-3}{2}\right)^2 + 6^2$<br>$\frac{4x^2+12x+9}{4} = \frac{4x^2-12x+9}{4} + 36$<br>$24x = 144$<br>$x = 6$ | ✓ susbt. in Pyth / *verv. in Pyth.*<br>✓ simplification / *vereenvoudiging*<br>✓ standard form linear equation<br>*Standaardvorm van lineêre vergelyking*<br>✓ *x*-value/*waarde* | (4) |

| | | | |
|---|---|---|---|
| 8.2.3 | $DM = 12$<br>$DQ = \sqrt{12^2 + 6^2}$<br>$= \sqrt{180}$<br>$= 6\sqrt{5}$ | ✓ DM<br>✓subst. in Pyth / *verv. in Pyth*<br>✓ answer / *antwoord* | (3) |
| | | | **[9]** |

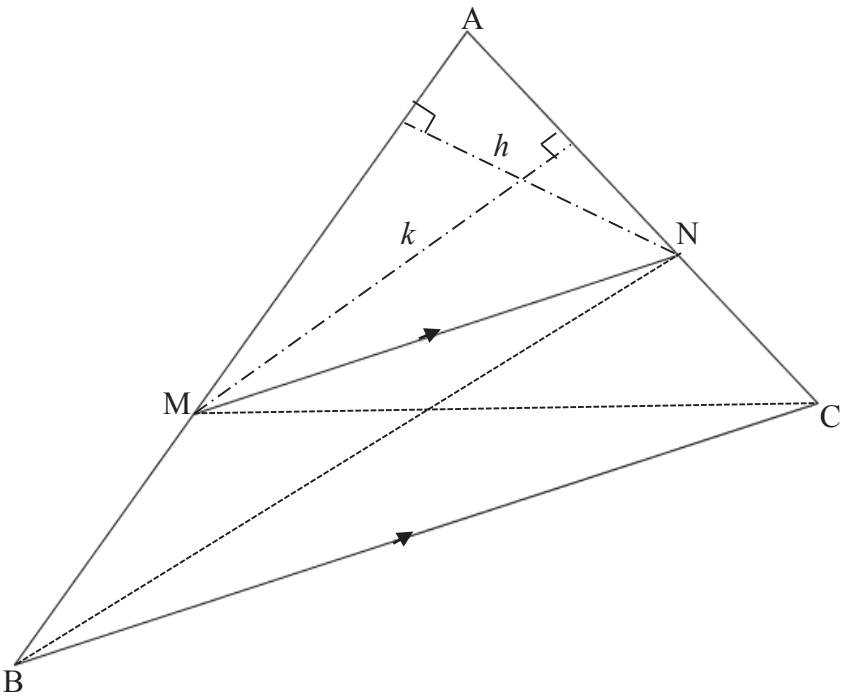
**QUESTION/*VRAAG 9***




| | | | |
|---|---|---|---|
| 9.1 | $\hat{A} = x$ [$\angle$at centre $= 2\angle$at circumf. / *Middelpunts*$\angle = 2 \times$ *Omtreks*$\angle$] | ✓ S ✓R | |
| | $\hat{C}_2 = x$ [$\angle s$ opp. = sides / $\angle$e teenoor gelyke sye] | ✓ S ✓R | |
| | $\hat{B}_1 = x$ [tan chord theorem / raaklyn-koord-stelling] | ✓ S ✓R | |
| | $\hat{T} = x$ [corresp. $\angle$s, / ooreenk. $\angle$e : TF$\|\|$BC] | ✓ S ✓R | (8) |
| 9.2 | $\hat{T} = \hat{A} = x$<br>$\therefore$ ATBE is a cyclic quad [converse $\angle$s same segment]<br>*ATBE is 'n koordevierhoek* [*omgekeerde* $\angle$*e in dieselfde seg*] | ✓ S<br>✓ R | (2) |
| | | | **[10]** |

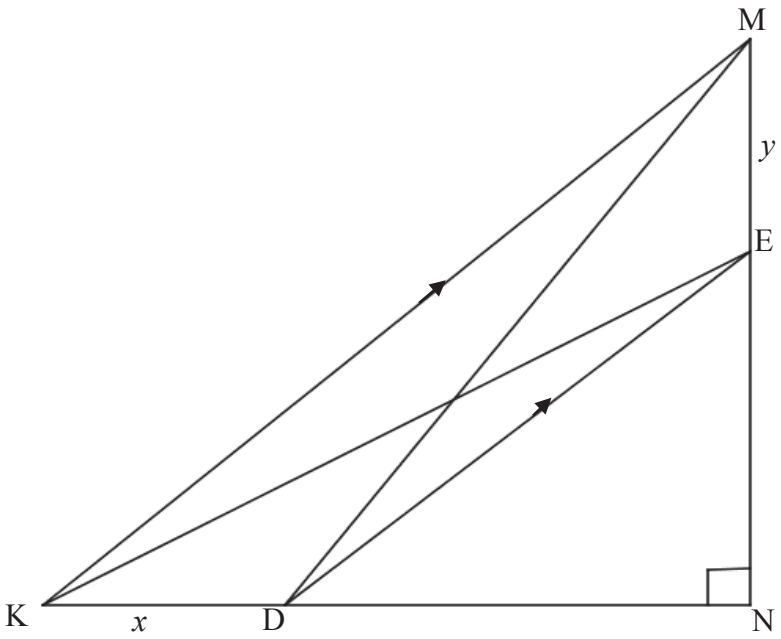
**QUESTION/*VRAAG 10***

10.1




| | |
|---|---|
| Contruction:Join BN and height $h$ from N $\perp$ AM and CM and height $k$ from M $\perp$ AN<br>*Konstruksie :Verbind BN en hoogte, h vanaf N $\perp$ AM*<br>*CM en hoogte, k vanaf M $\perp$ AN* | ✓ constr / *konstr.* |
| $\frac{\text{Area }\Delta\text{ AMN}}{\text{Area }\Delta\text{ BMN}} = \frac{\frac{1}{2}\times \text{AM}\times h}{\frac{1}{2}\times \text{BM}\times h}$ [same height/dieselfde hoogte ] | ✓ S ✓R |
| $= \frac{\text{AM}}{\text{BM}}$ | |
| $\frac{\text{Area }\Delta\text{ AMN}}{\text{Area }\Delta\text{ CMN}} = \frac{\frac{1}{2}\times \text{AN}\times k}{\frac{1}{2}\times \text{NC}\times k}$ [same height / dieselfde hoogte] | ✓ S |
| $= \frac{\text{AN}}{\text{NC}}$ | ✓ R |
| Area $\Delta$BMN = Area $\Delta$ CMN [same height, same base MN $\|\|$ BC] / [*dieselfde hoogte, dieselfde basis MN $\|\|$ BC*] | |
| $\therefore \frac{\text{AM}}{\text{BM}} = \frac{\text{AN}}{\text{NC}}$ | (5) |

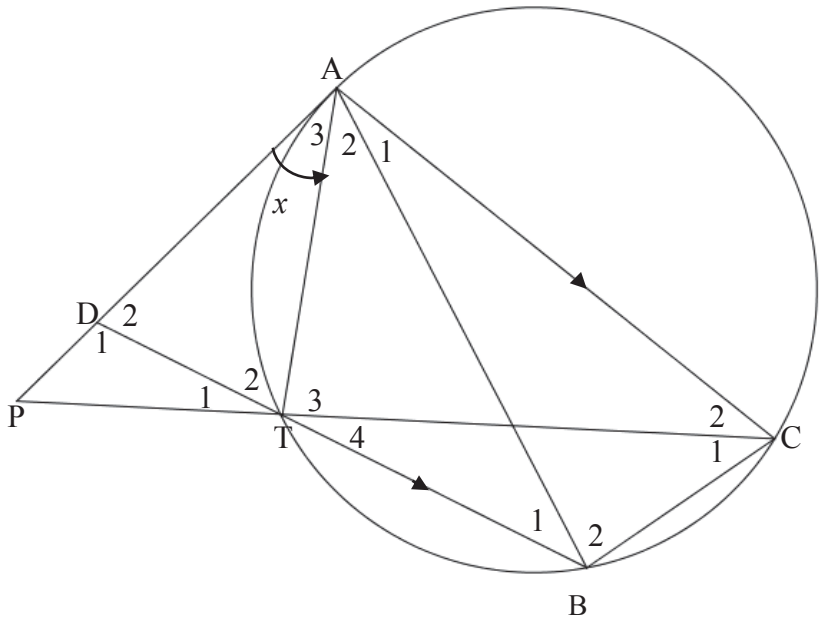
10.2




| | | |
|---|---|---|
| 10.2.1 | $\frac{ND}{x}=\frac{2}{1}$ [given/gegee]<br>$ND=2x$<br>and/en<br>$\frac{NE}{y}=\frac{2}{1}$ [prop theorem DE ‖ KM or / line drawn ‖ to one side of a Δ] / [verhouding stelling DE ‖ KM of / lyn ‖ getrek aan een sy van 'n Δ]<br>$\therefore NE=2y$<br>$KM^2=KN^2+MN^2$ [Pyth theorem/stelling]<br>$=(3x)^2+(3y)^2$<br>$=9x^2+9y^2$ | ✓S ✓R<br><br>✓subst in Pyth theo<br>*verv. in Pyth stel*<br><br>✓simplification/<br>*vereenvoudiging* (4) |

| | | |
|---|---|---|
| 10.2.2 | $DM^2 = DN^2 + MN^2$ [Pyth]<br>$= (2x)^2 + (3y)^2$<br>$KE^2 = KN^2 + NE^2$ [Pyth]<br>$= (3x)^2 + (2y)^2$<br>$DM^2 + KE^2 = 4x^2 + 9y^2 + 9x^2 + 4y^2$<br>$= 13(x^2 + y^2)$<br>$\frac{DM^2 + KE^2}{KM^2} = \frac{13(x^2 + y^2)}{9(x^2 + y^2)}$<br>$= \frac{13}{9}$ | ✓subst in Pyth / *verv. in Pyth*<br>✓subst in Pyth / *verv. in Pyth*<br>✓value of / *waarde van* $DM^2 + KE^2$<br>✓$\frac{13(x^2 + y^2)}{9(x^2 + y^2)}$ (4) |
| | | **[13]** |

**QUESTION/*VRAAG 11***




| | | | |
|---|---|---|---|
| 11.1 | $\hat{A}_1 = \hat{B}_1$ | $[\text{alt } \angle\text{s, AC} \parallel \text{DB}] / [verw.\angle e, AC \parallel DB]$ | ✓ S/R |
| | $\hat{A}_3 = \hat{B}_1$ | $[\text{tan chord}] / [Raaklyn - koord]$ | ✓S ✓R |
| | $\therefore \hat{A}_1 = \hat{A}_3$ | | |
| | $\hat{T}_2 = A\hat{C}B$ | $[\text{ext } \angle \text{ of a cyclic quad}] / [buite \angle van koordev]$ | ✓S ✓R |
| | $\hat{D}_2 = \hat{B}_2$ | $[\text{3rd } \angle\text{s}] / [3de \angle e]$ | ✓S |
| | **OR/OF** | | **OR/*OF*** |
| | $\hat{A}_1 = \hat{B}_1$ | $[\text{alt } \angle\text{s, AC} \parallel \text{DB}] / [verw.\angle e, AC \parallel DB]$ | ✓ S/R |
| | $\hat{A}_3 = \hat{B}_1$ | $[\text{tan chord}] / [Raaklyn - koord]$ | ✓S ✓R |
| | $\therefore \hat{A}_1 = \hat{A}_3$ | | |
| | $\hat{T}_2 = A\hat{C}B$ | $[\text{ext } \angle \text{ of a cyclic quad}] / [buite \angle van koordev]$ | ✓S ✓R |
| | $\therefore \Delta ABC \| \| \| \Delta ADT$ | $[\angle\angle\angle]$ | ✓R (6) |

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| | | |
|---|---|---|
| 11.2 | $\hat{T}_4 = \hat{T}_1$ [vert opp ∠s]/[*regoorst.∠e*]<br>$\hat{A}_1 = \hat{T}_4$ [∠s in same seg]/[*∠e in dieselfde segment*]<br>$A_1 = \hat{A}_3$ [proven]/[*Bewys*]<br>$\therefore \hat{T}_1 = \hat{A}_3$<br>∴ PT is a tangent to circle ADT [converse tan chord]<br>*PT is 'n raaklyn aan sirkel ADT* [*omgekeerde raaklyn – koord*] | ✓ S/R<br>✓S ✓R<br><br><br>✓R<br>(4) |
| 11.3 | $\hat{A}_3 = \hat{T}_1$ [proven]/[*bewys*]<br>$\hat{P} = \hat{P}$ [common]/[*gemeen*]<br>$P\hat{T}A = \hat{D}_1$ [3rd ∠s]/[*3de ∠e*]<br>$\Delta APT \|\|\| \Delta TPD$ [∠∠∠] | ✓S<br>✓S<br><br>✓R |
| | **OR/OF**<br>$\hat{A}_3 = \hat{T}_1$ [proven]/[*Bewys*]<br>$\hat{P} = \hat{P}$ [common]/[*gemeen*]<br>$P\hat{T}A = \hat{D}_1$ [3rd ∠s]/[*3de ∠e*] | ✓S<br>✓S<br>✓S<br>(3) |
| 11.4 | $\frac{AP}{TP} = \frac{PT}{PD}$ [$\|\|\|$ Δs]<br>$AP.PD = PT^2$<br>$AP(AP - AD) = PT^2$<br>$AP\left(AP - \frac{2}{3}AP\right) = PT^2$<br>$AP.\frac{AP}{3} = PT^2$<br>$AP^2 = 3PT^2$ | ✓S/R<br>✓simplification<br>*vereenvoudiging*<br>✓PD i.t.o AP and AD<br>*PD i.t.v AP en AD*<br>✓ subst in AD<br>*verv. in AD*<br>(4) |
| | | **[17]** |

**TOTAL/*TOTAAL*: 150**

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