Province of the
EASTERN CAPE
EDUCATION

NATIONAL
SENIOR CERTIFICATE
NASIONALE
SENIOR SERTIKAAT

GRADE/*GRAAD* 12

SEPTEMBER 2018

TECHNICAL MATHEMATICS P2
TEGNIESE WISKUNDE V2
MARKING GUIDELINE/*NASIENRIGLYN*

MARKS/*PUNTE*: 150

This marking guideline consists of 15 pages./*Hierdie nasienriglyn bestaan uit 15 bladsye*.

NOTE:

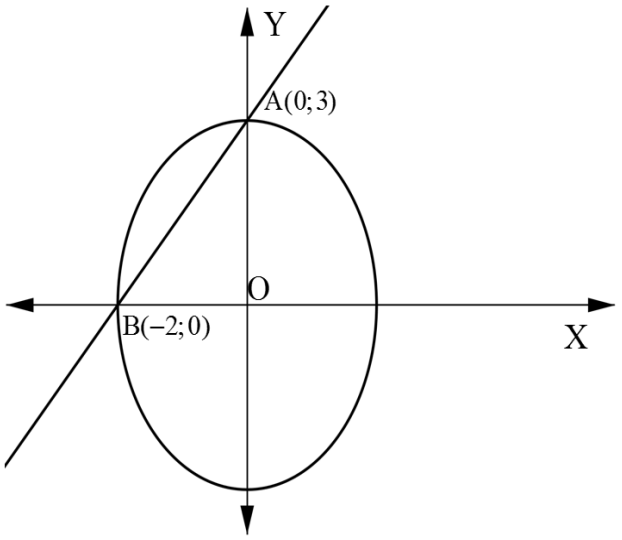
- Continuous accuracy (CA) applies in ALL aspects of the marking guideline.
- After two mistakes, do not apply CA marking.
- Assuming values/answers in order to solve a problem is unacceptable.

LET WEL:

- *Volgehoue akkuraatheid (CA) is deurgaans in ALLE aspekte van die nasienriglyn van toepassing.*
- *Na twee foute word CA nie toegepas nie.*
- *Aanvaarding van waardes/antwoorde om 'n probleem op te los, is onaanvaarbaar.*

Symbol/*Simbool*	**Explanation/*Verduideling***
M	Method/*Metode*
MA	Method with accuracy/*Metode met akkuraatheid*
A	Accuracy/A*kkuraat*
CA	Consistent accuracy/*Deurlopende akkuraatheid*
S	Simplification or Statement/*Vereenvoudiging of bewering*
R	Reason/*Rede*

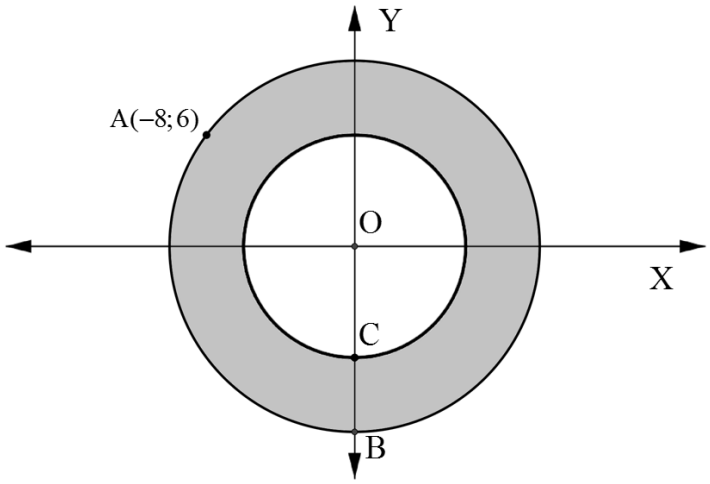
QUESTION/*VRAAG 1*



1.1	$AB = \sqrt{(x_2 - x_1)^2 + (y_2 - y_1)^2}$ $= \sqrt{(0+2)^2 + (3-0)^2}$ $= \sqrt{4+9}$ $\approx 3,61$	✓**MA** formula/*formule* & substitution/*vervanging* ✓**S** Simplification/*vereenvoudiging* ✓**CA** Answer in decimal format /*antwoord in desimale formaat*	(3)
	OR/*OF* $AB^2 = OB^2 + OA^2$ (Pyth) $= 2^2 + 3^2$ $= 4+9$ $AB = \sqrt{13}$ $\approx 3,61$	✓**MA** Pythagoras ✓**S** Simplification/*vereenvoudiging* ✓**CA** Answer in decimal format /*antwoord in desimale formaat*	(3)
1.2	$m_{AB} = \frac{y_2 - y_1}{x_2 - x_1}$ $= \frac{3-0}{0+2}$ $= \frac{3}{2}$ ∴ Equation of the line/ *Vergelyking van lyn*: $y - y_1 = m(x - x_1)$ $y - 0 = \frac{3}{2}(x+2)$ $y = \frac{3}{2}x + 3$	✓**MA** formula/*formule* & substitution/*vervanging* ✓**M** Formula equation of line/ *Formule vergelyking van lyn* ✓**A** Substitute pt A or B/ *vervang pt A of B* ✓**CA** Answer in standard form / *antwoord in standaardvorm*	(4)

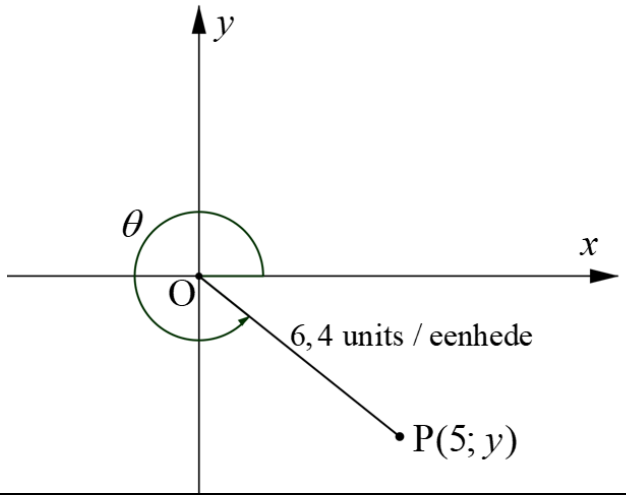
1.3	$\tan \hat{OBA} = m_{BA}$ $\tan \hat{OBA} = \frac{3}{2}$ $\hat{OBA} = 56,31°$ $\hat{OAB} = 90° - 56,31° = 33,69°$	✓**A** subst. into correct formula/*vervang in korrekte formule* ✓**CA** value of/*waarde van* $\hat{OBA}$ ✓ **CA** value of/*waarde van* $\hat{OAB}$	(3)
1.4	Scalene triangle **OR** Right angled triangle *Ongelyksydige driehoek **OF** Reghoekige driehoek*	✓**A**	(1)
1.5	$a = 2$ $b = 3$ $\therefore$ Equation of ellipse/Vergelyking van ellips $\frac{x^2}{4} + \frac{y^2}{9} = 1$	✓**A** value of/*waarde van a* ✓**A** value of/*waarde van b* ✓**CA** Equation of ellipse/*vergelyking van ellips*	(3)
			[14]

QUESTION/*VRAAG 2*



2.1	$r^2 = x^2 + y^2$ $= (-8)^2 + 6^2$ $= 64 + 36$ $= 100$ $\therefore r = 10$ $\therefore$ B(0 ; − 10)	✓**M** Calculating/*bereken r* ✓**CA** Value of/*waarde van r* ✓**A** $x = 0$ ✓**CA** $y = -10$ (neg. value/*waarde*) (4)
2.2	Radius of smaller circle /*Radius van kleiner sirkel* = 10 – 4 = 6 $\therefore$equation of smaller circle/*vergelyking van kleiner sirkel* $x^2 + y^2 = 36$	✓**M** radius of smaller circle/*radius van kleiner sirkel* ✓**CA** Equation of smaller circle/*vergelyking van kleiner sirkel* (2)
2.3	Bigger circle/ *Groter sirkel*: $m_{radius} = \frac{y_2 - y_1}{x_2 - x_1}$ $= \frac{6-0}{-8-0}$ $= -\frac{3}{4}$ $\therefore m_{\text{ tangent/raaklyn}} = \frac{4}{3}$ $\therefore$equation of tangent/*vergelyking van raaklyn*: $y - y_1 = m(x - x_1)$ $y - 6 = \frac{4}{3}(x + 8)$ $y = \frac{4}{3}x + \frac{32}{3} + 6$ $= \frac{4}{3}x + \frac{50}{3}$	✓**MA** gradient of radius of bigger circle/ *gradiënt van radius van groter sirkel* ✓**CA** Gradient of tangent/*gradiënt van raaklyn* ✓**A** Subst. A into equation/*vervang A in vergelyking* ✓**CA** Standard form of equation/ *standaardvorm van vergelyking* (4)
		[10]

QUESTION/*VRAAG* 3



3.1 3.1.1

$x^2 + y^2 = r^2$ (Pyth)

$5^2 + y^2 = 6,4^2$

$y = -\sqrt{40,96 - 25}$

$= -\sqrt{15,96}$

$= -3,99...$

≈ -4 units

✓**A** Pythagoras
✓**A** Substitution/*vervanging*
✓**CA** Simplification/*vereenvoudiging*
✓**CA** Rounded answer/*afgeronde antw*oord (neg. value/*waarde*)

(4)

3.1.2

$\cot\theta - \text{cosec}\,\theta \times \sin^2\theta$

$= \frac{5}{-4} - \frac{6,4}{-4} \times \left(\frac{-4}{6,4}\right)^2$

$= \frac{-5}{4} + \frac{6,4}{4} \times \frac{16}{40,96}$

$= 5$

✓**CA** cot θ
✓**CA** cosec θ
✓**CA** Sin θ
✓**CA** Simplification of/ *Vereenvoudiging van* $\sin^2\theta$
✓**CA** Answer

(5)

3.1.3

$\cos\theta = \frac{5}{6,4}$

$\theta = 360° - \cos^{-1}\left(\frac{5}{6,4}\right)$

$= 360° - 38,624...°$

$= 321,4°$

✓**A** Ratio/*verhouding*
✓**CA** Reference ∠/*verwysings* ∠
✓**A** 4th Quadrant/*kwadrant*
✓**CA** Answer, rounded/*antwoord afgerond*

(4)

3.2

$\sin(180° + x)\tan(360° + x)\cos(180° - x)$

$= (-\sin x)(\tan x)(-\cos x)$

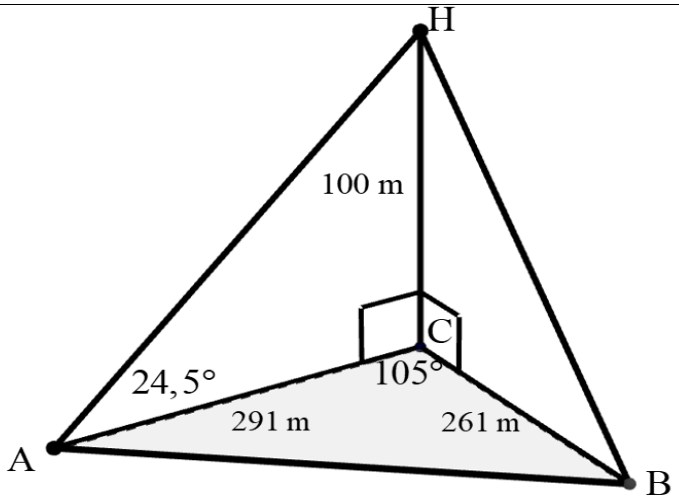
$= (-\sin x)\left(\frac{\sin x}{\cos x}\right)(-\cos x)$

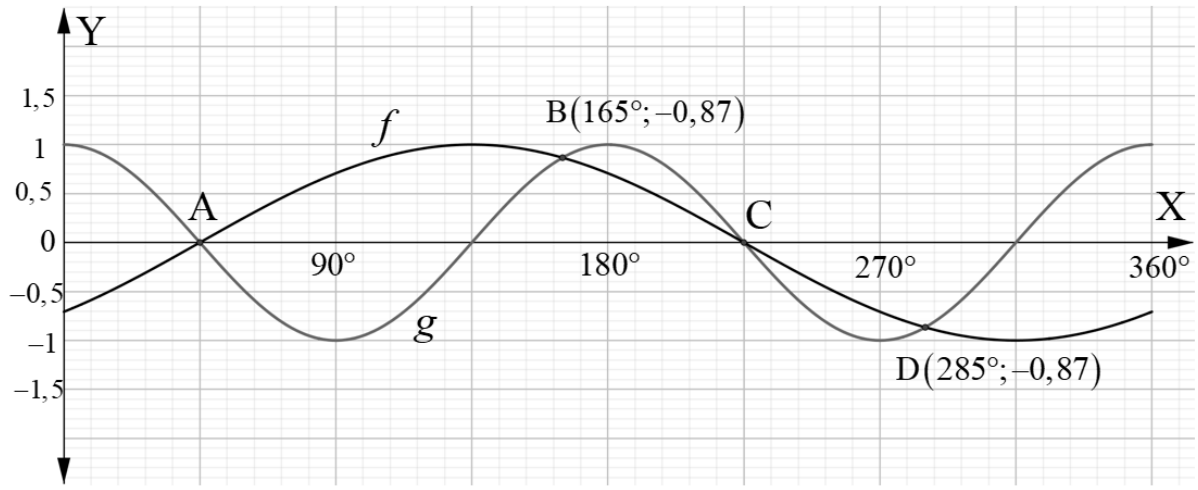
$= \sin^2 x$

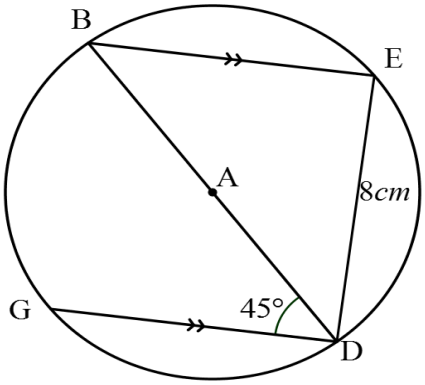
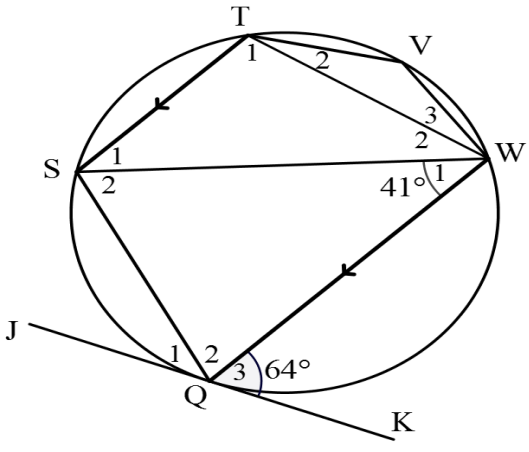
✓**A** $\sin x$
✓**A** $\tan x$
✓**A** $-\cos x$
✓**A** $\frac{\sin x}{\tan x}$
✓**CA** answer/*antwoord*

(5)

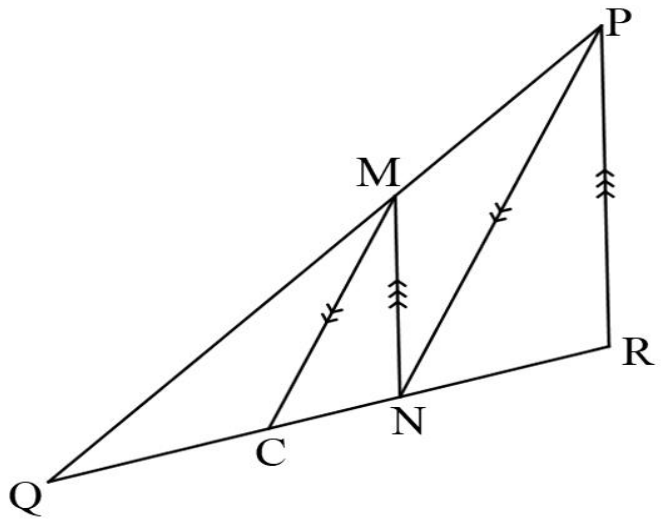
3.3	$\tan^2 5x$	✓**A** $\tan^2 5x$	(1)
3.4	$2\tan(x-23°)+5=0$ $\tan(x-23°)=-\frac{5}{2}$ $\text{Ref}\angle = \tan^{-1}(2,5) = 68,19...°$ $x-23° = 180° - 68,19...°$ OR 360°- 68,19...° $x = 111,8...° + 23°$ OR $291,80...° + 23°$ $= 134,8°$ or $314,8°$	✓**A** RHS = -2,5 ✓**CA** Ref $\angle$ / *verwys* $\angle$ ✓**CA** 2nd quadrant / *kwadrant* ✓**CA** 3rd quadrant / *kwadrant* ✓**A** Adding/ *tel* 23° by ✓**CA** Answers / *antwoorde*	(6)
			[25]

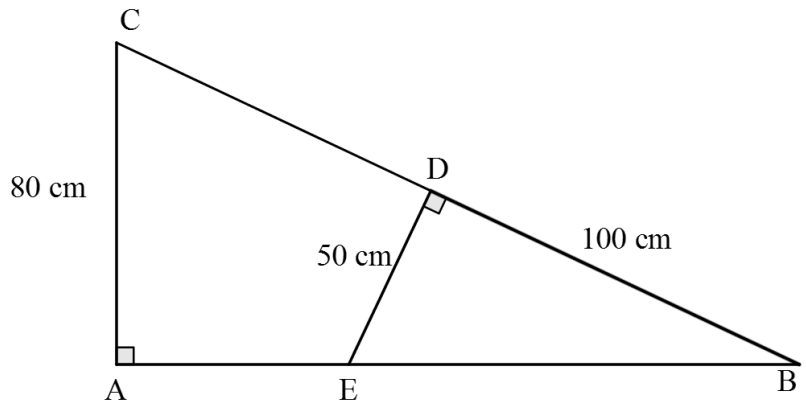
QUESTION/*VRAAG* 4			
			
4.1	In ΔAHC: $\frac{AH}{HC}=\text{cosec}24{,}5°$ $AH=\frac{100}{\sin 24{,}5°}$ ≈ 241 m	✓**A** Ratio/*verhouding* ✓**A** Substitution/*vervanging* ✓**CA** Rounded answer/*afgeronde antwoord*	(3)
4.2	In ΔBHC: $\tan B=\frac{HC}{BC}$ $=\frac{100}{261}$ $=0{,}383141...$ $\hat{B}=\tan^{-1}(0{,}383141...)$ $\approx 21°$	✓**A** Ratio/*verhouding* ✓**A** Substitution/*vervanging* ✓**CA** Rounded answer/*afgeronde antwoord*	(3)
4.3	$AB^2=AC^2+BC^2-2AC\times BC\cos A\hat{C}B$ $=219^2+261^2-2\times 219\times 261\times\cos 105°$ $=145669{,}6756$ $AB=381{,}6669...$ ≈ 382 m	✓**A** cos rule/*reël* ✓**CA** substitution/*vervanging* ✓**CA** simplification/*vereenvoudiging* ✓**CA** answer/*antwoord*	(4)
4.4	Area ΔABC $=\frac{1}{2}ab\sin C$ $=\frac{1}{2}\times 261\times 219\times\sin 105°$ $=3000847{,}5$ $\approx 3\,000\,848$ m^2	✓**A** Area rule/*reël* ✓**CA** Substitution/*vervanging* ✓**CA** value of Area/*waarde van Oppv.*	(3)
			[13]

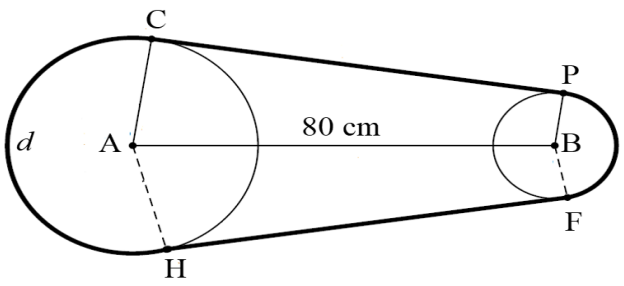
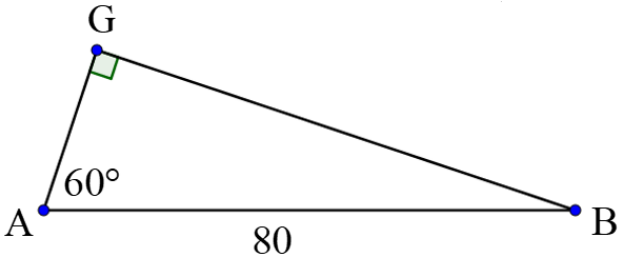
QUESTION/*VRAAG* 5			
			
5.1	Period/Periode = $\frac{360°}{2} = 180°$	✓**A** Answer/*antwoord*	(1)
5.2	A(45°; 0) & C(225°; 0)	✓**A** A(45°; 0) ✓**A** C(225°; 0)	(2)
5.3	$90° < x < 180°$	✓**A** End points / *endpunte* ✓**A** Notation / *notasie*	(2)
5.4	$45° < x < 165°$ and/*en* $225° < x < 285°$	✓**A**✓**A** first/*eerste interval* ✓**A**✓**A** second/*tweede interval*	(4)
5.5	$y \in [-3;3]$ *or* $-3 \leq y \leq 3$	✓**A** critical values / *kritiese waardes* ✓**A** notation / *notasie*	(2)
			[11]

QUESTION/VRAAG 6				
6.1	Supplementary / *Supplementêre* **OR/*OF*** Add up to 180°		✓completed statement/ *voltooide bewering*	(1)
6.2				
	6.2.1	$\hat{E} = 90°$ (∠ in semi-circle/sirkel)	✓**S** ✓**R**	(2)
	6.2.2	$B\hat{D}E = 45°$ (co-int∠s ;BE//GD) (binne∠e; BE//GD)	✓**S** ✓**R**	(2)
	6.2.3	BE = ED = 8 cm (sides opp. = angles) (*sye teenoor = hoeke*)	✓**S** ✓**R**	(2)
	6.2.4	$B\hat{G}D = 90°$ (∠ in semi-circle) ∴ BGDE is a rectangle (All ∠s = 90°) ∴ BGDE is a square (diagonals bisect/halveer at 45°) ∴ GD = 8 cm (all sides/sye =)	✓**SR** ✓**SR** ✓**SR** ✓**S**	(4)
6.3				
	6.3.1	$\hat{S}_1 = 41°$ (alt ∠s/verw∠e ; TS // WQ)	✓**S** ✓**R**	(2)
	6.3.2	$\hat{V} = 139°$ (opp ∠s of cyclic quad) (teenoorst ∠e van kdvh)	✓**S** ✓**R**	(2)
	6.3.3	$\hat{S}_2 = 64°$ (tan-chord/koord)	✓**S** ✓**R**	(2)
				[17]

QUESTION/VRAAG 7

7.1		Divides the other two sides proportionally/*Verdeel aan ander twee sye eweredig*	✓completed statement/ *voltooi bewering*	(1)
				
7.2	7.2.1	Let/*Laat* QC = *x*. $\frac{QC}{CN}=\frac{QM}{MP}$ (prop th/ewer st; MC // PN) $\frac{x}{2,86}=\frac{3}{2}$ $x=\frac{3\times 2,86}{2}$ $=4,3$ cm	✓**S** ✓**R** Proportionality/ *eweredigheid* MC // PN ✓**S** Set up proportion/*Op stel van eweredigheid* ✓**S** Simplification / *vereenvoudiging* ✓**S** Answer/*antwoord*	(5)
	7.2.2	Let/*Laat* NR = *y*. $\frac{QN}{NR}=\frac{QM}{MP}$ (prop th/ewer. st; MN // PR) $\frac{7,15}{y}=\frac{3}{2}$ $y=\frac{2\times 7,15}{3}$ $=4,8$ cm	✓**R** Proportionality/ *eweredigheid* MN // PR ✓**S** Set up proportion/*Op stel van eweredigheid* ✓**S** Simplification / *vereenvoudiging* ✓**S** Answer / *antwoord*	(4)

7.3				
	7.3.1	$\hat{B}$ is common/gemeenskaplik $B\hat{D}E = 90° = \hat{A}$ $\therefore \Delta BDE \text{ /// } \Delta BAC$ (AAA)	✓**S** ✓**S** ✓**R**	(3)
	7.3.2	$BE = \sqrt{50^2 + 100^2}$ (Pyth) ≈ 112 cm Let/*Laat* AE = x cm $\frac{BD}{BA} = \frac{DE}{AC}$ (/// Δs) $\frac{100}{x+112} = \frac{50}{80}$ $x + 112 = \frac{80 \times 100}{50}$ $x = 48$ cm	✓**S** BE = 112 ✓**S** Proportionality/ *eweredigheid* ✓**S** setup proportion/ *stel op eweredigheid* ✓**S** value of AE/waarde van AE	(4)
	7.3.3	$\frac{\text{Area } \Delta BDE}{\text{Area } \Delta BAC} = \frac{\frac{1}{2} \times DE \times DB}{\frac{1}{2} \times AC \times AB}$ $= \frac{50 \times 100}{80 \times 160}$ $= 0,39$	✓**S** formulae / *formule* ✓**S** substitution/ *vervanging* ✓**S** value of ratio/ *waarde van verhouding*	(3)
	7.3.4	Area AEDC = Area ABC – Area DBE = 6400 – 2500 = 3900 cm^2	✓**MA** ✓**CA** value of area / *waarde van opperv*	(2)
				[22]

QUESTION/VRAAG 8				
8.1				
	8.1.1	reflex $C\hat{A}E = \frac{2}{3} \times 360°$ $= 240°$	✓**A** Multiply by/*maal met* $\frac{2}{3} \times 360°$	(1)
	8.1.2	obtuse $C\hat{A}E = 360° - 240°$ $= 120°$ $C\hat{A}B = 60°$	✓**S** ✓**S**	(2)
	8.1.3	$d = s = r\theta$ $= 50 \times 240° \times \frac{\pi}{180}$ $= \frac{200\pi}{3}$ ≈ 209 cm	✓**A** Formula / *formule* ✓**A** Multiply *by/maal met* $\frac{\pi}{180°}$ ✓**A** Substitution/ vervaning ✓**CA** Answer/antwoord ✓**CA** Rounding/afronding	(5)
	8.1.4	 $A\hat{C}P = 90°$ (tan ⊥ radius) $\hat{G} = 90°$ (corrsp ∠s; BG || CP) $\frac{GB}{80} = sin\,60°$ $GB = \frac{\sqrt{3}}{2} \times 80$ $= 40\sqrt{3}$ ≈ 69 cm CP = GB = 69 cm	✓**A** $\hat{G} = 90°$ Ratio/ *verhouding* ✓**A** Ratio / verhouding ✓**CA** Simplification / vereenvoudiging ✓**CA** Answer / antwoord ✓**A** CP = GB	(5)
		OR/*OF* GA = AC – BP (opp sides of rectangle/ *teenoost sye van reghoek*) = 50 – 10 = 40 cm GB ≈ 69 cm (Pyth) CP = GB = 69 cm	**OR/*OF*** ✓**A** $\hat{G} = 90°$ Ratio / verhouding ✓**A** GA = 40 cm ✓**CA** Pythagoras ✓**CA** Answer / *antwoord* ✓**A** CP = GB	(5)

	8.1.5	Length of belt /lengte van band = CH + HF + FP + CP = 209 +69 + 21 + 69 = 368 cm	✓**A** HF = CP ✓**CA** Answer / *antwoord*	(2)
8.2	$d = 19$ $x = 13$ $4h^2 - 4dh + x^2 = 0$ $4h^2 - 4(19)h + 13^2 = 0$ $4h^2 - 76h + 169 = 0$ $h = \frac{-b \pm \sqrt{b^2 - 4ac}}{2a}$ $= \frac{76 \pm \sqrt{(-76)^2 - 4(4)(169)}}{2(4)}$ $= \frac{76 \pm \sqrt{3072}}{8}$ $\approx$ 3 cm and/en 16 cm	✓**A** Formula / formule ✓**A** Substitusie / *vervanging* ✓**M** Standard form / *standaardvorm* ✓**A** Quadratic formula / *kwadraties formule* ✓**CA** Subsitutisie / *vervanging* ✓**CA** Answers/*antwoorde* ✓**A** Rounding/*Afgeronding*	(7)
				[22]

QUESTION/*VRAAG* 9				
9.1	9.1.1	$\omega = 2\pi n$ $= 2\pi(35)$ $= 70\pi \approx 219,9$ rad/s	✓**A** Formula / *formule* ✓**A** Substitution / *vervanging* ✓**CA** Answer / *antwoord*	(3)
	9.1.2	$D = 40$ cm $= 0,4$ m $v = \pi Dn$ $= \pi(0,4)(35)$ $= 14\pi$ $= 43,98$ m/s	✓**A** Convert to/*herlei tot* m ✓**A** Formula / *formule* ✓**CA** Substitution / *vervanging* ✓**CA** Answer / *antwoord*	(4)
9.2		$V_{\text{rectangluar/reghoekig}} = l \times b \times h$ $= 5 \times 7 \times 12$ $= 420$ cm^3 $V_{\text{cylinder/silinder}} = \pi r^2 h$ $420 = \pi r^2 (60)$ $r^2 = \frac{420}{60\pi} = 2,228...$ $r = 1,492...$ diameter/middellyn $= 2r$ $\approx 2,99$ cm	✓**A** Subst. into formula / *formule rect* ✓**CA** Answer / *antwoord* ✓**A** Formula / *formule* cylinder/*silinder* ✓**CA** Substitution / *vervanging* ✓**CA** Value of diameter / *waarde vir middellyn*	(5)
				[12]
QUESTION/*VRAAG* 10				
		$A_T = a\left(\frac{o_1 + o_n}{2} + o_2 + o_3 + o_4 + ... + o_{n-1}\right)$ $= 5\left(\frac{8+3}{2} + 10 + 9 + 9 + 4\right)$ $= 187,5\,\text{cm}^2$ **OR/OF** $A_T = a(m_1 + m_2 + m_3 + \ldots + m_n)$ $= 5\left(\frac{8+10}{2} + \frac{10+9}{2} + \frac{9+9}{2} + \frac{9+4}{2} + \frac{4+3}{2}\right)$ $= 187,5\,\text{cm}^2$	✓formula ✓value of *a* ✓substitution ✓Area **OR/OF** ✓formula / *formule* ✓value of /*waarde van a* ✓substitution/*vervanging* ✓Area	(4)
			TOTAL/ *TOTAAL*:	**150**