Province of the
EASTERN CAPE
EDUCATION

EXAMINATIONS AND ASSESSMENT CHIEF DIRECTORATE
Home of Examinations and Assessment, Zone 6, Zwelitsha, 5600
REPUBLIC OF SOUTH AFRICA, Website: www.ecdoe.gov.za

2020 NSC CHIEF MARKER'S REPORT

SUBJECT:		**TECHNICAL MATHEMATICS**
PAPER:		**2**
DURATION OF PAPER:		**3 hours**

SECTION 1: (General overview of Learner Performance in the question paper as a whole)

The number of Eastern Cape NSC, SC and MEO candidates that wrote the final paper NSC Technical Mathematics 2020 paper was 1920.

A sample of 100 scripts was collected during the marking process. The selected sample comprises of scripts that were moderated by the Internal Moderator and/or Chief Marker, and/or the Senior Marker as well as some unmoderated scripts.

The graphical representation in the report will be bases on the 100 sampled candidates' responses which were selected as depicted in the next table.

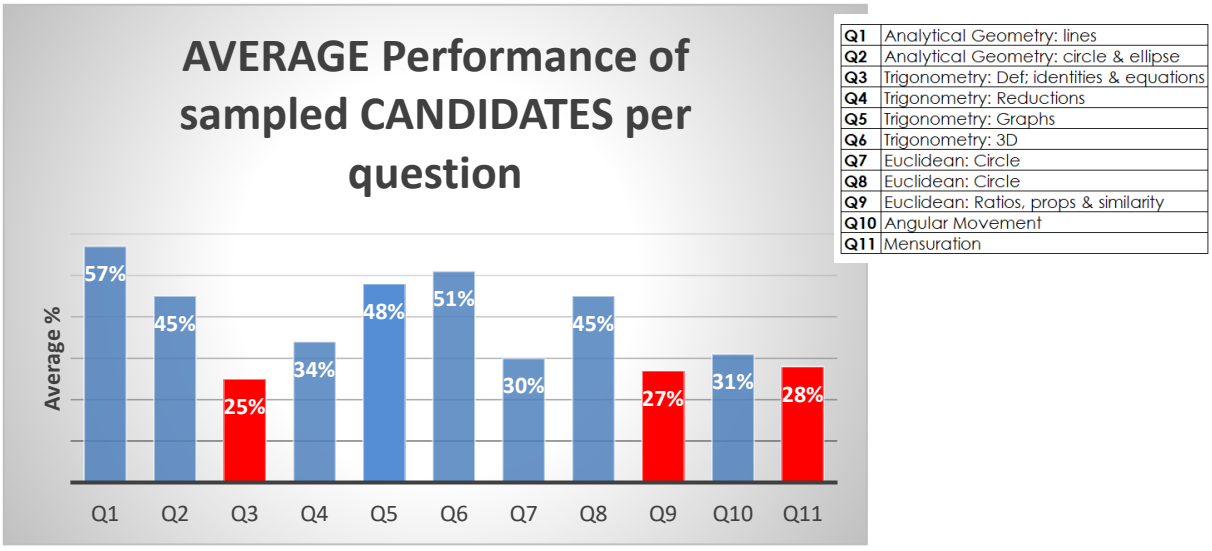
	[0; 44]	[45; 59]	[60; 74]	[75; 89]	[90; 104]	[105; 119]	[120; 150]	TOTAL
Required	15	15	20	20	20	5	5	100
Actual	36	18	18	15	6	5	2	100
Perc	36%	18%	18%	15%	6%	5%	2%	

The 2020 cohort performed poorer than the past two years. There was a drop of almost 6% in the raw pass rate in comparison with 2019 and almost a 13% drop in comparison with 2018 raw pass rate.

There are too many centres where all the learners are failing the paper and, in some instances, where the highest mark achieved by the centre is below 10% of the paper, that is less than 15 marks out of 150. On top of that, too many candidates achieved no marks for the question paper

Currently the Technical Mathematics Paper 2 fails the aim of the NCS as quoted in the CAPS documents:
"*(d) The National Curriculum Statement Grades R – 12 aims to produce learners that are able to:*
• *identify and solve problems and make decisions using critical and creative thinking;*"
The bulk of the learners still performs at level 1

The average performance of the sampled 100 candidates is depicted in the graph below



Question 1 was the best performing question as expected, with Question 3, 9 and 11 being the worst performing questions of the sampled candidates. The performance per questions were relatively close to each other. Question 1 and 6 with an average above 50%. Questions 2, 5 and 8 in the 40% with Questions 4, 7 and 10 with average percentage in the range of 30%, whilst the poorest performing questions were performing in the upper 20%. Question 3 (Trig definitions and equations) replaced Question 9 (Ratios, proportions, and similarities) as the worse performing question.

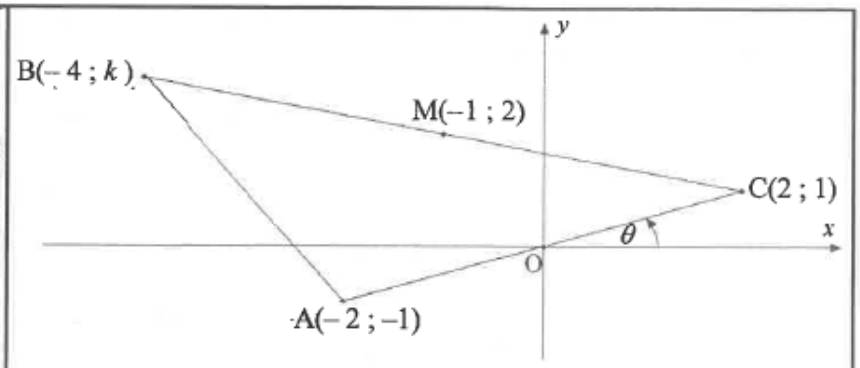
Analytical Geometry was the best performing topic, followed by Trigonometry and Euclidean Geometry with Angular Movement and Measurements completing at the back.

SECTION 2: Comment on candidates' performance in individual questions
(It is expected that a comment will be provided for <u>each question</u>).

QUESTION 1 [Total marks 15]

The picture below shows the triangular rooftop of a building. The diagram next to the picture represents the triangular rooftop in the Cartesian plane with origin O.
In the diagram, A(–2 ; –1), B(– 4 ; k) and C(2 ; 1) are the vertices of ΔABC with M(–1 ; 2) the midpoint of BC.
The angle of inclination, θ, is the angle between AC and the positive x-axis.

PICTURE



DIAGRAM

1.1 Determine:

1.1.1 The numerical value of k (2)

1.1.2 The gradient of AC (2)

1.1.3 The size of θ (in degrees) (2)

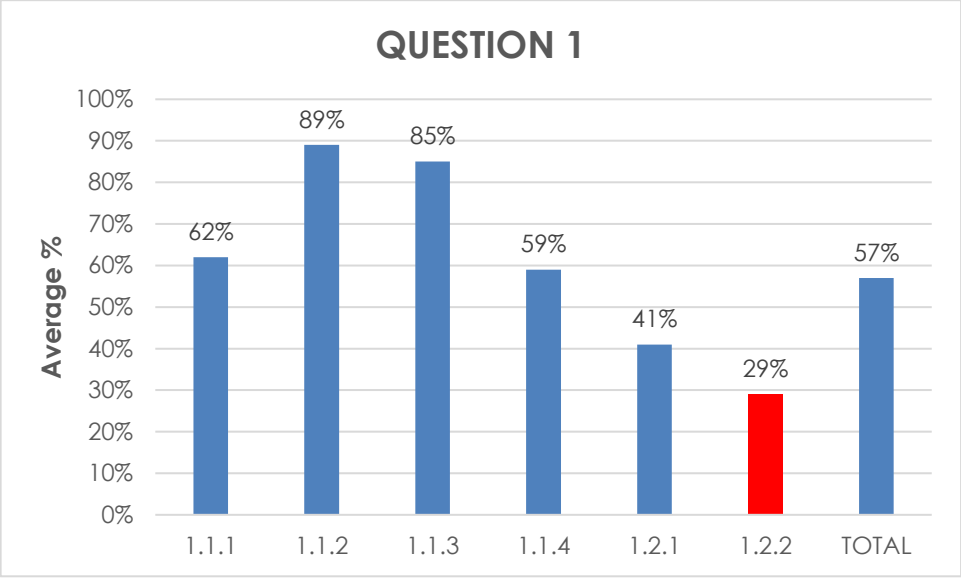
1.1.4 The equation of straight line BC in the form $y = ...$ (3)

1.2 If O is the midpoint of AC, use analytical geometry methods to show that:

1.2.1 MO ∥ BA (3)

1.2.2 $MO = \frac{1}{2} BA$ (3)

[15]



- Easy question to lead the paper in
- Learners found Q1.2.2 to be the most difficult to answer in this question
- Average percentage for this question is 57% from the sample candidates
- Q1.1.1 not well answered as one would have expected
- Q1.1.2 & Q1.1.3 were well answered
- Q1.2.2 was the weakest answered question

Common errors and misconceptions

i.) Q1.1.1- many learners struggled to apply the reverse application of the midpoint formula

ii.) Many learners lost the conclusion marks in Q1.2

iii.) Learners incorrectly switch between the distance and gradient formulae

iv.) Learners confuse the concepts of parallel and perpendicular

v.) Determining the equation of straight line – learners substitute coordinates from different point, e.g., x-coordinate of B and the y-coordinate of C

vi.) In the gradient formula, far too many learners switch the coordinates, instead of calculating $\frac{\Delta y}{\Delta x}$, they calculate incorrectly $\frac{\Delta x}{\Delta y}$. These are basic mistakes that can be avoided.

vii.) Q1.2.2 – learners calculate BA, but instead of calculating MO they calculate $\frac{1}{2}BA$. Further, many candidates calculate everything correctly, but they did not conclude

Other candidates did not realise the application of the distance formula, whilst some learners did not apply Analytical methods, and could not score any marks

Suggestions for improvement

i.) Expose learners to "reverse" mathematics – e.g., they must know how to calculate the midpoint, M, for example of AB. The "reverse" mathematics is when for example the coordinates of M and say A are given and now, they must calculate the coordinates of B.

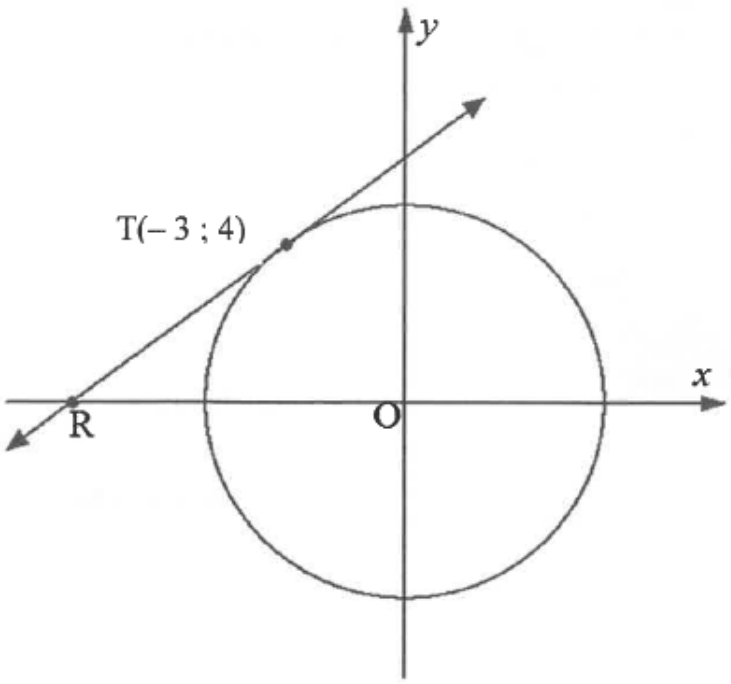
ii.) Expose learners to "*show that …*" type of questions.

iii.) Clearly distinguish between when are lines parallel and when they are perpendicular and how do you show that lines are parallel or perpendicular

iv.) Learners must be reminded to conclude when they are asked to show that some statement is true

QUESTION 2 [Total marks 11]

2.1 The diagram below shows a circle with centre O at the origin.
Point T(–3 ; 4) lies on the circle. Tangent RT to the circle passes through T.



2.1.1 Calculate the length of the diameter of the circle. (2)

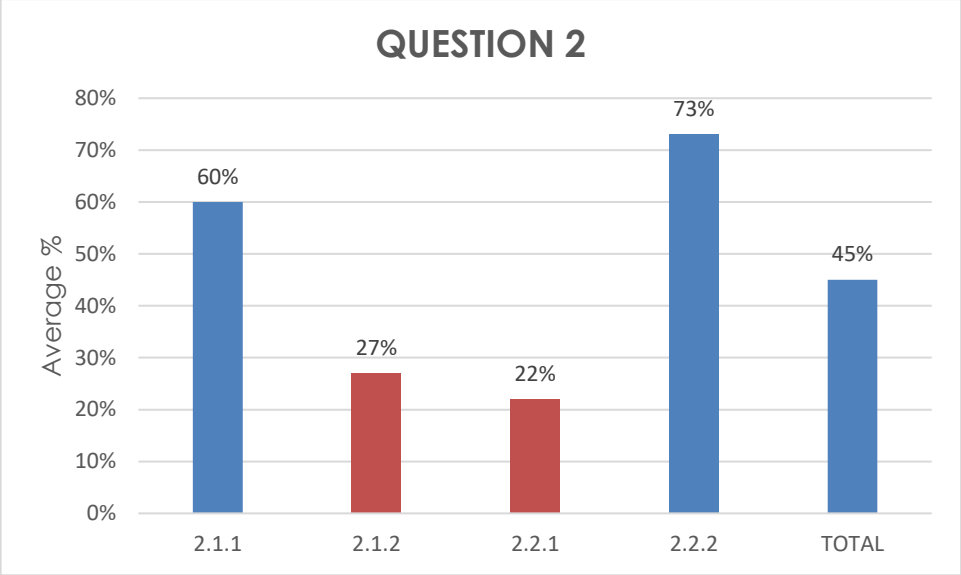
2.1.2 Prove, showing ALL calculations, that the straight line defined by the equation $4y - 3x - 25 = 0$ and which passes through point T is the tangent to the circle. (5)

2.2 Given the ellipse with the following properties:

- Centre at the origin
- Distance between the x-intercepts is 12 units
- Range of $-3{,}5 \leq y \leq 3{,}5$

2.2.1 Write the equation of the ellipse in the form $\frac{x^2}{a^2}+\frac{y^2}{b^2}=1$ (1)

2.2.2 Sketch the graph of the ellipse. Clearly show the intercepts with the axes. (3)
[11]



- Many candidates did not attempt Q2.1.2
- Most got the equation of the ellipse incorrect in Q2.2.1
- Sketching the ellipse in Q2.2.2 was the best answered sub-question
- A matter of concern is that there were many candidates that could not sketch the ellipse correctly and even candidates that scored zero for the paper, which are not part of the sampled candidates

Common errors and misconceptions

i.) Q2.1.1 – many calculates calculate the equation of the circle only and did not deduce the length of the diameter

ii.) Q2.1.2 – many lost the mark for not concluding that the determined equation is a tangent

iii.) Q2.2.1 – Many did not write it in the required format. A common mistake was that learners wrote the required equation as $\frac{6^2}{a^2}+\frac{3,5^2}{b^2}=1$ or in the format $\frac{a^2}{x^2}+\frac{b^2}{y^2}=1$ (although correct format given in the question paper). Many assumed that $a=12$

Suggestions for improvement

i.) Emphasise the difference between radius and diameter and their relationship

ii.) Teachers should expose learners to the different ways of asking the same question

a. Normal – determine the equation of a tangent to the circle through a point, straight forward as per CAPS document

b. In this paper – given a straight line – prove that the line is the tangent

iii.) Basic concepts like domain and range must not be neglected in the teaching process

QUESTION 3 [Total marks 18]

3.1 If $P = 146{,}31°$ and $Q = 91{,}58°$, determine the value of $\sqrt{\frac{2}{\tan(P+Q)}}$ (2)

3.2 Given: $\tan\beta = -\frac{2}{3}$ where $\cos\beta > 0$ and $\sin\theta = \frac{5}{13}$ where $\theta \in [90° ; 360°]$

Determine, with the aid of diagrams and WITHOUT using a calculator, the value of EACH of the following:

3.2.1 $2\cot\beta + 1$ (2)

3.2.2 $\sin\beta \cdot \sec\theta$ (6)

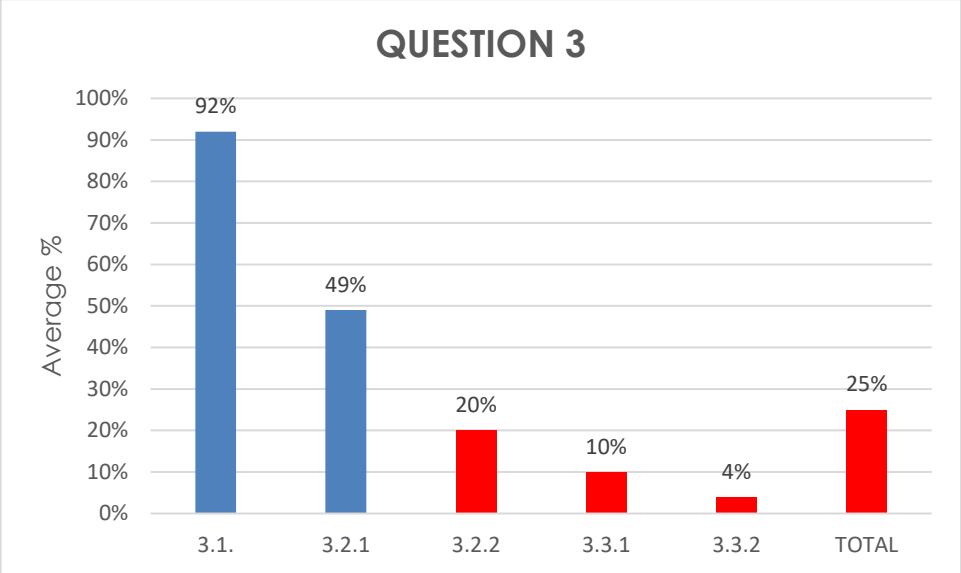
3.3 Given: $\sin A + 2\cos^2 A = 2$

3.3.1 Express the above equation in simplified form, in terms of $\sin A$. (3)

3.3.2 Hence, or otherwise, determine the value(s) of A if:

$\sin A + 2\cos^2 A = 2$ for $A \in [0° ; 180°)$ (5)

[18]



This was the worst performing question in the paper

Q3.1 – The expectation was a 100% achievement, seeing that is a calculator usage problem.

Q3.2 – candidates were caught off guard to draw two diagrams in different quadrants

Q3.2.2; Q3.3 – the worst performing sub-questions with Q3.3.2 the worst performing sub-question in the paper

Common errors and misconceptions

i.) Q3.1 – many struggled to use the calculator with confidence

ii.) Q3.2 – many could not draw the diagrams correctly – not realising that it should be separate diagrams. Many did not even attempt to draw the diagrams. Many drew them in the 2nd quadrant.

iii.) Q3.3 – many learners failed to understand that they need to factorise trigonometric equations (of which it was the first time to have two different solutions) – this was an unexpected quadratic equation, leading to solving simple trigonometry equation

iv.)	Q3.3.1 – Misunderstanding of the concept "*in terms of*" - many candidates made sin A the subject of the formula – many did not recognise the square identity
v.)	Q3.3.2 - this question depended on Q3.3.1 and if they messed it up, they lost marks - many learners do not clearly understand the zero property – many learners only gave the one factor equal to zero
Suggestions for improvement	
i.)	Calculator usage is one of Specific aims of Technical Mathematics, which must specially be practice by all learners on a regular basis.
ii.)	All learners should be in the possession of a scientific calculator and deliberate intervention in using these calculators should be implemented. Calculator worksheets with a variety of problems should address this problem.
iii.)	Drawing a trigonometry sketch diagram from given information needs serious attention
iv.)	The meaning of write "*in terms of*" something must be clarified to the learners
v.)	The application of the zero property (If ab = 0 then a = 0 or b = 0) also need reemphasizing and its application across different sections of Technical Mathematics
vi.)	Learners to know all the square identities or at least be able to identify the correct identity from the formula sheet

QUESTION 4 [Total marks 11]

4.1 Simplify EACH of the following:

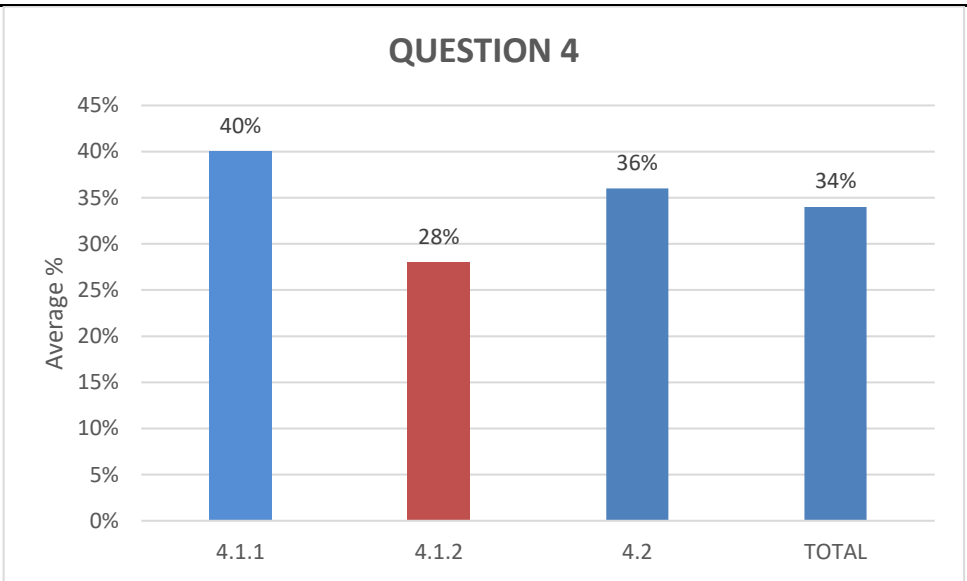
4.1.1 $1-\cos^2(2\pi-\theta)$ (2)

4.1.2 $\cos\alpha(\cot\alpha+\tan\alpha)$ (4)

4.2 Prove the following identity:

$$\frac{1}{\sin(180°+x)\cdot\sin(360°-x)}-\cot^2(180°-x)=1 \quad (5)$$

[11]



Question performed at an average percentage of 34% of the sampled candidates

Many did not answer this question

Question 4.1.2 was the worst performing sub-question of the sampled candidates

Common errors and misconceptions

i.) Q4.1.1 many learners left their answer as $\sin^2(2\pi-\theta)$

ii.) Q4.2 – many learners reduced $\cot^2(180°-x)=-\cot^2 x$, not realising the influence of squaring the ratio – many confuse the identity of $\cot x$ and $\tan x$

iii.) Learners do not know their identities AND they do not make use of the formula sheet with the given identities

Suggestions for improvement

i.) Practise enough problems on a regular basis

ii.) Revise identities

iii.) Explain all the relevant formulae on the formula sheet to the learners

QUESTION 5 [Total marks 11]

Given: Functions f and g defined by $f(x) = -\tan x$ and $g(x) = 2\sin x$ for $0° \leq x \leq 180°$

5.1 Draw sketch graphs of f and g on the same set of axes. Clearly indicate ALL the intercepts with the axes, turning points, asymptotes and end points. (6)

5.2 Write down:

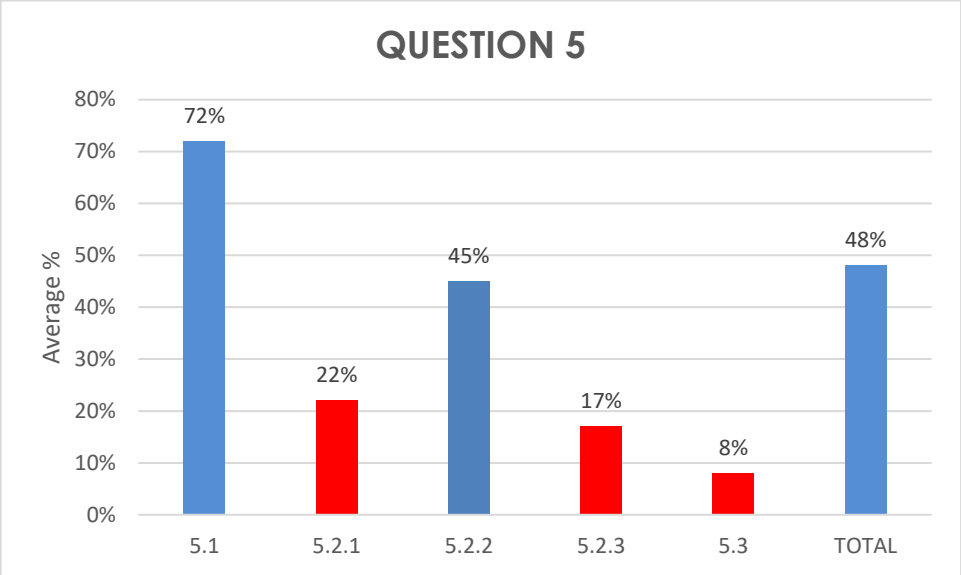
5.2.1 The range of f (1)

5.2.2 The period of g (1)

5.2.3 The equation of the asymptote of h if $h(x) = f(x + 30°)$ (1)

5.3 Determine the value(s) of x for which $f(x) \cdot g(x) < 0$ (2)

[11]



Q5.3 – worst performing sub-question – many do not have an idea how to answer this question

Q5.2.1 – this was a surprisingly under-performing question, to read off the range of the tan-graph

Q5.2.3 – poorly answered

Common errors and misconceptions

i.) Q5.1 – Many decided on a 30° intervals, where intervals of 45° would have been better, especially for the tan-graph

ii.) Q5.2.1 – many candidates indicated the interval they drew on their graph

iii.) Q5.2.2 – many wrote the period to be 180°, confusing it with the domain for the functions

iv.) Q5.2.3 – many did not realise that $y = f(x + 30°)$ means a shift to the left

v.) Q5.3 - many did not even give their answer in interval notation

Suggestions for improvement

i.) Clearly distinguish the difference between the domain and period of a function and again range and domain as well

ii.) Learners should be taught how to use the table-mode of the calculator to draw accurate graphs

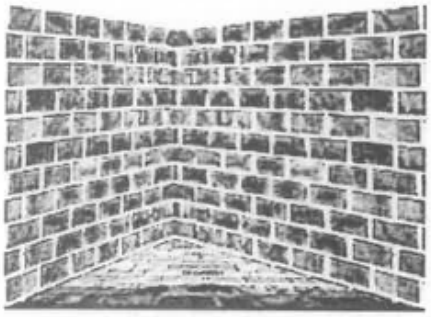
iii.) Horizontal shifting of graphs must be revised

iv.)	Please note the inconsistency in the CAPS document – page 18 the Overview and the Grade 11 Curriculum Statement
v.)	Grade 11 Curriculum statement only refers to $y = \tan x$, $y = \sin(x + p)$ and $y = \cos(x + p)$ (pg. 37)
vi.)	However, the overview goes even further that $y = \tan kx$ and $y = \tan(x + p)$ must also be studied
vii.)	The Examination guidelines of 2018 refers only to draw $y = \mathrm{a}\tan x$ and learners must know the effects of *p* in $y = \tan(x + p)$
viii.)	So, for teaching purposes consult the Overview and Curriculum statements of the CAPS document, but for examination purposes the Examination Guidelines must also be referred to
ix.)	Interpretation of graphs must be constantly incorporated in graphs revision worksheets

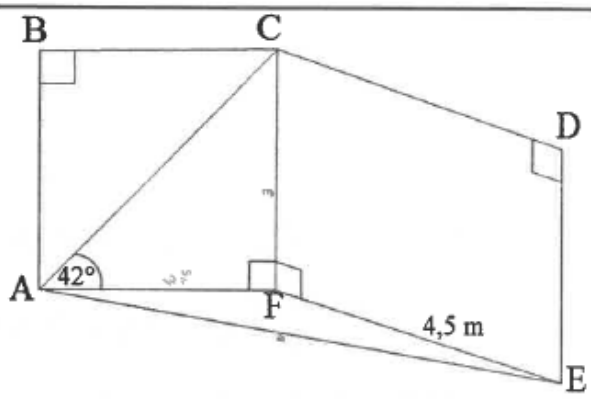
QUESTION 6 [Total marks 10]

The picture below shows two vertical rectangular walls that are not perpendicular to each other. The diagram next to it represents the two rectangular walls, ABCF and CDEF. Points A, F and E lie in the same horizontal plane and form a triangular section AFE of the floor.

Furthermore, $\hat{FAC} = 42°$, FE = 4,5 m and $CF = \frac{2}{3}FE$



PICTURE OF TWO WALLS



DIAGRAM

6.1 Write down the length of CF. (1)

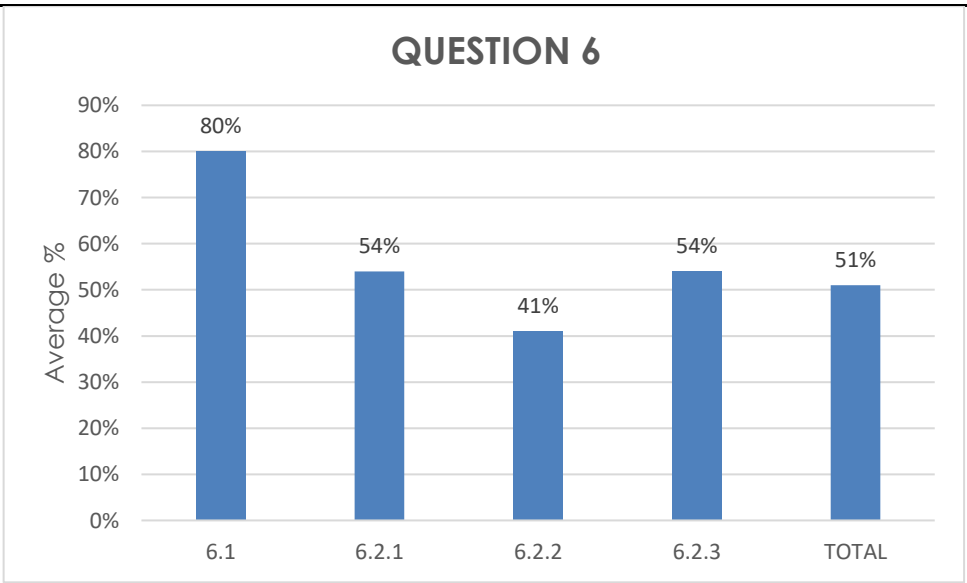
6.2 Determine, correct to ONE decimal place:

6.2.1 The length of AF (2)

6.2.2 The size of $\hat{AFE}$ if AE = 6 m (4)

6.2.3 The area of ΔAFE (3)

[10]



The sampled candidates scored at an average of 51% in this question.

Although this questions' average percentage is below the average percentage of Question 1, it is the best performing question, since not one sub-question was under-performing, if we take 30% as the pass rate per question.

Common errors and misconceptions

i.) Q6.1 – many candidates could not answer this easy question, reflecting in the number of scripts where candidates scored zero for the paper

ii.)	Q6.2.2 – learners struggled to decide which formula to use or they apply the formula incorrectly
i.)	Many learners managed to substitute correctly into the cosine rule but struggled to apply BODMAS correctly. Others realise that they made a mistake when the calculator gave them an error but struggled to recover. Many did not realise the $A\hat{F}E$ was obtuse.
iii.)	Many candidates are rounding too early
Suggestions for improvement	
ii.)	Learners must be taught how to decide which formula (cosine or sine rule) to select
iii.)	Many learners were struggling to make cos the subject of the formula
iv.)	Learners must work as accurately as possible and only round their final answer

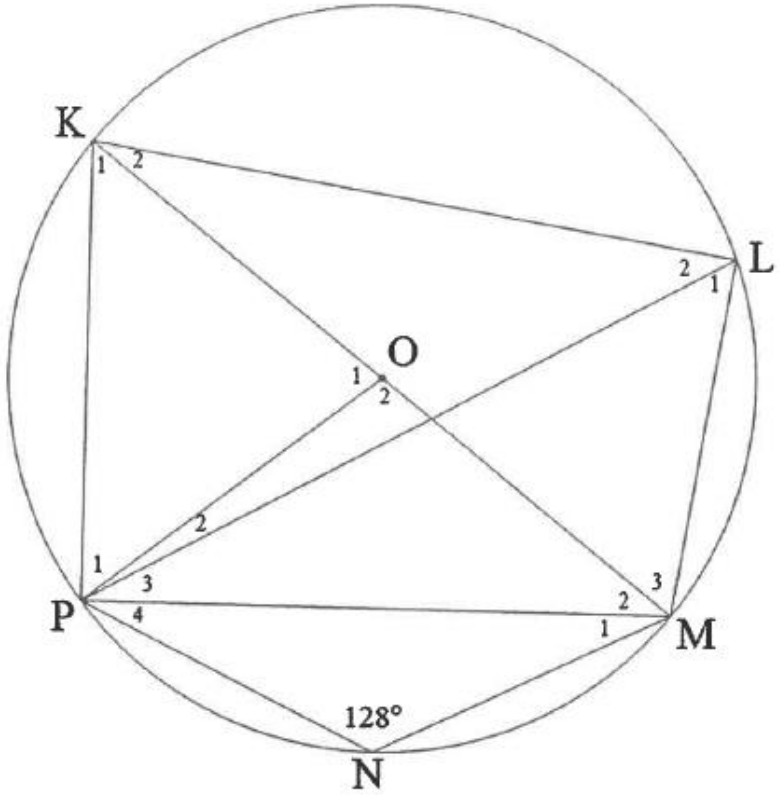
QUESTION 7 [Total marks 12]

7.1 Complete the following theorem:

Angles subtended by a chord of a circle, … , are equal. (1)

7.2 In the diagram below, O is the centre of circle KLMNP.
KOM is a diameter of the circle and chords LP and PM are drawn.

$\hat{N} = 128°$

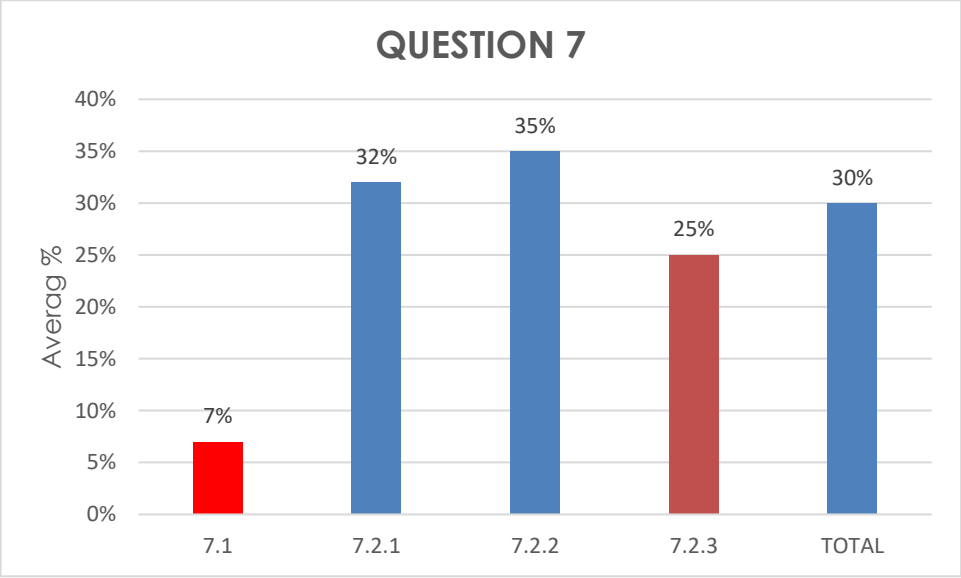


Determine, stating reasons, the size of EACH of the following angles:

7.2.1 $\hat{K}_1$ (2)

7.2.2 $\hat{L}_2$ (5)

7.2.3 $\hat{P}_2$ if $\hat{P}_3 = 29°$ (2)
[10]



- The average for this question is only 30%
- Q7.1 - Completing the theorem statement was the worst answered sub-question

Common errors and misconceptions

i.) Q7.2.1 – many learners did not recognise that KPNM is the cyclic quadrilateral and assumed that POMN is the cyclic quadrilateral and hence calculate $\widehat{O_1} = 128°$ and $\widehat{O_2} = 52°$

ii.) Candidates assumed $\widehat{L_2} = \widehat{P_3}$ because they are alternate angles

iii.) Many candidates stop by stating two angles are equal without indicating the size of the angle.

iv.) Many candidates did not realise the isosceles triangles KOP and POM

Suggestions for improvement

i.) Learners must be able to complete a theorem statement – these questions are classified as level 1 questions – use the proper reasons as provided by the Examination Guidelines

ii.) Knowing your theorems are the first path to success to Euclidean Geometry and identifying the applicable theorem in a diagram

iii.) If the centre of a circle is given, learners should immediately check if there are radii involve and mark it equal

iv.) Learners should realise that the solution to the problem lies in the given statement – they must identify the key words and which theorems/axioms/corollaries are applicable to those keywords

v.) Afterwards, learners should try to fill in the diagram using that acquired information

vi.) When setting out their solution, they need to be aware that marks are allocated as per Statement and Reason, or as a Statement/Reason combo

vii.) Normally when the Reason refers to a grade 11 or grade 12 concept, it carries a mark on its own, otherwise for a reason lower than that is carries a mark for the Statement/Reason

viii.) When the question requires to determine the size of an angle, it means there must be value attach to the angle

QUESTION 8 [Total marks 14]

8.1 Complete the following theorem:

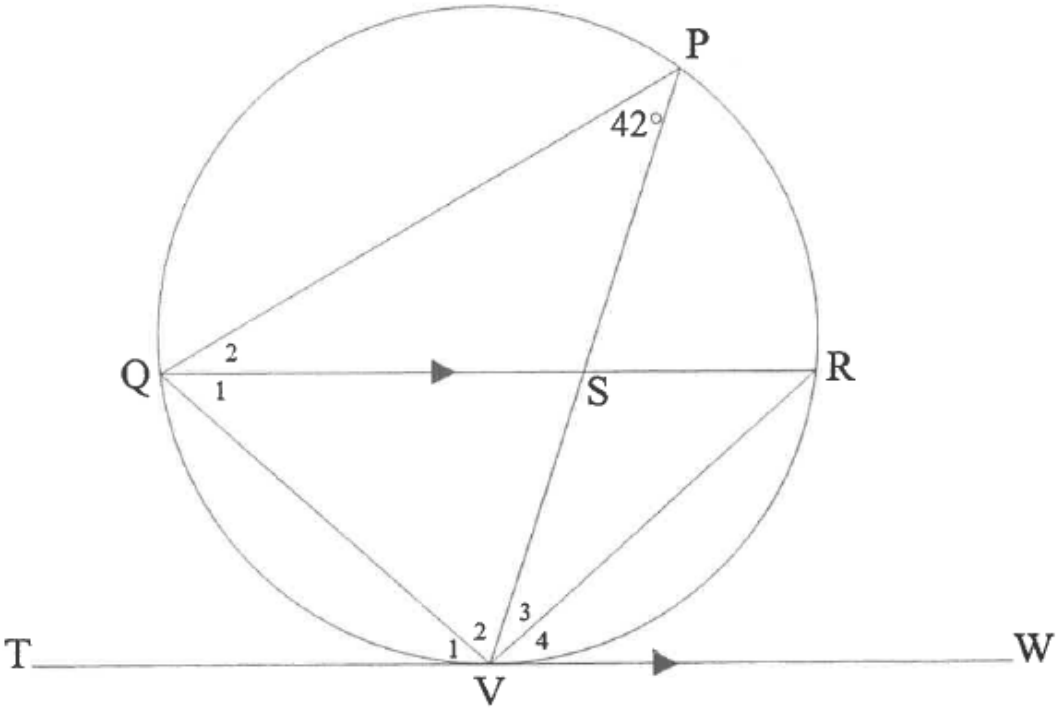
The angle between the tangent to a circle and the chord drawn from the point of contact is equal to … (1)

8.2 In the diagram below, TVW is the tangent to circle PRVQ at V.

Chords PV and QR intersect at point S.

TW || QR

$\hat{P} = 42°$



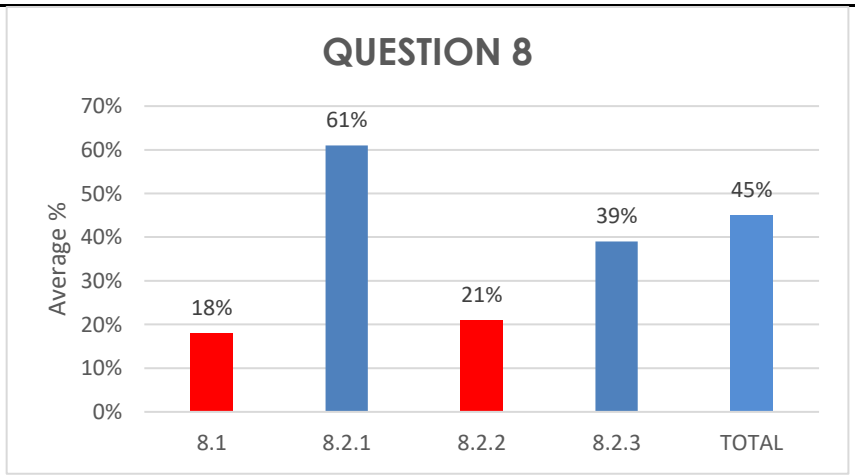
Determine, with reasons:

8.2.1 FOUR other angles each equal to 42° (6)

8.2.2 Whether QR is a diameter of the circle (2)

8.2.3 The size of $\hat{Q}_2$ if $\hat{V}_2 = 67°$ (3)

[12]



- Average of the question was 45% for the sampled candidates
- Q8.1 was also answered badly because it required the candidate the complete the theorem statement
- Many candidates attempted Q8.2.1, but did not score full marks

Common errors and misconceptions

i.) Q8.2.1 – reference to incorrect theorems was a common mistake – not indicating the parallel lines with alternate angles, some learners even mention corresponding angles

ii.) Q8.2.2 – many candidates only indicate that QR is the diameter or not without any justification

iii.) Q8.2.3 – many candidates did not use the given information that $\hat{V}_2 = 67°$. This boils down to reading the question properly

Suggestions for improvement

i.) The same suggestions for Question 7 hold for Question 8

ii.) Learners need a lot of practice to master these skills – it is not a once off event, but continual practice events

iii.) Difference between alternate and corresponding angles are simple concepts that learners must be drilled on

iv.) Expose learners also too, how do you prove or show that a chord is a diameter

v.) This problem can also be used as a small investigation to find out if it only holds for $\hat{P} = 42°$:

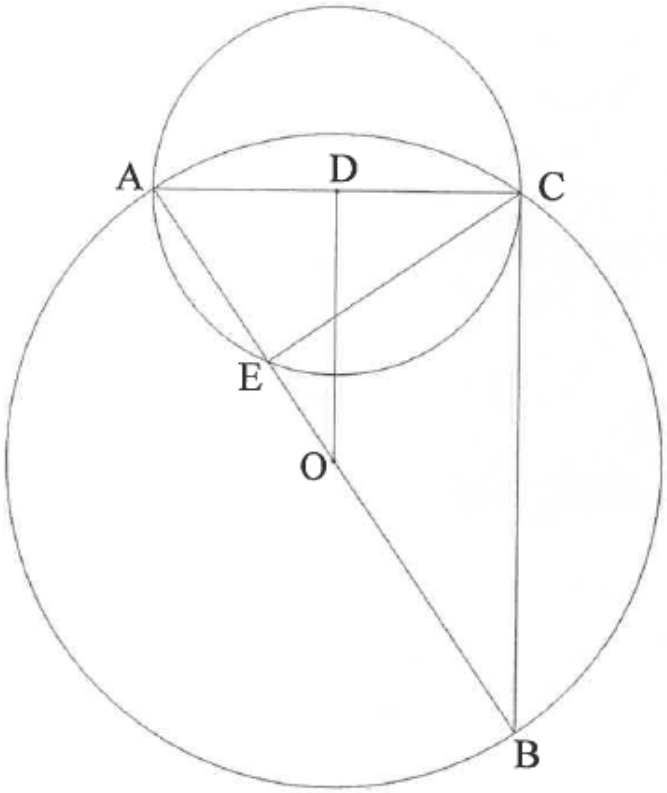
- Request learners to draw tangent TVW with V the point of contact
- Q any random point on the circumference of the circle
- Draw QR ∥ TVW
- Choose any point P on the circumference of the circle
- Complete the diagram as in the questions
- Learners to measure their angles
- Findings:
  - Learners will find four (4) angles equal with different sizes
  - Everybody should deduce that QV = VR
- Learners could now be asked to verify their answers by using appropriate theorems and the logical set out of their solutions, moving from the given to what needs to be calculated.

QUESTION 9 [Total marks 15]

9.1 Complete the following theorem:

The perpendicular bisector of a chord of a circle passes through … (1)

9.2 In the diagram below, O is the centre of circle ABC and D is the centre of circle ACE.
AB and AC are the diameters of the larger and smaller circles respectively.
BC is the tangent of the smaller circle at C.
DO = 6 units and AC = 8 units.



9.2.1 Give TWO different reasons why $\hat{ACB} = 90°$ (2)

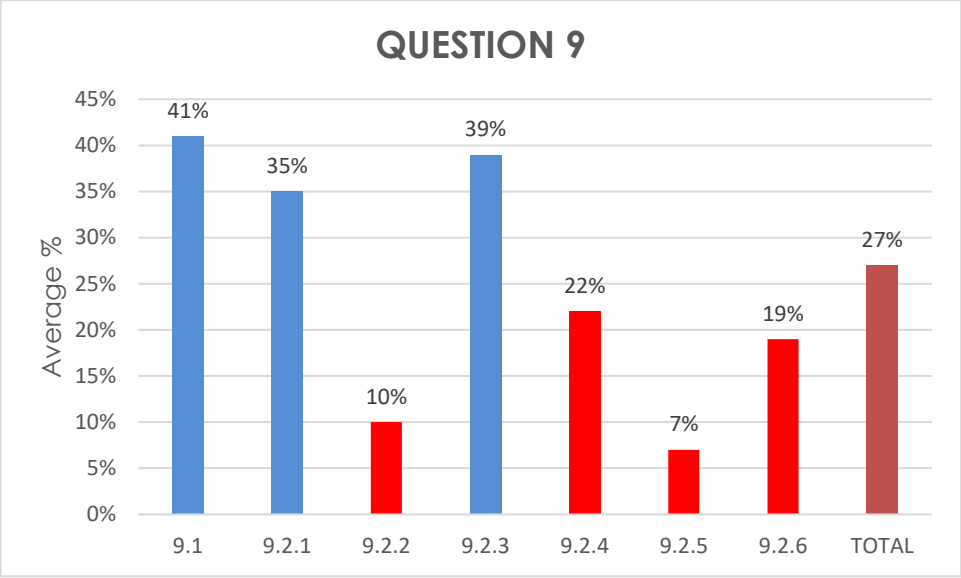
9.2.2 Give a reason why DO || CB. (1)

9.2.3 Determine the length of diameter AB. (4)

9.2.4 Prove that $\Delta ABC \,|||\, \Delta ACE$. (3)

9.2.5 Show that $AC^2 = AB \times AE$ (1)

9.2.6 Determine the length of AE. Leave your answer in simplified surd form. (3)
[15]



Second worst question answered according to sampled candidates

Q9.2.2; 9.2.5 ; 9.2.6 & 9.2.4 – the worst performing sub-questions

Q9.2.5 – in most cases the question was left unanswered

Common errors and misconceptions

i.) Q9.2.1 – most candidates only spotted the angle in the semi-circle. They did not see the radius perpendicular to the tangent

ii.) Q9.2.2 – candidates did not recognise the midpoint theorem – many observed that $OD \perp AC$ but could not provide the reason and to move from there

iii.) Q9.2.4 – learners mixed the proving of similarity with congruence and even provides congruency reasons for similarity

iv.) Q9.2.5 – the proportion was not provided

v.) Q9.2.6 – many learners did not realise that they can use the statement provided in Q9.2.5 to solve this question

Suggestions for improvement

i.) If learners see the word centres, then immediately they must look for the radius or radii, before the look for the theorems relating to the centre

ii.) Teach learners to "break-up" the diagram – this means ample exercises for the eyes to get use to

iii.) To prove similar triangles – when using angles, there will always be a common angle – learners must start there, then they must look for which other angle is equal and then the third angle will be equal automatically, further they do not have to mention the third angle provide they provide the reason AAA

iv.) There are only two ways of proving triangles similar, either showing the angles of the triangles are equal or that the sides are in proportion

v.) When learners have the following information $\Delta ABC \ \|\|\| \ \Delta ACE$ they should be able to immediately deduce that $\frac{AB}{AC} = \frac{AC}{AE} = \frac{BC}{CE}$. Teachers to make sure that learners understand this

vi.) Further, expose learners to more of these types of questions

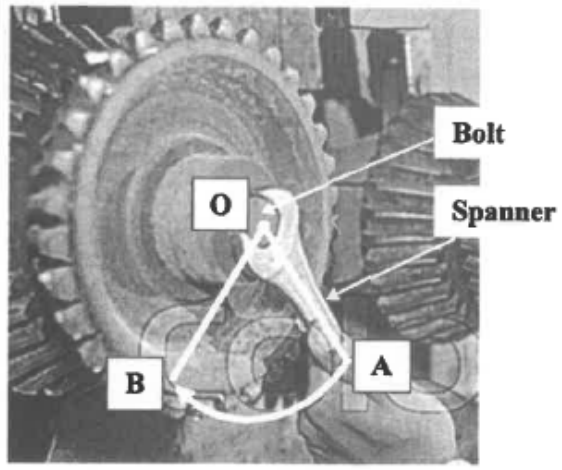
QUESTION 10 [Total marks 16]

10.1 The picture below shows a spanner being used to tighten a bolt by rotating it in a clockwise direction from point A to point B. The diagram next to the picture represents the rotation of the spanner. Point O, the centre of the bolt, is also the centre of rotation of the spanner.

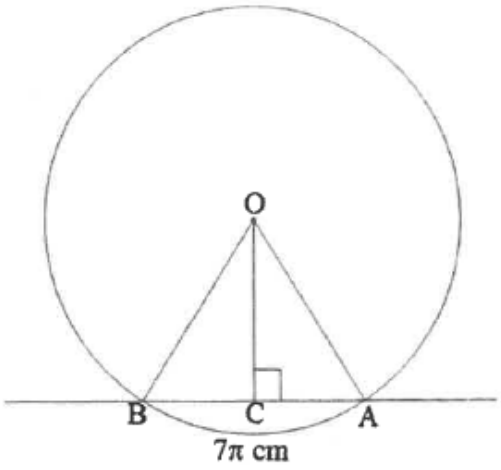
OA represents the distance from the centre of the bolt to the end of the spanner.

Reflex angle $\hat{AOB} = \frac{29}{18}\pi$ and $OC \perp BA$.

Arc length of AB is 7π cm.



PICTURE



DIAGRAM

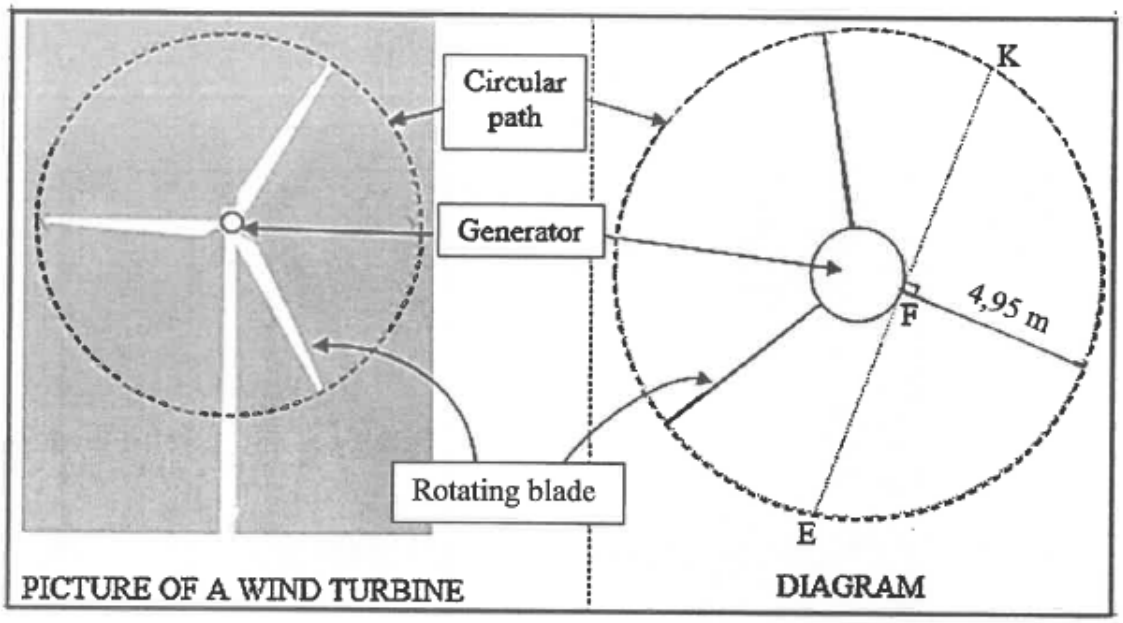
10.1.1 Write down, in radians, the size of acute $\hat{AOB}$. (1)

10.1.2 Hence, convert the size of acute $\hat{AOB}$ to degrees. (2)

10.1.3 Determine the length of OA. (3)

10.1.4 Calculate the area of major sector AOB. (3)

10.2 The picture below shows a wind turbine that is used to harness the power of wind to produce electricity.
The diagram below represents the rotating blades. The tips of the blades form a circular path when rotated. The smaller circle represents the generator.
The three identical rotating blades each have a length of 4,95 m (from point F to the tip of the blade).
KE = 10,5 m is a chord of the larger circle and is also a tangent to the smaller circle at point F.

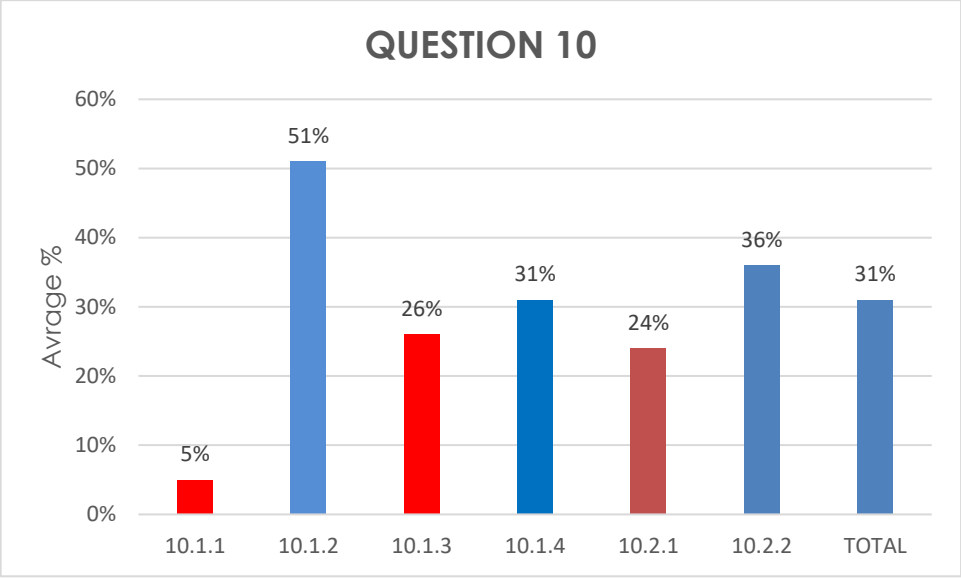


Calculate:

10.2.1 The length of the diameter of the circular path formed by the tips of the rotating blades (3)

10.2.2 The number of revolutions per minute if the circumferential velocity of the tip of a rotating blade is $6{,}61\pi$ metres per second (4)

[16]



Common errors and misconceptions

i.) Q10.1.1 – many candidates did not recognise that $\frac{29\pi}{18}$ refers to the reflex angle or it is a matter not recognising the difference between an acute and reflex angle.

ii.) Q10.1.3 – many candidates did not realise that the angle should be in radians

iii.) Q10.1.4 – many candidates calculated the minor sector instead of the major sector, not realising the reflex angle comes in to play – some candidates chose both formula and work with them both

iv.) Q10.2.1 – some candidates assumed that KE is the diameter of the circle – many did not realise that they can use the segment, diameter, and height formula $4h^2 - 4dh + x^2 = 0$

v.) Q10.2.2 – many candidates wrote down the correct formula but did the substitution incorrectly. Further, some also forgot to convert the final answer to minutes

Suggestions for improvement

i.) Maybe, the examiners should have indicated the reflex angle on the diagram

ii.) When using the formulae $s = r\theta$ and $Area = \frac{rs}{2} = \frac{r^2\theta}{2}$,the formula sheet clearly states that the angle(θ) must be in RADIANS or otherwise convert everything to degrees, remembering that everything must be in the same units – learners must select a formula and not to use both

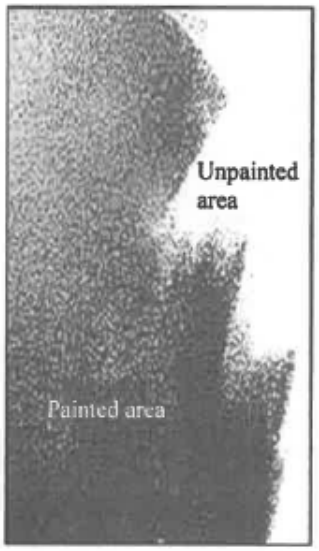
iii.) Concepts of minor and major segments also need attention

QUESTION 11 [Total marks 21]

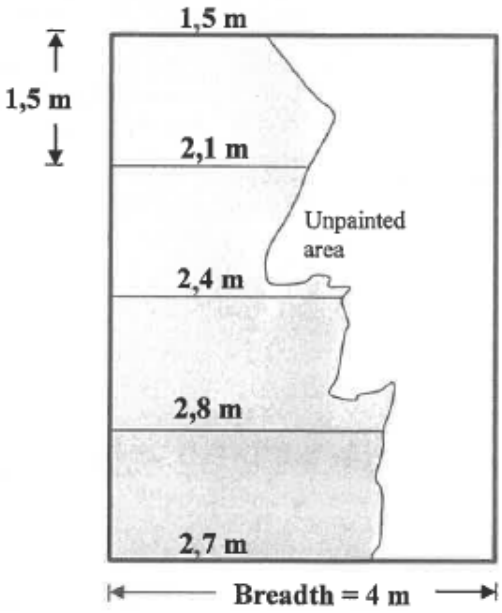
11.1 The picture below shows a partially painted rectangular wall which has a breadth of 4 m.
The diagram below represents the situation above.
The height of the unpainted part of the wall is divided into four equal parts that are each 1,5 m long, as shown in the diagram.

The ordinates of the parts are:

1,5 m; 2,1 m; 2,4 m; 2,8 m and 2,7 m



PICTURE OF THE WALL



DIAGRAM

The following formula may be used:

Area of a rectangle = length × breadth

11.1.1 Determine the height of the wall. (1)

11.1.2 Calculate the painted area of the wall by using the mid-ordinate rule. (4)

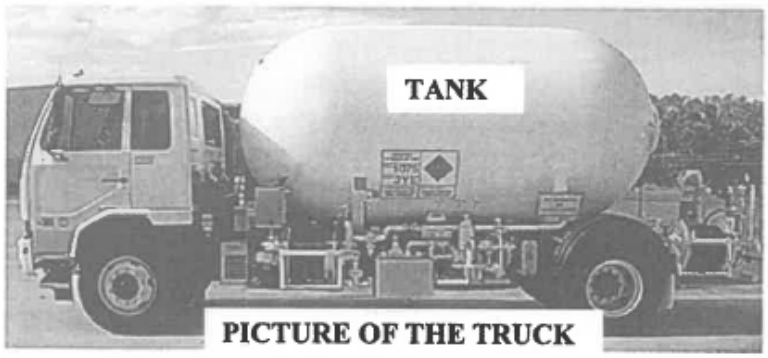
11.1.3 A one-litre tin of paint, which is sufficient to cover an area of 3,26 m^2, costs R156,36.

Determine whether the minimum cost of the paint required to paint the remaining section of the wall will exceed R600.

NOTE: The paint is only sold in one-litre tins. (6)

11.2 The picture below shows a truck used for transporting liquid in a tank. The shape of the tank consists of a right cylindrical section in the middle with hemispheres at each end.

The diagram below shows the side view of the tank which is 6 m long.
The cylindrical section of the tank is 3 m long and the length of the radius of both the cylindrical and hemispherical sections of the tank is equal to 1,5 m.



PICTURE OF THE TRUCK

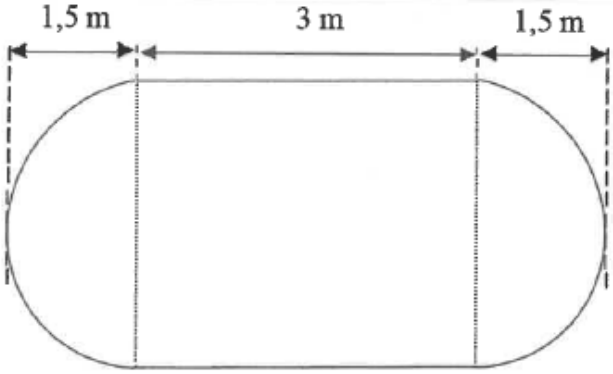


DIAGRAM: Side view of the tank

The following formulae may be used:

Total surface area of a right cylinder $= 2\pi r^2 + 2\pi r h$

Area of a rectangle = length × breadth

Volume of a right cylinder $= (\pi r^2) \times$ **height**

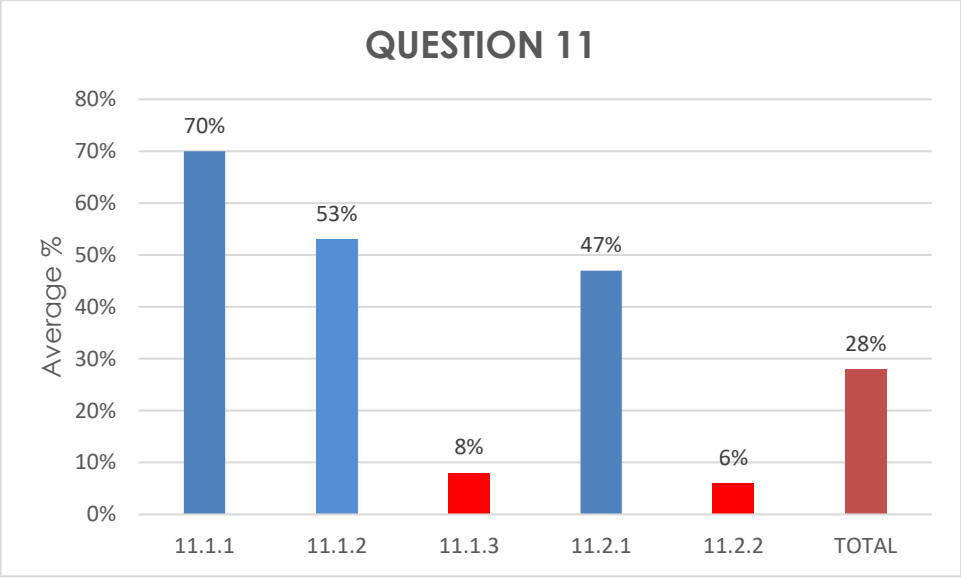
Surface area of a sphere $= 4\pi r^2$

Volume of a sphere $= \frac{4}{3}\pi r^3$

11.2.1 Calculate the total surface area of the tank. (5)

11.2.2 A right cylindrical tank has the same volume and the same radius as the tank of the truck. Show whether the height of this right cylindrical tank is more than three times its radius. (5)

[21]



Many candidates did not attempt Q11.2

Q11.1.3 and Q11.2.2 were the worst performing question for the sampled candidates and the second worst performing question in the paper

Q11.1.2 – many candidates applied the ordinates of the formula incorrectly, although providing the correct formula

Q11.1.3 – many candidates did not know how to approach this problem

Q11.2.1 – many candidates could not calculate the total surface area, but the marking guideline favoured them, to score marks

Common errors and misconceptions

i.) Q11.1.1 – candidates took the "breath" as an ordinate

ii.) Q11.1.2 – many candidates are still applying the mid-ordinate rule incorrectly

iii.) Q11.1.3 – many candidates did not realise that the unpainted area is calculated by calculating the area of the wall and then subtracting the painted area – further, most learners did not calculate the number of tins needed as a whole number

iv.) Q11.2.1 – many selected the first option for the surface area = $2\pi r^2 + 2\pi rh$ not realising that $2\pi r^2$ means the top and bottom areas of a cylinder

Suggestions for improvement

i.) The application of the mid-ordinate rule must be drilled into the learners – these are easy marks to get

ii.) Expose learners to more practical modelling problems

iii.) The CAPS is clear that the Total surface area of a CLOSE right-angled prism is given by, $SA = 2 \times area\ of\ base + circumference\ of\ base\ \times height$

iv.) For an OPEN right-angled prism, $SA = area\ of\ base + circumference\ of\ base\ \times height$

v.) For Q11.21. – a possible approach is, $SA = circumference\ of\ cylinder + 2\ \times hemispheres$

vi.) Hemisphere is the half of a sphere

vii.) Thus, $Surface\ Area = 2\pi r + 2 \times \frac{1}{2} \times 4\pi r^2 = 2\pi rh + 4\pi r^2$

viii.) In a similar way we can approach the volume:

ix.) $Volume_{cylinderical\ tank} = Volume_{cylinder} + 2 \times Volume_{hemisphere} = \pi r^2 h + 2 \times \frac{1}{2} \times \frac{4}{3}\pi r^3$

x.) Further, learners must know that in these type of problems that $\pi = 3{,}1415\ldots$

OVERALL COMMENTS

- The academic year 2020 was a huge challenge for all candidates. However, many learners from rural areas, felt the impact more as many could not benefit from the online and virtual support due to the lack of relevant resources and/or connectivity (poor or no network connectivity).
- Even after schools finally opened, some teachers did not return to complete the curriculum, support candidates by revising, etc. until the country was at Alert Level 1 and that made them to lose a lot.
- The above as well as other factors made the overall performance of learners to be poor (even poorer than the previous years 2018 and 2019).
- Many candidates proved not to be fully prepared:
    - There was a reasonable number (more than 15 %) of the learners who did not attempt questions in one or even two topics.
    - Even many of the learners who attempted did not collect marks from even level 1 and 2 questions in certain topics especially grade 11 content.
    - What is indicated in the above bullet is an indication that many schools and teachers may not have had time to revise grade 11 content as they pushed to just complete the prescribed grade 12 curriculum which also proves it was rushed.
- It is disheartening to see that a centre achieved zero percent pass rate and learners achieving single digit totals in the paper
- As the marking team we want Teachers to encourage learners always to attempt all questions. With CA marking learners can score marks because different skills are tested in a particular question
- When using formulae, learners must make sure the units are the same.
- In most cases for Technical Mathematics the angles are in radians, especially in the topic Angular Movement as provided and mentioned on the formula sheet
- Understanding all the formulae on the formula sheet will be a big advantage to the learners, so that they are able to identify the applicable formulae
- It is thus our responsibility to expose our learners to Complex problems as well, although many of them can not even get the level 1 and 2 questions properly mastered
- Finally, there was also a lot of reading expected in questions 1, 6, 10 and 11 which may have made some candidates (especially those whose LOLT is not HL) not to understand and attempt some of these questions.
- As a marking team we also feel that there were more complex questions asked and with the performance of the learners in this paper, we will be asking for adjustments of marks. The final percentage will depend after consultation with the Paper One team.

basic education

Department:
Basic Education
REPUBLIC OF SOUTH AFRICA

SENIOR CERTIFICATE/ NATIONAL SENIOR CERTIFICATE

GRADE 12

TECHNICAL MATHEMATICS P2

NOVEMBER 2020

MARKS: 150

TIME: 3 hours

This question paper consists of 13 pages and 2 information sheets.

* T M A T E 2 *

EASTERN CAPE

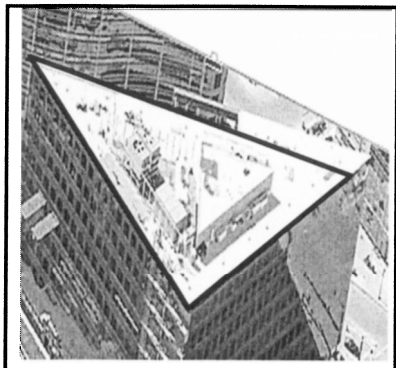
INSTRUCTIONS AND INFORMATION

Read the following instructions carefully before answering the questions.

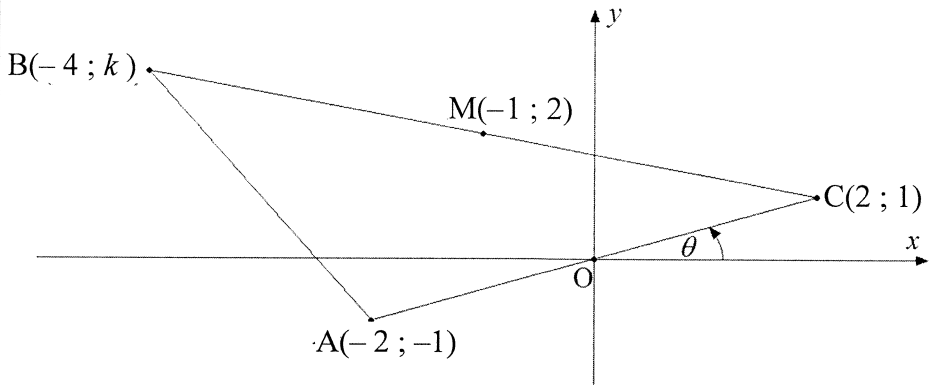
1. This question paper consists of 11 questions.
2. Answer ALL the questions in the SPECIAL ANSWER BOOK provided.
3. Clearly show ALL calculations, diagrams, graphs, etc. that you have used to determine your answers.
4. Answers only will NOT necessarily be awarded full marks.
5. If necessary, round off answers to TWO decimal places, unless stated otherwise.
6. Diagrams are NOT necessarily drawn to scale.
7. You may use an approved scientific calculator (non-programmable and non-graphical), unless stated otherwise.
8. An information sheet with formulae is included at the end of the question paper.
9. Write neatly and legibly.

QUESTION 1

The picture below shows the triangular rooftop of a building. The diagram next to the picture represents the triangular rooftop in the Cartesian plane with origin O.
In the diagram, A(–2 ; –1), B(– 4 ; k) and C(2 ; 1) are the vertices of ΔABC with M(–1 ; 2) the midpoint of BC.
The angle of inclination, θ, is the angle between AC and the positive x-axis.



PICTURE



DIAGRAM

1.1 Determine:

1.1.1 The numerical value of k (2)

1.1.2 The gradient of AC (2)

1.1.3 The size of θ (in degrees) (2)

1.1.4 The equation of straight line BC in the form $y = ...$ (3)

1.2 If O is the midpoint of AC, use analytical geometry methods to show that:

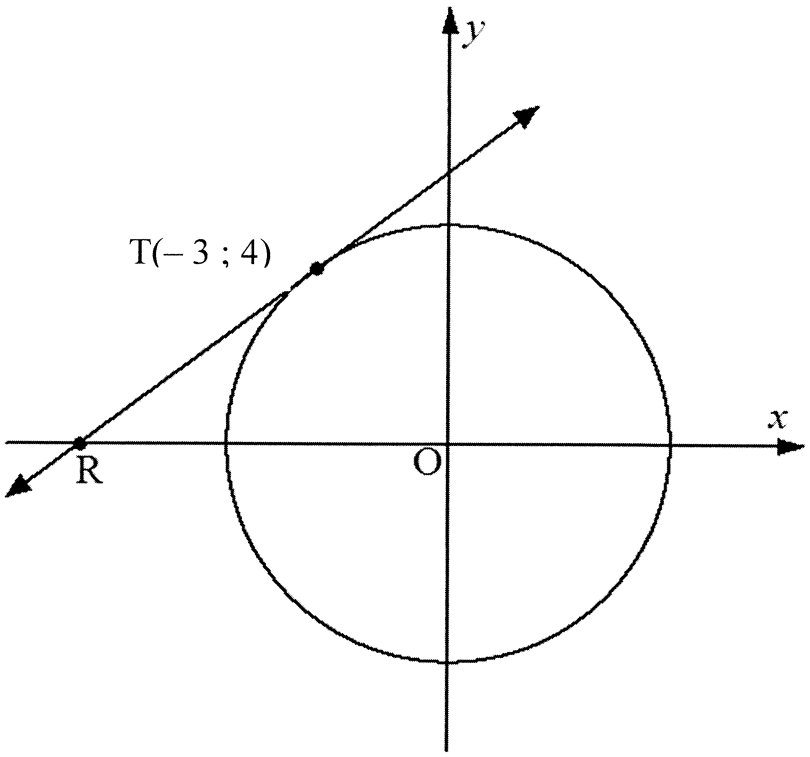
1.2.1 MO $\parallel$ BA (3)

1.2.2 $MO = \frac{1}{2}BA$ (3)

[15]

QUESTION 2

2.1 The diagram below shows a circle with centre O at the origin.
Point T(–3 ; 4) lies on the circle. Tangent RT to the circle passes through T.



2.1.1 Calculate the length of the diameter of the circle. (2)

2.1.2 Prove, showing ALL calculations, that the straight line defined by the equation $4y - 3x - 25 = 0$ and which passes through point T is the tangent to the circle. (5)

2.2 Given the ellipse with the following properties:

- Centre at the origin
- Distance between the x-intercepts is 12 units
- Range of $-3{,}5 \leq y \leq 3{,}5$

2.2.1 Write the equation of the ellipse in the form $\frac{x^2}{a^2}+\frac{y^2}{b^2}=1$ (1)

2.2.2 Sketch the graph of the ellipse. Clearly show the intercepts with the axes. (3)
[11]

QUESTION 3

3.1 If $P = 146,31°$ and $Q = 91,58°$, determine the value of $\sqrt{\frac{2}{\tan(P + Q)}}$ (2)

3.2 Given: $\tan\beta = -\frac{2}{3}$ where $\cos\beta > 0$ and $\sin\theta = \frac{5}{13}$ where $\theta \in [90° ; 360°]$

Determine, with the aid of diagrams and WITHOUT using a calculator, the value of EACH of the following:

3.2.1 $2\cot\beta + 1$ (2)

3.2.2 $\sin\beta \cdot \sec\theta$ (6)

3.3 Given: $\sin A + 2\cos^2 A = 2$

3.3.1 Express the above equation in simplified form, in terms of $\sin A$. (3)

3.3.2 Hence, or otherwise, determine the value(s) of A if:

$\sin A + 2\cos^2 A = 2$ for $A \in [0° ; 180°)$ (5)

[18]

QUESTION 4

4.1 Simplify EACH of the following:

4.1.1 $1 - \cos^2(2\pi - \theta)$ (2)

4.1.2 $\cos\alpha(\cot\alpha + \tan\alpha)$ (4)

4.2 Prove the following identity:

$$\frac{1}{\sin(180° + x)\cdot\sin(360° - x)} - \cot^2(180° - x) = 1$$ (5)

[11]

QUESTION 5

Given: Functions f and g defined by $f(x) = -\tan x$ and $g(x) = 2\sin x$ for $0° \leq x \leq 180°$

5.1 Draw sketch graphs of f and g on the same set of axes. Clearly indicate ALL the intercepts with the axes, turning points, asymptotes and end points. (6)

5.2 Write down:

5.2.1 The range of f (1)

5.2.2 The period of g (1)

5.2.3 The equation of the asymptote of h if $h(x) = f(x + 30°)$ (1)

5.3 Determine the value(s) of x for which $f(x) \cdot g(x) < 0$ (2)

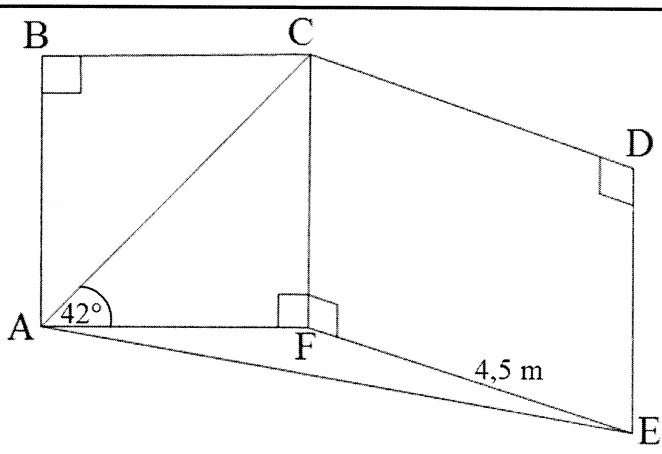
[11]

QUESTION 6

The picture below shows two vertical rectangular walls that are not perpendicular to each other. The diagram next to it represents the two rectangular walls, ABCF and CDEF.
Points A, F and E lie in the same horizontal plane and form a triangular section AFE of the floor.

Furthermore, $\hat{FAC} = 42°$, FE = 4,5 m and $CF = \frac{2}{3}FE$

PICTURE OF TWO WALLS



DIAGRAM

6.1 Write down the length of CF. (1)

6.2 Determine, correct to ONE decimal place:

6.2.1 The length of AF (2)

6.2.2 The size of $\hat{AFE}$ if AE = 6 m (4)

6.2.3 The area of ΔAFE (3)

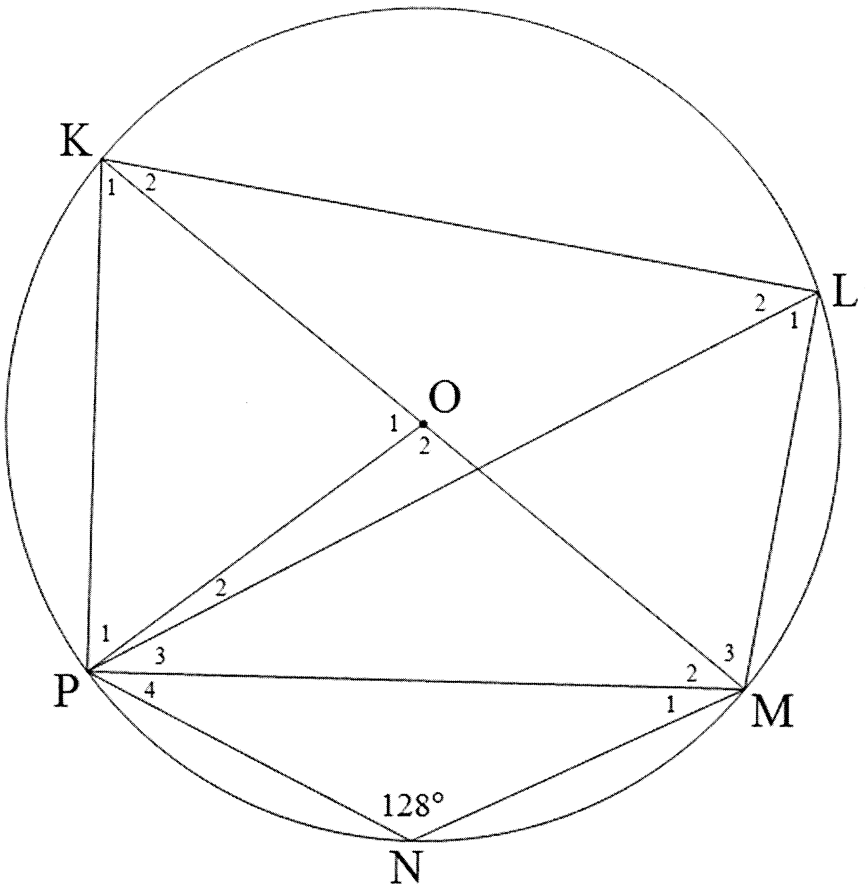
[10]

QUESTION 7

7.1 Complete the following theorem:

Angles subtended by a chord of a circle, … , are equal. (1)

7.2 In the diagram below, O is the centre of circle KLMNP.
KOM is a diameter of the circle and chords LP and PM are drawn.
$\hat{N} = 128°$



Determine, stating reasons, the size of EACH of the following angles:

7.2.1 $\hat{K}_1$ (2)

7.2.2 $\hat{L}_2$ (5)

7.2.3 $\hat{P}_2$ if $\hat{P}_3 = 29°$ (2)
[10]

QUESTION 8

8.1 Complete the following theorem:

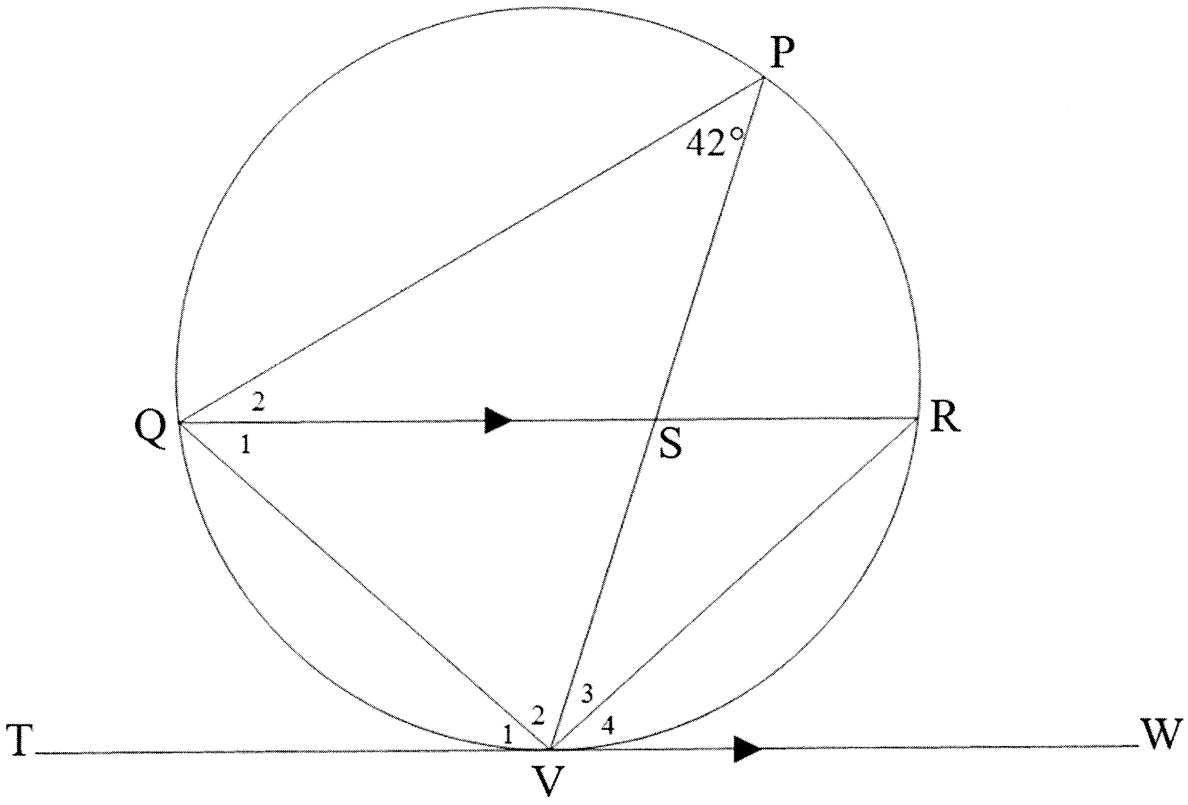
The angle between the tangent to a circle and the chord drawn from the point of contact is equal to … (1)

8.2 In the diagram below, TVW is the tangent to circle PRVQ at V.

Chords PV and QR intersect at point S.

TW || QR

$\hat{P} = 42°$



Determine, with reasons:

8.2.1 FOUR other angles each equal to 42° (6)

8.2.2 Whether QR is a diameter of the circle (2)

8.2.3 The size of $\hat{Q}_2$ if $\hat{V}_2 = 67°$ (3)

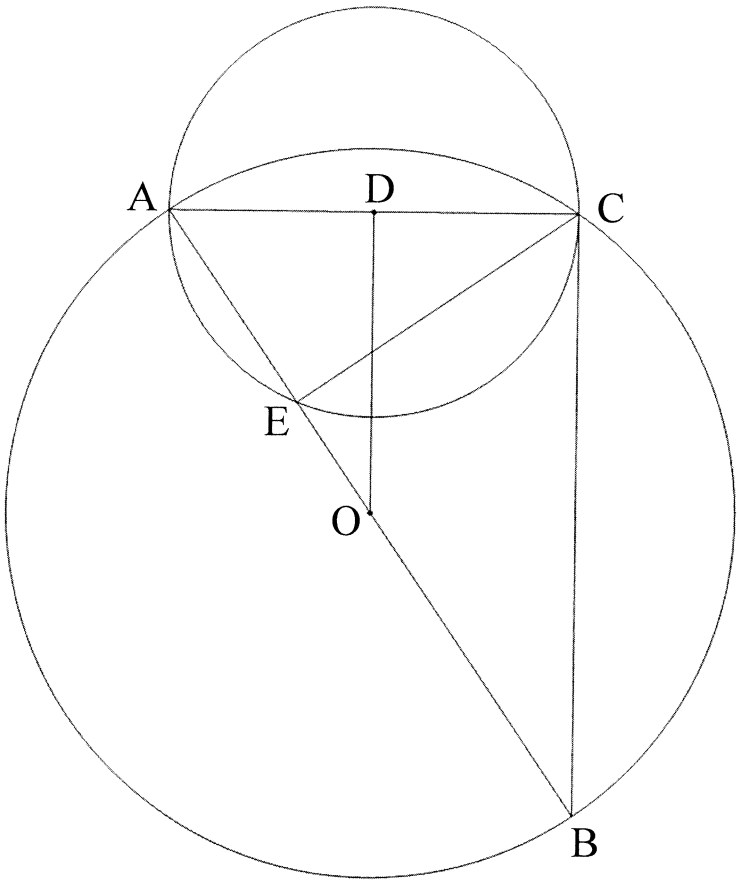
[12]

QUESTION 9

9.1 Complete the following theorem:

The perpendicular bisector of a chord of a circle passes through ... (1)

9.2 In the diagram below, O is the centre of circle ABC and D is the centre of circle ACE.
AB and AC are the diameters of the larger and smaller circles respectively.
BC is the tangent of the smaller circle at C.
DO = 6 units and AC = 8 units.



9.2.1 Give TWO different reasons why $A\hat{C}B = 90°$ (2)

9.2.2 Give a reason why DO || CB. (1)

9.2.3 Determine the length of diameter AB. (4)

9.2.4 Prove that $\Delta ABC \,|||\, \Delta ACE$. (3)

9.2.5 Show that $AC^2 = AB \times AE$ (1)

9.2.6 Determine the length of AE. Leave your answer in simplified surd form. (3)
[15]

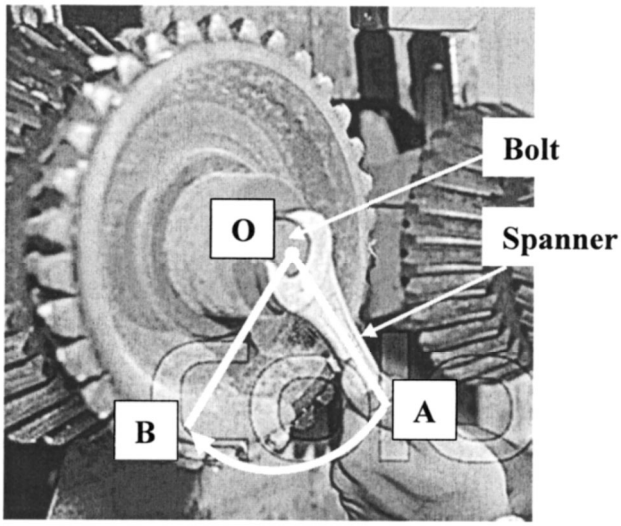
QUESTION 10

10.1 The picture below shows a spanner being used to tighten a bolt by rotating it in a clockwise direction from point A to point B. The diagram next to the picture represents the rotation of the spanner. Point O, the centre of the bolt, is also the centre of rotation of the spanner.

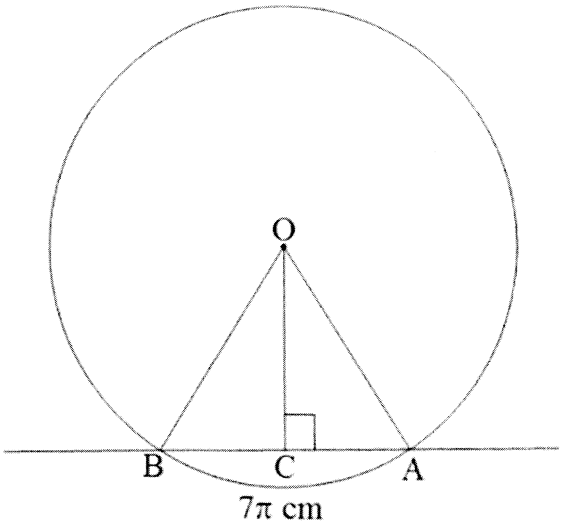
OA represents the distance from the centre of the bolt to the end of the spanner.

Reflex angle $\hat{AOB} = \frac{29}{18}\pi$ and $OC \perp BA$.

Arc length of AB is 7π cm.



PICTURE



DIAGRAM

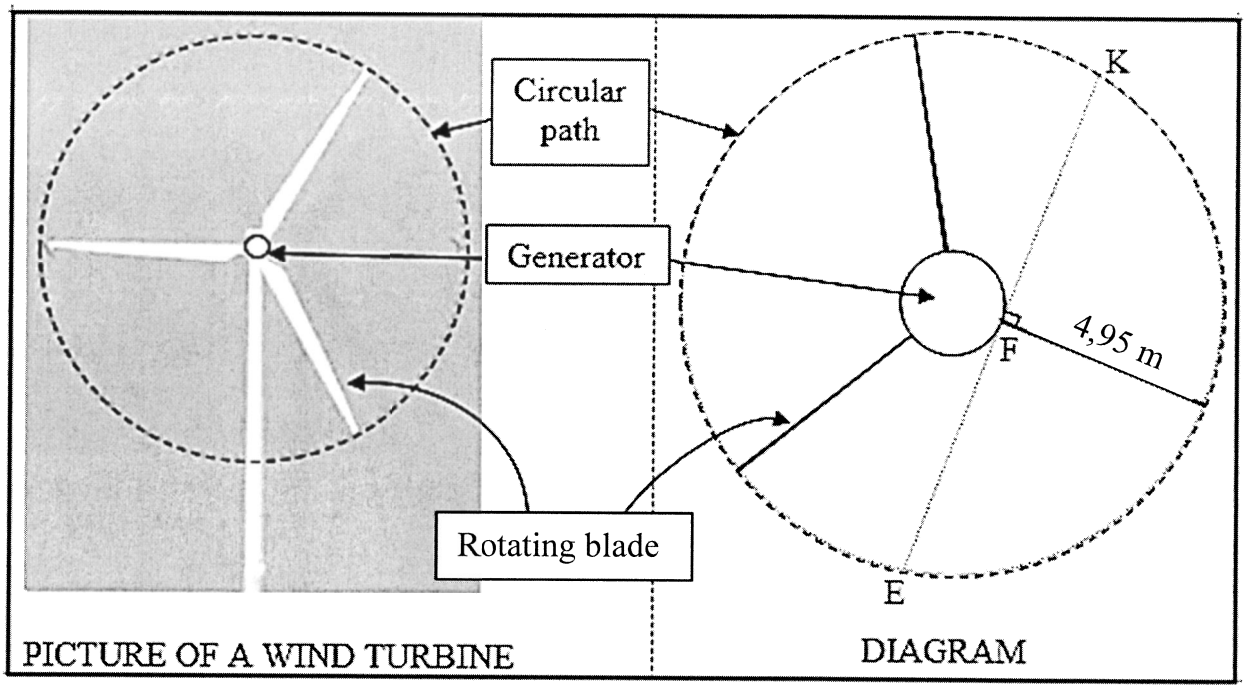
10.1.1 Write down, in radians, the size of acute $\hat{AOB}$. (1)

10.1.2 Hence, convert the size of acute $\hat{AOB}$ to degrees. (2)

10.1.3 Determine the length of OA. (3)

10.1.4 Calculate the area of major sector AOB. (3)

10.2 The picture below shows a wind turbine that is used to harness the power of wind to produce electricity.
The diagram below represents the rotating blades. The tips of the blades form a circular path when rotated. The smaller circle represents the generator.
The three identical rotating blades each have a length of 4,95 m (from point F to the tip of the blade).
KE = 10,5 m is a chord of the larger circle and is also a tangent to the smaller circle at point F.



Calculate:

10.2.1 The length of the diameter of the circular path formed by the tips of the rotating blades (3)

10.2.2 The number of revolutions per minute if the circumferential velocity of the tip of a rotating blade is $6,61\pi$ metres per second (4)

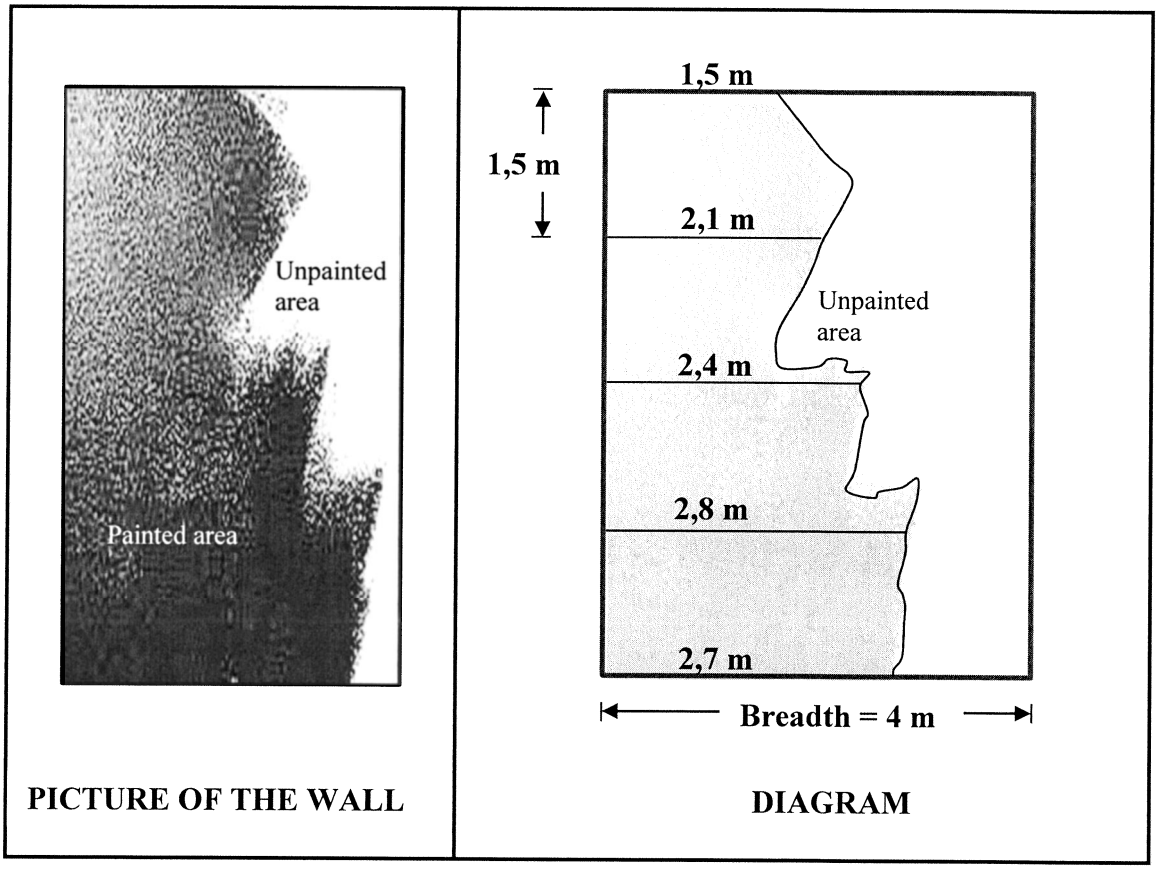
[16]

QUESTION 11

11.1 The picture below shows a partially painted rectangular wall which has a breadth of 4 m.
The diagram below represents the situation above.
The height of the unpainted part of the wall is divided into four equal parts that are each 1,5 m long, as shown in the diagram.

The ordinates of the parts are:

1,5 m; 2,1 m; 2,4 m; 2,8 m and 2,7 m



PICTURE OF THE WALL **DIAGRAM**

The following formula may be used:

Area of a rectangle = length × breadth

11.1.1 Determine the height of the wall. (1)

11.1.2 Calculate the painted area of the wall by using the mid-ordinate rule. (4)

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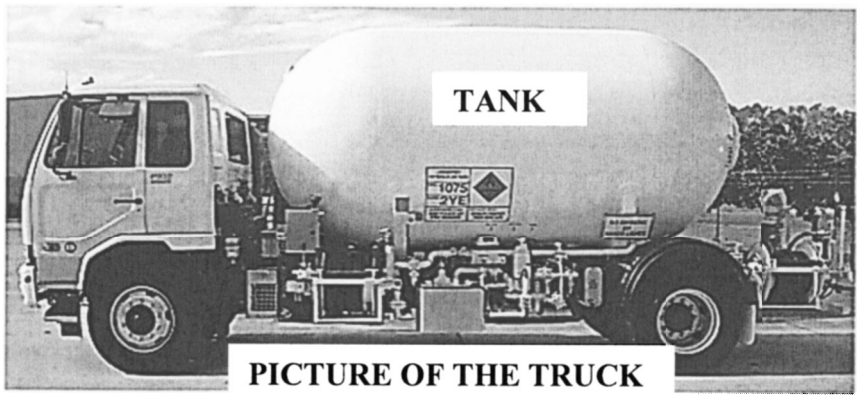
Determine whether the minimum cost of the paint required to paint the remaining section of the wall will exceed R600.

NOTE: The paint is only sold in one-litre tins. (6)

11.2 The picture below shows a truck used for transporting liquid in a tank. The shape of the tank consists of a right cylindrical section in the middle with hemispheres at each end.

The diagram below shows the side view of the tank which is 6 m long.
The cylindrical section of the tank is 3 m long and the length of the radius of both the cylindrical and hemispherical sections of the tank is equal to 1,5 m.



PICTURE OF THE TRUCK

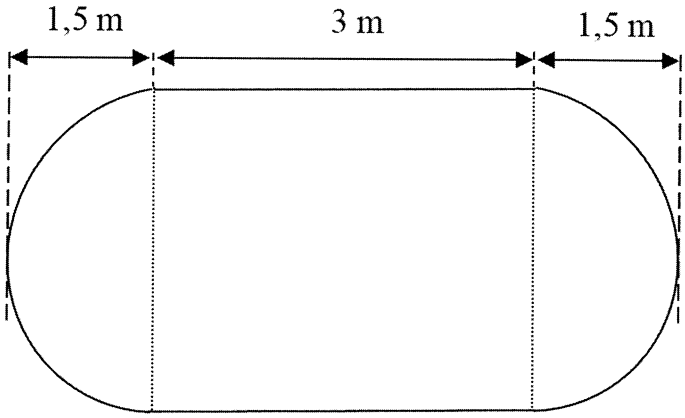


DIAGRAM: Side view of the tank

The following formulae may be used:

Total surface area of a right cylinder $= 2\pi r^2 + 2\pi r h$

Area of a rectangle = length × breadth

Volume of a right cylinder $= (\pi r^2) \times$ **height**

Surface area of a sphere $= 4\pi r^2$

Volume of a sphere $= \frac{4}{3}\pi r^3$

11.2.1 Calculate the total surface area of the tank. (5)

11.2.2 A right cylindrical tank has the same volume and the same radius as the tank of the truck. Show whether the height of this right cylindrical tank is more than three times its radius. (5)

[21]

TOTAL: 150

INFORMATION SHEET: TECHNICAL MATHEMATICS

$$x = \frac{-b \pm \sqrt{b^2 - 4ac}}{2a} \qquad x = -\frac{b}{2a} \qquad y = \frac{4ac - b^2}{4a}$$

$$a^x = b \Leftrightarrow x = \log_a b \ , \qquad a > 0 \ , \ a \neq 1 \ \text{and} \ b > 0$$

$$A = P(1+ni) \qquad A = P(1-ni) \qquad A = P(1-i)^n \qquad A = P(1+i)^n$$

$$i_{eff} = \left(1 + \frac{i}{m}\right)^m - 1$$

$$f'(x) = \lim_{h \to 0} \frac{f(x+h) - f(x)}{h}$$

$$\int x^n dx = \frac{x^{n+1}}{n+1} + C \quad , \quad n \neq -1$$

$$\int \frac{1}{x} dx = \ln x + C, \qquad x > 0 \qquad\qquad \int a^x dx = \frac{a^x}{\ln a} + C \quad , \quad a > 0$$

$$d = \sqrt{(x_2 - x_1)^2 + (y_2 - y_1)^2} \qquad \text{M}\left(\frac{x_1 + x_2}{2}; \frac{y_1 + y_2}{2}\right)$$

$$y = mx + c \qquad y - y_1 = m(x - x_1) \qquad m = \frac{y_2 - y_1}{x_2 - x_1} \qquad m = \tan\theta$$

$$\frac{x^2}{a^2} + \frac{y^2}{b^2} = 1$$

In ΔABC: $\dfrac{a}{\sin A} = \dfrac{b}{\sin B} = \dfrac{c}{\sin C}$ $\qquad a^2 = b^2 + c^2 - 2bc \cdot \cos A$

Area of $\Delta ABC = \dfrac{1}{2} ab \cdot \sin C$

$$\sin^2\theta + \cos^2\theta = 1 \qquad 1 + \tan^2\theta = \sec^2\theta \qquad \cot^2\theta + 1 = \text{cosec}^2\theta$$

$\pi rad = 180°$

Angular velocity $= \omega = 2\pi n = 360°n$ where n = rotation frequency

Circumferential velocity = $v = \pi Dn$ where D = diameter and n = rotation frequency

$s = r\theta$ where r = radius and θ = central angle in radians

Area of a sector $= \frac{rs}{2} = \frac{r^2\theta}{2}$ where r = radius, s = arc length and θ = central angle in radians

$4h^2 - 4dh + x^2 = 0$ where h = height of segment, d = diameter of circle and x = length of chord

$A_T = a(m_1 + m_2 + m_3 + \ldots + m_n)$ where a = equal parts, $m_1 = \frac{o_1 + o_2}{2}$ and n = number of ordinates

OR

$A_T = a\left(\frac{o_1 + o_n}{2} + o_2 + o_3 + o_4 + \ldots + o_{n-1}\right)$ where a = equal parts, $o_i = i^{th}$ ordinate and n = number of ordinates

basic education

Department:
Basic Education
REPUBLIC OF SOUTH AFRICA

SENIOR CERTIFICATE/*SENIOR SERTIFIKAAT*
NATIONAL SENIOR CERTIFICATE/
NASIONALE SENIOR SERTIFIKAAT

GRADE 12/*GRAAD* 12

TECHNICAL MATHEMATICS P2/*TEGNIESE WISKUNDE V2*
NOVEMBER 2020
FINAL / *FINALE*

MARKING GUIDELINE/*NASIENRIGLYN*

MARKS/*PUNTE*: 150

CODE	EXPLANATION
A	Accuracy/Akkuraatheid
AO	Answer Only/Slegs antwoord
CA	Consistent accuracy/Volgehoue akkuraatheid
I	Identity/Identiteit
F	Correct Formula/Korrekte formule
M	Method/Metode
NPR	No penalty for rounding/Geen penalisering vir afronding
NPU	No penalty for omitting units/*Geen penalisering vir eenhede weggelaat*
R	Rounding/*Afronding*
RE	Reason/Rede
S	Simplification/Vereenvoudiging
SF	Substitution in correct formula/*Vervanging in korrekte formule*
ST	Statement/*Bewering*
ST/RE	Statement with Reason/*Bewering met rede*

Date approved/ **Datum goedgekeur** **25 November 2020**	**EXTERNAL MODERATOR**	**INTERNAL MODERATORS**
	MA HENDRICKS	**NS MUTHIGE & N TOM**
	MA HENDRICKS External Moderator UMALUSI	

These marking guidelines consist of 27 pages./
Hierdie nasienriglyne bestaan uit 27 bladsye.

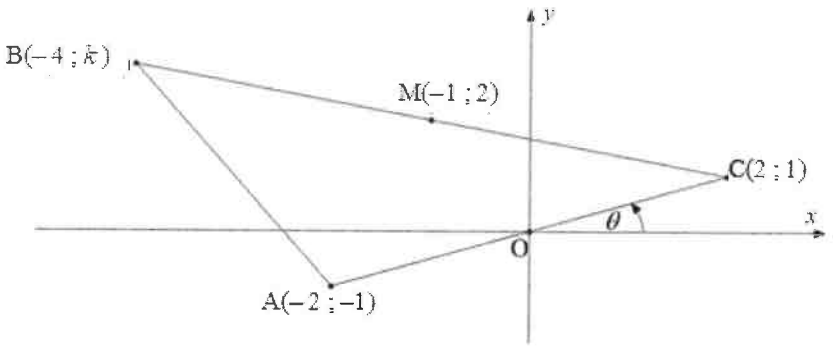
NOTE:

- If a candidate answers a question **TWICE**, only mark the **FIRST** attempt.
- Consistent accuracy to be applied as indicated on the marking guidelines.
- # Questions where Tolerance Range will be applied are **Q3.2.2, Q4.2, Q10.1.1/2 and Q11.2.1**

LET WEL:

- *Indien 'n kandidaat 'n vraag **TWEE** keer beantwoord, sien slegs die **EERSTE** poging na.*
- *Volgehoue akkuraatheid sal toegepas word soos op die nasienriglyne aangedui .*
- *# Vrae waar Toleransie Wydte toegepas word is **V3.2.2, V4.2, V10.1.2 en V11.2.1.***

QUESTION/*VRAAG* 1



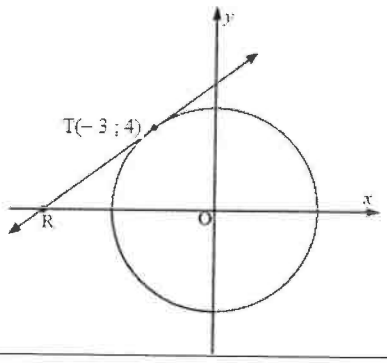
1.1.1	$\frac{k+1}{2} = 2$ $\therefore\ k = 3$ **OR/*OF*** $\frac{k-2}{-4-(-1)} = \frac{2-1}{-1-2}$ $\frac{k-2}{-3} = \frac{1}{-3}$ $\therefore\ k\ = 3$	✓**M** **A** ✓ value of/*waarde van k* **CA** **OR/*OF*** ✓**M** **A** ✓ value of/*waarde van k* **CA** **AO: Full marks/*Volpunte*** (2)
1.1.2	$m_{AC} = \frac{y_2 - y_1}{x_2 - x_1}$ $= \frac{1-(-1)}{2-(-2)}$ or / *of* $\frac{1-0}{2-0}$ or / *of* $\frac{-1-0}{-2-0}$ $= \frac{1}{2}$	✓ **SF** **A** ✓ **S** **CA** **AO: Full marks/*Volpunte***
1.1.3	$m_{AC} = \tan\theta$ $\tan\theta = \frac{1}{2}$ $\theta \approx 26{,}57°$	✓ **SF** **CA** ✓ angle/*hoek* **CA** **NPR** (2)

MA HENDRICKS
External Moderator
DMALUSI

1.1.4	$m_{BC} = \frac{2-1}{-1-2} = -\frac{1}{3}$ $y - 1 = -\frac{1}{3}(x-2)$ *or / of* $1 = -\frac{1}{3}(2) + c$ $y = -\frac{1}{3}x + \frac{2}{3} + \frac{3}{3}$ $c = \frac{5}{3}$ $\therefore y = -\frac{1}{3}x + \frac{5}{3}$ **OR/*OF*** $m_{BC} = \frac{2-1}{-1-2} = -\frac{1}{3}$ $y - 2 = -\frac{1}{3}(x-(-1))$ *or / of* $2 = -\frac{1}{3}(-1) + c$ $y = -\frac{1}{3}x - \frac{1}{3} + 2$ $c = \frac{5}{3}$ $\therefore y = -\frac{1}{3}x + \frac{5}{3}$ **OR/*OF*** $m_{BC} = \frac{2-1}{-1-2} = -\frac{1}{3}$ $y - 3 = -\frac{1}{3}(x-(-4))$ *or / of* $3 = -\frac{1}{3}(-4) + c$ $y = -\frac{1}{3}x - \frac{4}{3} + 3$ $c = \frac{5}{3}$ $\therefore y = -\frac{1}{3}x + \frac{5}{3}$	✓ gradient/*gradiënt* **A** ✓ **SF** **CA** ✓equation/*vergelyking* **CA** **OR/*OF*** ✓ gradient/*gradiënt* **A** ✓ **SF** **CA** ✓equation/*vergelyking* **CA** **OR/*OF*** ✓ gradient/*gradiënt* **CA** ✓ **SF** **CA** ✓equation/*vergelyking* **CA** (3)

1.2.1	$m_{MO} = \frac{y_2 - y_1}{x_2 - x_1} = \frac{2-0}{-1-0} = -2$ $m_{AB} = \frac{y_2 - y_1}{x_2 - x_1} = \frac{3-(-1)}{-4-(-2)} = -2 = m_{MO}$ $\therefore MO \parallel BA$	✓ gradient/*gradiënt* m_{MO} **A** ✓ gradient/*gradiënt* m_{AB} **CA** ✓ M equal gradients /*gelyke gradiente* **A** (3)
1.2.2	$MO = \sqrt{(-1-0)^2 + (2-0)^2}$ $= \sqrt{5}$ $AB = \sqrt{(-4-(-2))^2 + (3-(-1))^2}$ $= \sqrt{20}$ $= 2\sqrt{5} = 2\,MO$ $\therefore MO = \frac{1}{2}AB$	✓ length of/*lengte van* MO **A** ✓length of AB/*lengte van* **CA** ✓ conclusion/ *gevlg* **CA** (3)
		[15]

QUESTION/*VRAAG* 2



2.1.1	$OT = \sqrt{(-3)^2 + (4)^2} = 5$ units/*eenhede* $\therefore$ diameter/*middellyn* $= 10$ units/*eenhede* **OR/OF** Diameter /*middellyn*$= 2OT = 2\sqrt{(-3)^2 + (4)^2} = 10$ units/*eenhede* **OR/OF** Diameter/*Middellyn* $= \sqrt{(-3-3)^2 + (4+4)^2} = 10$ units/*eenhede*	✓ length of radius/*lengte van radius* **A** ✓ length of diameter/ *lengte van middlyn* **CA** **OR/OF** ✓ **SF** **A** ✓ length of diameter/ *lengte van middlyn* **CA** **OR/OF** ✓ **SF** **A** ✓ length of diameter/ *lengte van middlyn* **CA** **AO: Full marks/** ***Volpunte*** (2)
2.1.2	$m_{OT} = -\frac{4}{3}$ $4y - 3x - 25 = 0$ $y = \frac{3}{4}x + \frac{25}{4}$ $\therefore m_{line} = \frac{3}{4}$ $m_{OT} \times m_{tan} = -\frac{4}{3} \times \frac{3}{4} = -1$ $4y - 3x - 25 = 0$ is a tangent at/*is die raaklyn by* **T** **OR/*OF***	✓ gradient/*gradiënt* of OT **A** ✓in the form/ *in die vorm* $y = mx + c$ **A** ✓ gradient/*gradiënt* of line/*van lyn* **CA** ✓ **M** **A** ✓ gradient/*gradiënt* product/*produk* **CA** **OR/*OF***

$x^2 + y^2 = (-3)^2 + (4)^2$

$\therefore x^2 + y^2 = 25$

$4y - 3x - 25 = 0$

$y = \frac{3}{4}x + \frac{25}{4}$

$x^2 + \left(\frac{3}{4}x + \frac{25}{4}\right)^2 = 25$

$x^2 + 6x + 9 = 0$

$(x+3)(x+3) = 0$ **OR/OF** $\Delta = (6)^2 - 4(1)(9)$

$\therefore x = -3$ $= 0$

$\therefore$ only ONE point of intersection, therefore is a tangent / *slegs EEN snypunt, dus 'n raaklyn*

✓ equation of a circle/ *verg. van sirkel* **A**

✓in the form/ *in die vorm* $y = mx + c$ **A**

✓ **M** **CA**

✓ value of/*waarde van x* ***or*** discriminant **CA**

✓ conclusion/ *gvlg* **CA**

OR/*OF*

$4y - 3x - 25 = 0$

$\therefore R\left(-\frac{25}{3};0\right)$

and/*en* $T(-3;4)$

$RT^2 = \left(-\frac{25}{3} - (-3)\right)^2 + (0-4)^2 = \frac{400}{9}$

$OT^2 = 25$

$OR^2 = \frac{625}{9}$

$RT^2 + OT^2 = \frac{400}{9} + 25 = \frac{625}{9} = OR^2$

$\therefore \hat{RTO} = 90°$

$\therefore$ the line is a tangent to a circle/*die lyn is 'n raaklyn aan die sirkel*

OR/*OF*

$m_{OT} = -\frac{4}{3}$ $m_{\perp} = \frac{3}{4}$

$y - 4 = \frac{3}{4}(x+3)$

$y = \frac{3}{4}x + \frac{25}{4}$

$4y - 3x - 25 = 0$

$\therefore$ this line is the tangent / *hierdie lyn is die raaklyn*

OR/*OF*

✓ coordinates of x-intercept of R/ *koordnte van x- afsnt* **A**

✓ value of/*waarde van* RT^2 **A**

✓ value of/*waarde van* OT^2 **CA**

✓ value of/*waarde van* OR^2 **CA**

✓ conclusion/*gevolg* **CA**

OR/*OF*

✓ gradient/*gradiënt* of OT **A**

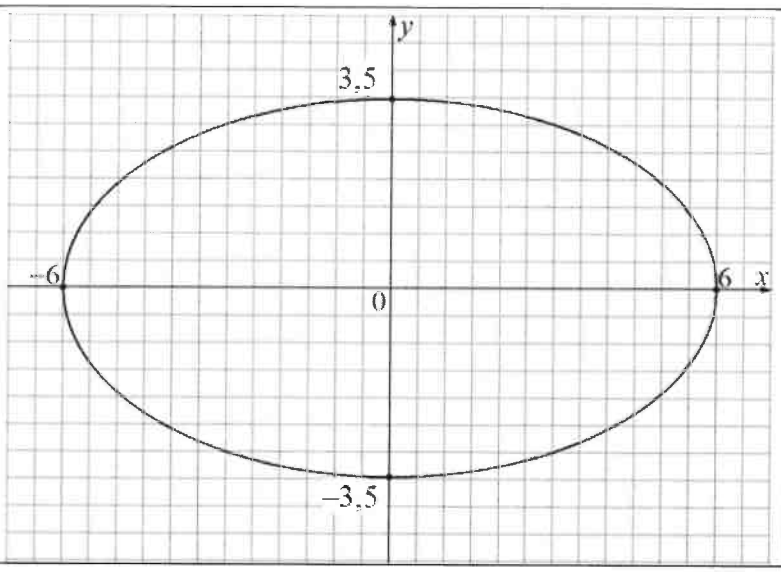
✓ gradient/*gradiënt* of $m_{\perp}$ OT **CA**

✓**SF** **CA**

✓ equation of line / *vergelyking /van lyn* **CA**

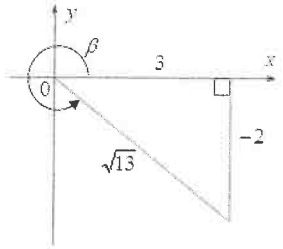
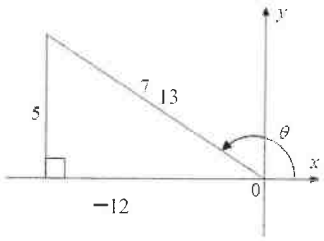
✓ conclusion / *gvlg* CA

(5)

2.2.1	$\frac{x^2}{6^2}+\frac{y^2}{(3,5)^2}=1$ **OR/*OF*** $\frac{x^2}{6^2}+\frac{y^2}{\left(\frac{7}{2}\right)^2}=1$	✓correct vorm/*korrekte vorm* **A** (1)
2.2.2		**CA from/*van Q/V* 2.2.1** ✓both *y*-intercepts/*beide y-afsnitte* **CA** ✓ both *x*-intercepts/ *beide x-afsnitte* **CA** ✓ elliptical shape/*ellips vorm* **CA** (3)
		[11]

QUESTION/*VRAAG* 3

3.1	$\sqrt{\dfrac{2}{\tan(P+Q)}} = \sqrt{\dfrac{2}{\tan(146{,}31° + 91{,}58°)}}$ $\approx 1{,}12$ **OR/*OF*** $\sqrt{\dfrac{2}{\tan(P+Q)}} = \sqrt{2\cot(P+Q)} = \sqrt{2\cot(146{,}31° + 91{,}58°)}$ $\approx 1{,}12$	✓ substitution/*verv* **A** ✓value/*waarde* **CA** **OR/*OF*** ✓ substitution/ *verv* **A** ✓value/*waarde* **CA** **NPR** **AO: Full marks/ *Volpunte*** (2)
3.2.1	$2\cot\beta + 1$ $= 2\left(-\dfrac{3}{2}\right) + 1$ $= -2$ **OR/*OF*** $\dfrac{2}{\tan\beta} + 1$ $= \dfrac{2}{\left(-\dfrac{2}{3}\right)} + 1$ $= -2$	✓ cot ratio/ *verhouding* **A** ✓ **S** **CA** **OR/*OF*** ✓tan ratio/ *verhouding* **A** ✓ **S** **CA** (2)

3.2.2 #.	$\tan\beta = -\frac{2}{3}$ $\qquad$ $\sin\theta = \frac{5}{13}$   $r^2 = (3)^2 + (2)^2$ $\qquad$ $x^2 + (5)^2 = (13)^2$ $\therefore r = \sqrt{13}$ $\qquad$ $\therefore x = -12$ $\sin\beta.\sec\theta = \left(-\frac{2}{\sqrt{13}}\right) \times \left(-\frac{13}{12}\right)$ $= \frac{13}{6\sqrt{13}}$ **OR/ OF** $\frac{\sqrt{13}}{6}$ *or / of* $\sin\beta.\left(-\sqrt{1+\tan^2\theta}\right) = \left(-\frac{2}{\sqrt{13}}\right) \times \left(-\sqrt{1+\left(\frac{5}{12}\right)^2}\right)$ $= \frac{13}{6\sqrt{13}}$ **OR/OF** $\frac{\sqrt{13}}{6}$	✓ tan diagram **A** ✓ sin diagram **A** ✓ values of/ *waardes van* x & r **CA** ✓ $\sin\beta$ ratio/*verh* **CA** ✓ $\cos\theta$ ratio/*verh* **CA** ✓ **S** **CA** *or / of* ✓ $\sin\beta$ ratio/*verh* **CA** ✓ $\tan\theta$ ratio/*verh* **CA** ✓ **S** **CA** **Accept/*aanvaar*** 0,60 (6)

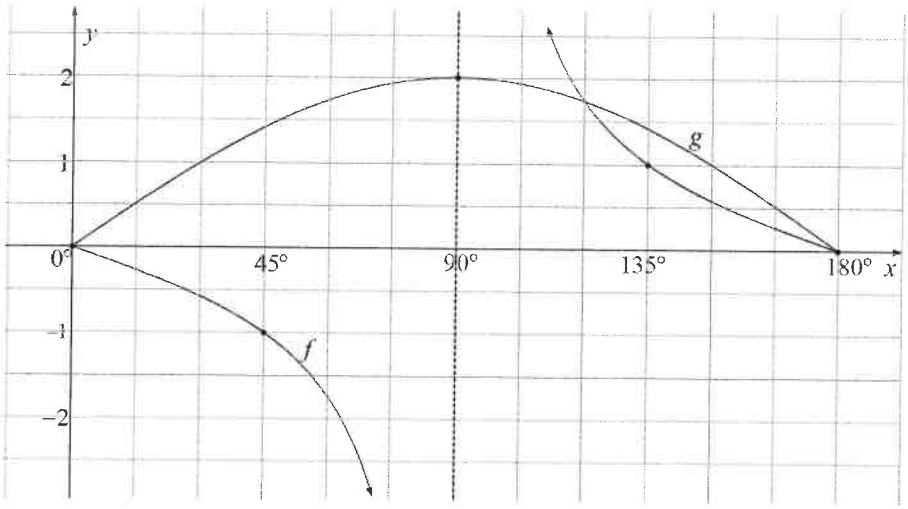
3.3.1	$\sin A + 2\cos^2 A = 2$ $\sin A + 2(1 - \sin^2 A) = 2$ $\sin A + 2 - 2\sin^2 A = 2$ $-2\sin^2 A + \sin A = 0$ **OR/*OF*** $2\sin^2 A - \sin A = 0$ **OR/*OF*** $\sin A + 2\cos^2 A = 2$ $\sin A + 2\cos^2 A - 2 = 0$ $\sin A - 2(1 - \cos^2 A) = 0$ $\sin A - 2(\sin^2 A) = 0$ $-2\sin^2 A + \sin A = 0$ **OR/*OF*** $2\sin^2 A - \sin A = 0$	✓ **I** **A** ✓ **S** **CA** ✓ Simplified vorm/*eenv vorm* **CA** **OR/*OF*** ✓ factors/*faktore* **CA** ✓ **I** **A** ✓ Simplified vorm/*eenv vorm* **CA** (3)
3.3.2	$\sin A + 2\cos^2 A = 2$ $-2\sin^2 A + \sin A = 0$ or / *of* $2\sin^2 A - \sin A = 0$ $\sin A(1 - 2\sin A) = 0$ or / *of* $\sin A(2\sin A - 1)$ $\sin A = 0$ or/*of* $\sin A = \frac{1}{2}$ ref./verw. $\angle = 0°$ or/*of* ref./verw. $\angle = 30°$ $\therefore A = 0°$ or/*of* $A = 30°$ or/of $A = 150°$	**CA from/*van* Q/V3.3.1** ✓ factors/*faktore* **CA** ✓ zero property/nul eienskap **CA** ✓ both reference angles/*beide verw. hoeke* **CA** ✓ 0° **CA** ✓ 30° and 150° **CA** (5)
		[18]

QUESTION/*VRAAG* 4

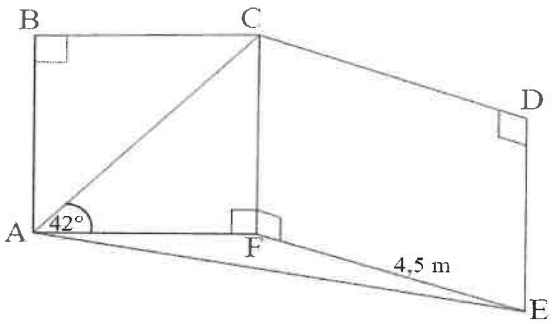
4.1.1	$1-\cos^2(2\pi-\theta) = 1-\cos^2\theta$ $= \sin^2\theta$ **OR/OF** $1-\cos^2(2\pi-\theta) = \sin^2(2\pi-\theta)$ $= \sin^2\theta$	✓ reduction/*reduksie* **A** ✓ **I** **A** **OR/OF** ✓ **I** **A** ✓ reduction/*reduksie* **A** **AO: Full marks/ *volpunte*** (2)
4.1.2	$\cos\alpha(\cot\alpha+\tan\alpha)$ $= \cos\alpha\left(\frac{\cos\alpha}{\sin\alpha}+\frac{\sin\alpha}{\cos\alpha}\right)$ $= \cos\alpha\left(\frac{\cos^2\alpha+\sin^2\alpha}{\sin\alpha\cdot\cos\alpha}\right)$ $= \frac{1}{\sin\alpha}$ *or / of* $\operatorname{cosec}\alpha$ **OR/*OF*** $\cos\alpha(\cot\alpha+\tan\alpha)$ $= \cos\alpha\left(\frac{\cos\alpha}{\sin\alpha}+\frac{\sin\alpha}{\cos\alpha}\right)$ $= \frac{\cos^2\alpha}{\sin\alpha}+\frac{\cos\alpha\cdot\sin\alpha}{\cos\alpha}$ $= \frac{1-\sin^2\alpha}{\sin\alpha}+\sin\alpha$ or / *of* $\frac{\cos^2\alpha}{\sin\alpha}+\sin\alpha$ $= \frac{1}{\sin\alpha}-\sin\alpha+\sin\alpha$ or / *of* $\frac{\cos^2\alpha+\sin^2\alpha}{\sin\alpha}$ $= \frac{1}{\sin\alpha}$ *or / of* $\operatorname{cosec}\alpha$ **OR/OF** $\cos\alpha(\cot\alpha+\tan\alpha)$ $= \cos\alpha\left(\frac{1}{\tan\alpha}+\frac{\tan\alpha}{\tan\alpha}\right) = \cos\alpha\left(\frac{1+\tan^2\alpha}{\tan\alpha}\right)$ $= \cos\alpha\left(\frac{\sec^2\alpha}{\tan\alpha}\right)$ $= \cos\alpha\left(\frac{\frac{1}{\cos^2\alpha}}{\frac{\sin\alpha}{\cos\alpha}}\right)$ $= \frac{1}{\sin\alpha}$ **OR / *OF*** $\operatorname{cosec}\alpha$	✓ both quotient Identities/ *beide kwosiënt-ident* **A** ✓ **S** **CA** ✓ **I** **A** ✓ **S** **A** **OR/*OF*** ✓ both quotient Identities/ *beide kwosiënt-ident* **A** ✓ **S** **CA** ✓ **I** **A** ✓ **S** **A** **OR/OF** ✓ $\frac{1}{\tan\alpha}$ **A** ✓ **I** $\sec^2\alpha$ **A** ✓ **S** **CA** ✓ **S** **A** (4)

4.2 #	$\text{LHS}/LK = \dfrac{1}{\sin(180° + x)\cdot\sin(360° - x)} - \cot^2(180° - x)$ $= \dfrac{1}{(-\sin x)\cdot(-\sin x)} - \cot^2 x$ $= \dfrac{1}{\sin^2 x} - \cot^2 x$ $= \text{cosec}^2 x - \cot^2 x$ $= 1 = \text{RHS}/RK$	✓ $-\sin x$ **A** ✓ $-\sin x$ **A** ✓ $\cot^2 x$ **A** ✓ **I** **A** ✓ **I** **A**
	OR/*OF*	**OR/*OF***
	$\text{LHS}/LK = \dfrac{1}{\sin(180° + x)\cdot\sin(360° - x)} - \cot^2(180° - x)$ $= \dfrac{1}{(-\sin x)\cdot(-\sin x)} - \cot^2 x$ $= \dfrac{1}{\sin^2 x} - \dfrac{\cos^2 x}{\sin^2 x}$ $= \dfrac{1-\cos^2 x}{\sin^2 x}$ $= \dfrac{\sin^2 x}{\sin^2 x}$ $= 1 = \text{RHS}/RK$	✓ $-\sin x$ **A** ✓ $-\sin x$ **A** ✓ $\cot^2 x$ **A** ✓ **I** $(\sin^2 x)$ **A** ✓ **S** CA (5)
		[11]

QUESTION/*VRAAG* 5

5.1		***f*:** ✓ shape/*vorm* **A** **Addendum** ✓ intercepts/*afsnitte* **A** ✓ asymptote/*asimptote* **A** ***g*:** ✓ shape/*vorm* **A** ✓ turning point/*draaipunt* **A** ✓ intercepts/*afsnitte* **A** (6)
5.2.1	$y \in$ {Real Numbers / Re*ële getalle*} **OR/*OF*** $y \in (-\infty ; \infty)$ **OR/*OF*** $y \in \mathbb{R}$	✓ range/*waardeversameling* **A** (1)
5.2.2	360°	✓ period/*periode* **CA** (1)
5.2.3	$x = 60°$	✓ equation of asymptote/*vergelyking van asimptote* **A** (1)
5.3	$0° < x < 90°$ **OR/*OF*** $x \in (0° ; 90°)$	✓both end points/*beide eind punte* **CA** ✓ notation/*notasie* **CA** (2)
		[11]

QUESTION/*VRAAG* 6



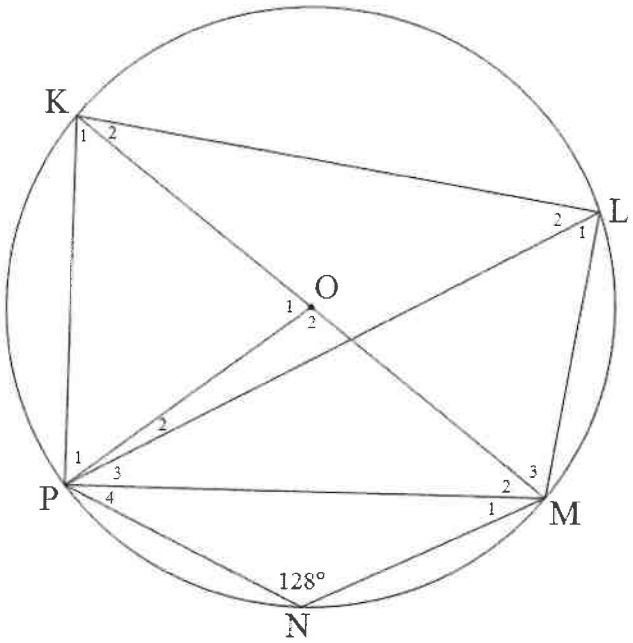
6.1	$CF = \frac{2}{3}(4{,}5\text{ m}) = 3\text{ m}$	✓ length of/*lengte van* CF **A** (1)
6.2.1	$\tan 42° = \frac{3\text{ m}}{AF}$ $AF = \frac{3\text{ m}}{\tan 42°}$ $\approx 3{,}3\text{ m}$ **OR/*OF*** $\tan 48° = \frac{AF}{3\text{ m}}$ $AF = 3\text{ m} \times \tan 48°$ $\approx 3{,}3\text{ m}$ **OR/*OF*** $\frac{AF}{\sin 48°} = \frac{3\text{ m}}{\sin 42°}$ $AF = 3\text{ m} \times \frac{\sin 48°}{\sin 42°}$ $\approx 3{,}3\text{ m}$	✓ tan ratio/ *verh* **CA** ✓approx. value of/ *benader. waarde van* AF **CA** **OR/*OF*** ✓ sin rule/ *reël* **CA** ✓approx. value of/ *benader.waarde van* AF **CA** **NPR** (2)
6.2.2	$AE^2 = AF^2 + FE^2 - 2AF \cdot FE\cos \hat{AFE}$ $(6)^2 = (3{,}3)^2 + (4{,}5)^2 - 2(3{,}3)(4{,}5)\cos \hat{AFE}$ $\cos \hat{AFE} = \frac{(3{,}3)^2 + (4{,}5)^2 - (6)^2}{2(3{,}3)(4{,}5)}$ $\cos \hat{AFE} \approx -0{,}1636....$ *ref / verwys* $\angle \approx 80{,}6°$ $\therefore \hat{AFE} \approx 180° - 80{,}6° = 99{,}4°$	✓ **F** **A** ✓ **SF** **CA** **from *vanaf* Q/*V* 6.2.1** ✓ **S** **CA** ✓ approx. value of/*benader. waarde van* $\hat{AFE}$ **CA** **NPR** (4)

6.2.3	$A_{\Delta AFE} = \frac{1}{2} AF \times FE \times \sin \hat{AFE}$ $= \frac{1}{2}(3,3) \times (4,5) \times \sin 99,4°$ $\approx 7,3\,m^2$ **OR/*OF*** $height = 3,3 \times \sin 80,6° \approx 3,26$ $A_{\Delta AFE} = \frac{1}{2} \times base \times height$ $= \frac{1}{2}(3,26) \times (4,5)$ $\approx 7,3\,m^2$	✓ **F** **A** ✓ **SF** **CA** **from Q 6.2.1 and Q6.2.2** ✓ area of/ *oppvlk* van ΔAFE **CA** **OR/*OF*** ✓ ⊥ height */hoogte* **A** ✓ **SF** **CA** ✓ area of/ *oppvlk* van ΔAFE **CA** **NPR** (3)
		[10]

QUESTION/*VRAAG* 7

7.1	on the same side of the chord/*aan dieselfde kant van die koord*	✓ **ST**	**A** (1)

7.2



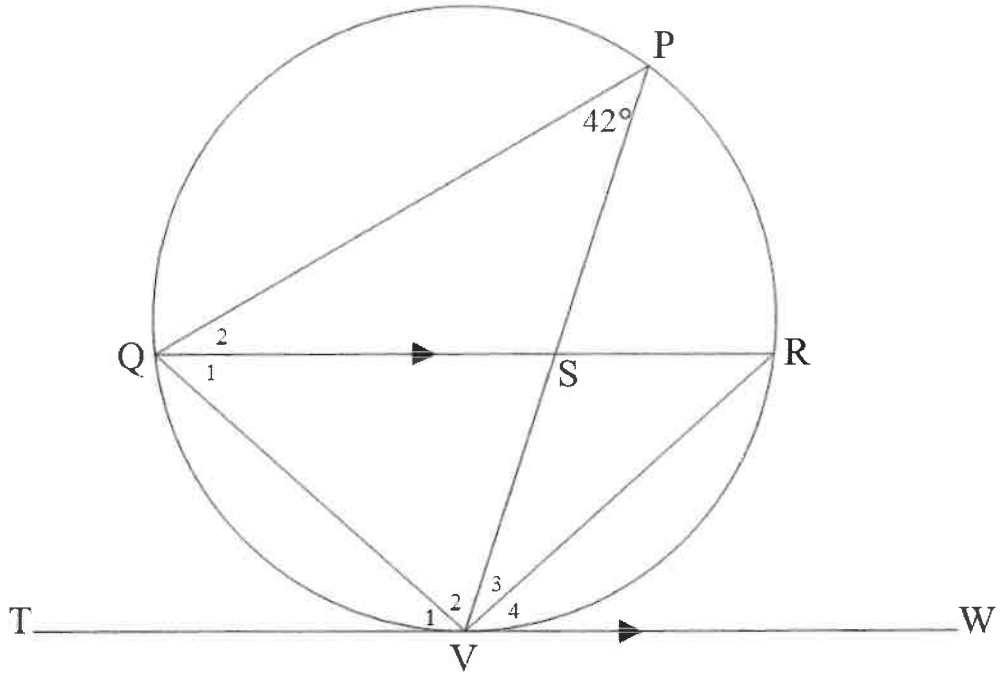
7.2.1	$\hat{K}_1 + 128° = 180°$ [Opp. ∠s of cyclicquad. / *teenoorst∠^{e} vankvhk*] $\hat{K}_1 = 52°$	✓ **ST** **A** ✓ **RE** **A** (2)
7.2.2	$K\hat{P}M = 90°$ [∠s in the semicircle/ *∠^{e} in halfsirkel*] $\therefore \hat{M}_2 = 180° - 52° - 90° = 38°$ [∠s of Δ / *∠^{e} van Δ*] $\therefore \hat{L}_2 = \hat{M}_2 = 38°$ [∠s in the same segment/*∠^{e} in dieslfde seg.*] **OR/*OF*** $\hat{O}_2 = 2\hat{K}_1 = 104°$ [∠ at centre = 2×∠at circum./*mdpts∠ = 2×omtrks∠*] $\hat{O}_1 = 180° - 104° = 76°$ [∠s on str. line/*∠^{e} op reguitlyn*] $\hat{L}_2 = \frac{1}{2}(76°) = 38°$ [∠ at centre = 2×∠at circum./*mdpts∠ = 2×omtrks∠*] **OR/*OF***	✓ **ST** ✓ **RE** **A** ✓ **ST** **CA** **From/vanaf Q/V7.2.1** ✓ **ST** ✓ **RE** **CA** **OR/*OF*** ✓ **ST** ✓ **RE** **CA** ✓ **ST** **CA** **from/*vanaf* Q/*V* 7.2.1** ✓ **ST** ✓ **RE** **CA** **OR/*OF***

	$\hat{P}_1 = \hat{K}_1 = 52°$ $(\angle \text{ opp} = \text{sides} / \angle \text{ } teenoor = sye)$ $\hat{O}_1 = 180° - 2(52°) = 76°$ $(\text{sum int} / \text{ } som\,binne \angle \Delta)$ $\hat{L}_2 = \frac{76°}{2} = 38°$ $\left[\begin{matrix}\angle \text{ at centre } = 2 \times \angle \text{at circum.}/ \\ mdpts\angle = 2 \times omtrks\angle\end{matrix}\right]$ **OR/*OF*** $\hat{K}_1 = \hat{L}_1 = 52°$ [∠s in the same segment/$\angle^e$ *in dieslfde seg.*] $\hat{L}_1 + \hat{L}_2 = 90°$ $\left(\angle\text{s in the semicircle/ } \angle^e \text{ } in\,die\,halfsirkel\right)$ $\hat{L}_2 = 90° - 52° = 38°$	✓ **ST** ✓ **RE** **CA** ✓ **ST** **CA** **from/*vanaf* Q/*V* 7.2.1** ✓ **ST** **CA** ✓ **RE** **CA** **OR/*OF*** ✓ **ST** ✓ **RE** **A** ✓ **ST** ✓ **RE** **A** ✓ **ST** **CA** (5)
7.2.3	$\hat{OPM} = \hat{M}_2 = 38°$ [∠s opposite equal sides/ *teen gelyke sye*] $\hat{P}_2 = 38° - 29° == 9°$ **OR/*OF*** $\hat{KPM} = 90°$ [∠s in the semicircle/ $\angle^e$ *in die halfsirkel*] $\hat{P}_2 = 90° - 52° - 29° = 9°$	✓ **ST/ RE** **A** ✓ **ST** **A** **OR/*OF*** ✓ **ST/RE** **A** ✓ **ST** **A** (2)
		[10]

QUESTION/*VRAAG* 8

8.1	the angle in the alternate segment/*die hoek in die oorstaande segment*	✓**ST**	**A** (1)

8.2



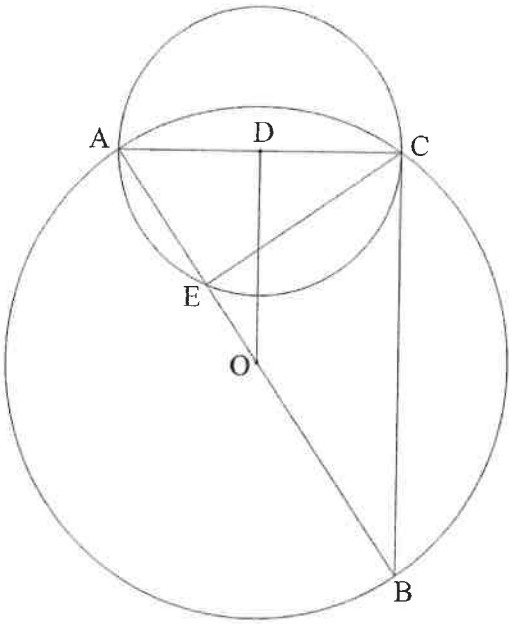
8.2.1	$\hat{R} = 42°$ [∠s in the same segment/ *∠e in dieselfde segment*]	✓ **ST** ✓ **RE**	**A** **A**
	$\hat{V}_1 = \hat{R} = 42°$ [tan - chord theorem/ *rklyn - koord st*]	✓ **ST** ✓ **RE**	**A** **A**
	$\hat{Q}_1 = \hat{V}_1 = 42°$ [alt.∠s / *verw ∠e* ; QR ∥ TW]	✓ **ST**	**A**
	$\hat{V}_4 = \hat{R} = 42°$ [alt.∠s / *verw ∠e* ; QR ∥ TW]	✓ **ST**	**A**
	OR / *OF*		
	$\hat{V}_4 = \hat{Q}_1 = 42°$ [tan - chord theorem/ *rklyn - koord st*]		(6)

8.2.2	$42° + \hat{QVR} + 42° = 180°$ [∠s on a straight line/] *op 'n reguitlyn*] $\therefore \hat{QVR} = 96° \neq 90°$ ∴ QR is not a diameter/*QR is nie 'n midddellyn* **OR/*OF*** $\hat{Q}_1 + \hat{R} = 42° + 42° = 84°$ [sum of ∠s of Δ] / *[somvan∠ e van Δ]* $\therefore \hat{QVR} = 96° \neq 90°$ ∴ QR is not a diameter/*QR is nie 'n midddellyn*	✓ Value of/ *waarde van* $\hat{QVR}$ **CA** ✓ Conclusion/*gevolgtrekking* **CA** **OR/*OF*** ✓ Value of/ *waarde van* $\hat{QVR}$ **CA** ✓ Conclusion/*gevolgtrekking* **CA** (2)
8.2.3	$\hat{Q}_2 = 180° - 42° - 42° - 67° = 29°$ [sum of ∠s of Δ / *som van ∠e van Δ*] **OR/*OF*** $\hat{V}_3 = 96° - 67° = 29°$ [proved in/ *bewys in* Q 8.2.2] $\hat{Q}_2 = \hat{V}_3 = 29°$ [∠s in the same segment/ *∠e in dieselfde segment*] **OR/*OF*** $67° + \hat{PQV} + 42° = 180°$ [sum of ∠s of Δ / *som van∠ e van Δ*] $\therefore \hat{PQV} = 71°$ $42° + \hat{Q}_2 = 71°$ $\therefore \hat{Q}_2 = 29°$	✓✓ **ST** **CA** ✓ **RE** **A** **OR/*OF*** ✓ **ST** **CA** ✓ **RE** **A** ✓ **ST** **CA** **OR/*OF*** ✓ **ST** **CA** ✓ **RE** **A** ✓ **ST** **CA** (3)
		[12]

QUESTION/*VRAAG* 9

9.1	the **CENTRE** of the circle/*die **MIDDELPUNT** van die sirkel*	✓ **ST**	**A** (1)

9.2



9.2.1	**Any TWO different reasons** ***Enige TWEE verskllende redes:*** • Tangent is perpendicular to a radius/diameter/*raaklyn is loodreg aan radius/middellyn* • Angle in the semicircle/*hoek in halfsirkel* • $\frac{1}{2}\hat{AOB} = \hat{ACD}$ [∠ at centre = 2×∠ at circum./ *mdpts∠ = 2×omtrks∠*]	✓**RE** ✓**RE**	**A** **A** (2)
9.2.2	Midpoint theorem/*middelpuntstelling* **OR/*OF*** Line joining the midpoints of two sides of a triangle/ *lyn wat die twee middelpunte van twee sye van 'n driehoek verbind* **OR/*OF*** $\hat{ADO} = \hat{ACB}\ (=90°)$ *DO* ∥ *CB* (corresponding ∠s/ *ooreenkmst ∠e*=) *OR / OF* (co-interior ∠s/*ko - binne ∠e*)	✓ **RE** **OR/*OF*** ✓ **RE**	**A** **A** (1)

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9.2.3	$AD = 4$ units/*eenh* $OA^2 = 6^2 + 4^2$ [Pythagoras Theorem /*stelling*] $OA = 2\sqrt{13}$ *or / of* $= \sqrt{52}$ *or / of* $\approx 7,21$ $AB = 2 \times 2\sqrt{13} = 4\sqrt{13}$ or/*of* $\approx 14,42$ **OR/*OF*** $BC = 2DO$ $= 2 \times 6 = 12$ $AB^2 = 12^2 + 8^2$ Th/*St* Pythagoras $AB = 4\sqrt{13}$ OR/*OF* $\approx 14,42$	✓ **ST** **A** ✓ **S** **A** ✓ length of/*lengte van* OA **CA** ✓ length of/*lengte van* AB **CA** **OR/*OF*** ✓ **ST** **A** ✓ **ST** **A** ✓ Pythagoras **CA** ✓ length of/*lengte van* AB **CA** **NPR** (4)
9.2.4	In ΔABC and ΔACE $\hat{A}$ is common / *gemene* $\angle$ $A\hat{C}B = A\hat{E}C$ [Both/ *beide* $= 90°$] $\hat{B} = A\hat{C}E$ [sum of/ *som* $\angle$s/*e* of/*van* Δ] $\therefore \Delta ABC \,\|\|\| \,\Delta ACE$ [$\angle\angle\angle$]	✓ **ST** **A** ✓ **ST** **A** ✓ **RE** *or/of* 3rd/de angle/*hoek* **A** (3)
9.2.5	$\frac{AC}{AE} = \frac{AB}{AC}$ [similar/*gelykvormige* Δs/*e*] $\therefore AC^2 = AE \times AB$	✓ proportion/*eweredig.* **A** (1)
9.2.6	$AC = 8$ units/*eenhede* and/ en $AB = 4\sqrt{13} \approx 14,42$ units/ *eenhede* $8^2 = AE \times 4\sqrt{13}$ $AE = \frac{8^2}{4\sqrt{13}} = \frac{16}{\sqrt{13}}$ **OR/*OF*** $= \frac{16}{13}\sqrt{13}$ *or / of* 4,44 units /*eenhede*	**CA FROM Q9.2.3** ✓ substitution/*vervanging* AB **CA** ✓ substitution/*vervanging* AC **A** ✓ length of/*lengte van* AE **CA** NPR (3) **[15]**

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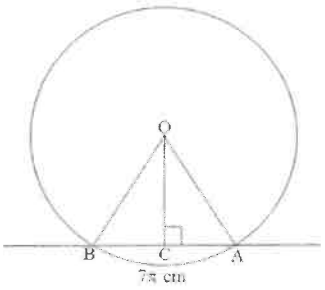
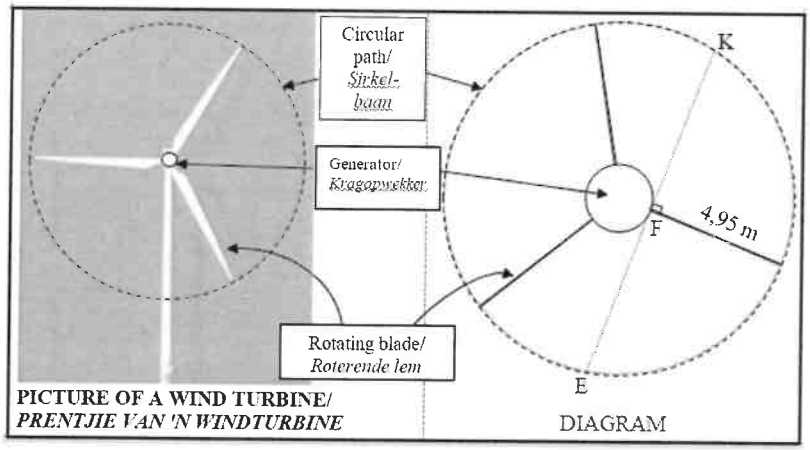
QUESTION/*VRAAG* 10

10.1



10.1.1 #	$A\hat{O}B = 2\pi - \frac{29}{18}\pi = \frac{7}{18}\pi$ OR/OF $\approx 0{,}39\ \pi$ OR/*OF* $\approx 1{,}22$	✓ ∠ in radians/ *radiale* **A** **NPR** (1)
10.1.2 #	$A\hat{O}B = \frac{7}{18}\pi \times \frac{180}{\pi} = 70°$	✓ **M** **A** ✓ degrees/grade **CA** **NPR** (2)
10.1.3	$s = r\theta$ $7\pi = r \times \frac{7}{18}\pi$ $r = 18$ cm $\quad \therefore$ OA = 18 cm	✓ **F** **A** ✓ **SF** **From/vanaf Q10.1.1** **CA** ✓ radius value/ *radius waarde* **CA** **NPR** (3)
10.1.4	Area of maj. sect./*opvl.van hf.sekt.* $= \frac{rs}{2}$ **OR/*OF*** $= \frac{r^2\theta}{2}$ $= \frac{18 \times 18 \times \frac{29}{18}\pi}{2}\text{cm}^2$ **OR/*OF*** $= \frac{(18)^2 \times \frac{29}{18}\pi}{2}\text{cm}^2$ $= 261\pi\text{ cm}^2 \approx 819{,}96\text{ cm}^2$ **OR/*OF*** Area of m. sect./ *opvl van h.sek.* $= \pi r^2 - \frac{rs}{2}$ **OR / *OF*** $= \pi r^2 - \frac{r^2\theta}{2}$ $= \pi(18)^2 - \frac{18 \times 7\pi}{2}\text{cm}^2$ **OR/*OF*** $= \pi(18)^2 - \frac{(18)^2 \times \frac{7}{18}\pi}{2}\text{cm}^2$ $= 261\pi\text{ cm}^2 \approx 819{,}96\text{ cm}^2$ **OR/OF** Area of m. sect./ *opvl van h.sek.* $= \frac{\theta}{360} \times \pi r^2$ $= \frac{290}{360} \times \pi(18)^2$ $= 261\pi\text{ cm}^2 \approx 819{,}96\text{ cm}^2$	✓ **F** **A** ✓ **SF** **CA** ✓ area of sector/ *oppervl van sektor* **CA** **OR/*OF*** ✓ **F** **A** ✓ **SF** **CA** ✓ area of sector/ *oppervl van sektor* **CA** ✓ **F** **A** ✓ **subst./ verv** **CA** ✓ area of sector/ *oppervl van sektor* **CA** **NPR** (3)

10.2

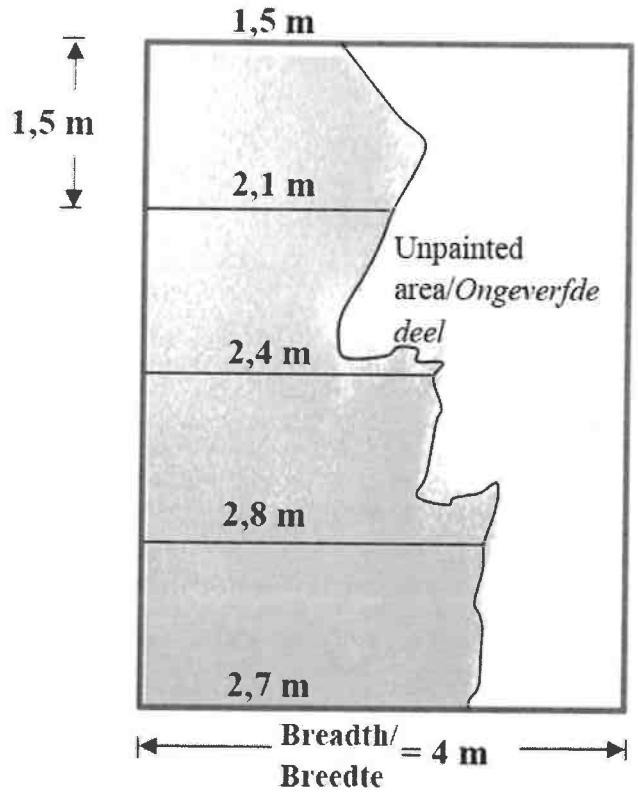


PICTURE OF A WIND TURBINE/
PRENTJIE VAN 'N WINDTURBINE

DIAGRAM

10.2.1	$4h^2 - 4dh + x^2 = 0$ $4(4,95)^2 - 4d(4,95) + (10,5)^2 = 0$ $98,01 - 19,8d + 110,25 = 0$ $-19,8d = -208,26$ $d \approx 10,52$ m **OR/*OF*** $d = h + \frac{x^2}{4h}$ $= 4,95 + \frac{(10,5)^2}{4(4,95)}$ $\approx 10,52$ m **OR/*OF*** Let/*Laat* $r = x + 4,95$ $(x + 4,95)^2 = x^2 + (5,25)^2$ $x^2 + 9,9x + 24,5025 = x^2 + 27,5625$ $9,9x = 3,06$ $x = \frac{17}{55}$ $d = 2\left(\frac{17}{55} + 4,95\right)$m $\approx 10,52$ m	✓ **F** **A** ✓ **SF** **A** ✓ diameter/*middellyn* **CA** **OR/*OF*** ✓ **F** **A** ✓ **SF** **A** ✓ diameter/*middellyn* **CA** **OR/*OF*** ✓ use of Pythagoras Theorem/*gebruik Pythagoras stelling* **A** ✓ value of/ *waarde van x* **A** ✓ diameter/*middellyn* **CA** **NPR** (3)
10.2.2	$v = \pi D n$ $6,61\pi$ m/s $= \pi(10,52\text{m}) \times n$ $\therefore n \approx 0,628$ *rev*/s $\approx 37,70$ *rev*/minute	✓ **F** **A** ✓ **SF** **CA** ✓ value of/ *waarde van n* **CA** ✓ conversion to per minute/*herleiding na per minuut* **CA** **NPR** (4)
		[16]

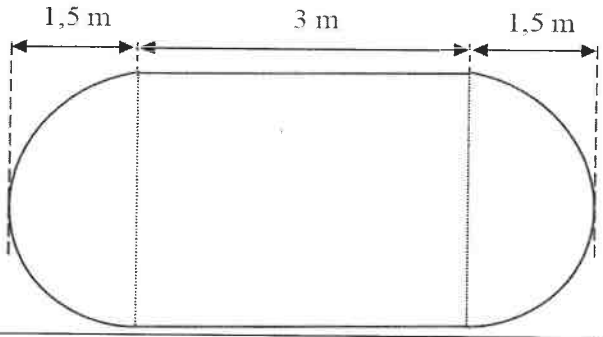
QUESTION/*VRAAG* 11



11.1.1	height/ *hoogte* $= 1,5 \times 4$ $= 6\,\text{m}$	✓ height/*hoogte* **A** (1)
11.1.2	$A_T = a(m_1 + m_2 + m_3 + ... + m_n)$ $= 1,5\left(\frac{1,5+2,1}{2} + \frac{2,1+2,4}{2} + \frac{2,4+2,8}{2} + \frac{2,8+2,7}{2}\right)\text{m}^2$ $= 1,5(1,8 + 2,25 + 2,6 + 2,75)\text{m}^2$ $= 14,1\ \text{m}^2$ **OR/*OF*** $A_T = a\left(\frac{o_1 + o_n}{2} + o_2 + o_3 + o_4 + ... + o_{n-1}\right)$ $= 1,5\left(\frac{1,5+2,7}{2} + 2,1 + 2,4 + 2,8\right)\text{m}^2$ $= 1,5(2,1 + 2,1 + 2,4 + 2,8)\text{m}^2$ $= 14,1\ \text{m}^2$	✓ **F** **A** ✓ value of/*waarde van a* **A** ✓ **SF** **A** ✓ painted area/*geverfde oppervl* **CA** **OR/*OF*** ✓ **F** **A** ✓ value of/*waarde van a* **A** ✓ **SF** **A** ✓ painted area/*geverfde oppervl* **CA** (4)

11.1.3	$A_{wall/muur} = l \times b$ $= 6\,\text{m} \times 4\,\text{m} = 24\,\text{m}^2$ $A_{\text{unpainted/ongeverfde}} = 24\,\text{m}^2 - 14{,}1\,\text{m}^2 = 9{,}9\,\text{m}^2$ Paint needed/*verf benodig* $= \dfrac{9{,}9\,\text{m}^2}{3{,}26\,\text{m}^2}$ $\approx 3{,}04\,\text{litres}$ $\therefore$ 4 tins Cost of paint/ *koste vanverf* $= 4 \times \text{R}156.36$ $= \text{R}625{,}44$ $\therefore$ YES, the minimum cost of the paint EXCEEDS R600/ *JA die minimum koste is MEER as R600*	**CA From/vanaf Q/V 11.1.1& Q/V11.1.2** ✓ area of wall/*oppervl van muur* **CA** ✓ unpainted area/ *ongeverfde oppervl* **CA** ✓**M** **A** ✓ tins needed/*blikkies benodig* **CA** ✓ total cost/*totale koste* **CA** ✓ conclusion/ *gevolgtrekking* **CA** (6)

11.2



11.2.1 #	$SA_{\text{cylindrical part / silindriese deel}} = 2\pi rh$ $= 2\pi(1,5)(3)\ \text{m}^2$ $= 9\pi\,\text{m}^2$ **OR/*OF*** $\approx 28,27\,\text{m}^2$ $SA_{\text{two hemispheres/twee hemisfere}} = 4\pi r^2$ $= 4\pi(1,5)^2\ \text{m}^2$ $= 9\pi\,\text{m}^2$ **OR/*OF*** $\approx 28,27\,\text{m}^2$ $SA_{\text{Total/Totaal}} = 9\pi\,\text{m}^2 + 9\pi\,\text{m}^2$ $= 18\pi\,\text{m}^2$ **OR/*OF*** $\approx 56,55\,\text{m}^2$ **OR/*OF*** $SA_{\text{Total/Totaal}} = 2\pi rh + 4\pi r^2$ $= 2\pi(1,5)(3) + 4\pi(1,5)^2\ \text{m}^2$ $= 9\pi\,\text{m}^2 + 9\pi\,\text{m}^2$ $= 18\pi\,\text{m}^2$ **OR**/OF $\approx 56,55\,\text{m}^2$	✓ **SF** **A** ✓ $SA_{\text{cylindrical part / silindriese deel}}$ **CA** ✓ **SF** **A** ✓ $SA_{\text{two hemispheres/twee hemisfere}}$ **CA** ✓ $SA_{\text{Total/Totaal}}$ **CA** **OR/*OF*** ✓ **M** **A** ✓ **SF** **A** ✓ **SF** **A** ✓ **S** **CA** ✓ $SA_{\text{Total/Totaal}}$. **CA** **NPR** (5)
11.2.2	$V_{\text{Total/Totaal}} = V_{\text{cylinder/silinder}} + V_{\text{sphere / sfeer}}$ $= \pi r^2 h + \frac{4}{3}\pi r^3$ $= \pi(1,5)^2(3) + \frac{4}{3}\pi(1,5)^3$ $= \frac{27}{4}\pi\,\text{m}^3 + \frac{9}{2}\pi\,\text{m}^3$ $= \frac{45}{4}\pi\ \text{m}^3 \approx 35,34m^3$ $V_{\text{tank}} = \pi r^2 h$ $\frac{45}{4}\pi = \pi(1,5)^2 h$ $h = 5$ ∴ height is more than three times the radius/ *Hoogte is meer as drie keer die radius* **OR/*OF***	✓ **SF** **A** ✓ **SF** **A** ✓ value of/ *waarde van* $V_{\text{Total/Totaal}}$ **CA** ✓ Value/ waarde h **CA** ✓ Conclusion **CA** **OR/*OF***

	$V_{\text{cylinder/silinder}} = \pi r^2 h$ $= \pi(1,5)^2(3)$ $= \frac{27}{4}\pi \text{m}^3$	✓ value of/ *waarde van* $V_{\text{cylinder/silinder}}$ **A**
	$V_{\text{sphere / sfeer}} = \frac{4}{3}\pi r^3$ $= \frac{4}{3}\pi(1,5)^3$ $= \frac{9}{2}\pi \text{m}^3$	✓ value of / *waarde van* $V_{\text{sphere / sfeer}}$ **A**
	$\therefore V_{\text{Total/Totaal}} = \frac{27}{4}\pi \text{m}^3 + \frac{9}{2}\pi \text{m}^3$ $= \frac{45}{4}\pi \text{ m}^3 \approx 142,63m^3$	✓value of / *waarde van* $V_{\text{Total/Totaal}}$ **CA**
	$V_{\text{tank/tenk}} = \pi r^2 h$ $\frac{45}{4}\pi = \pi(1,5)^2 h$ $h = 5$ $\therefore$ height is more than 3 times the radius/ *Hoogte is meer as drie keer die radius*	✓ Value/ waarde h **CA** ✓conclusion/ *gevolgtrekking* **CA** **NPR** (5)
		[21]

TOTAL/*TOTAAL*: 150

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