



Province of the
EASTERN CAPE
EDUCATION

**NATIONAL SENIOR
CERTIFICATE/
*NASIONALE SENIOR
SERTIFIKAAT***

GRADE/*GRAAD* 12

JUNE/*JUNIE* 2021

**TECHNICAL MATHEMATICS P2/
TEGNIесе WISKUNDE V2
MARKING GUIDELINE/*NASIENRIGLYN*
(*EXEMPLAR/EKSEMPLAAR*)**

MARKS/*PUNTE*: 150

This marking guideline consists of 15 pages./
Hierdie nasienriglyn bestaan uit 15 bladsye.

NOTE:

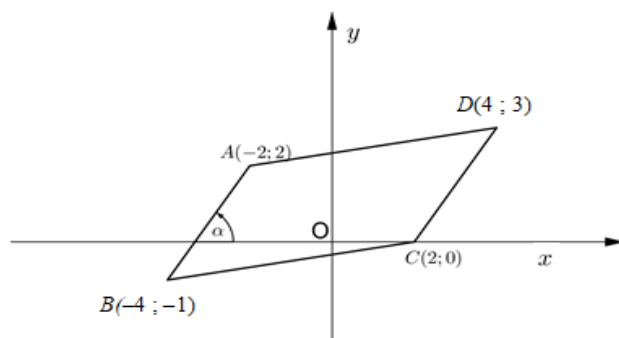
- Continuous accuracy (CA) applies ONLY where indicated in this marking guideline.
- Assuming values/answers in order to solve a problem is unacceptable.

LET WEL:

- *Volgehoue akkuraatheid (CA) is SLEGS van toepassing soos aangedui in hierdie nasienriglyn.*
- *Aanvaarding van waardes/antwoorde om 'n problem op te los, is onaanvaarbaar.*

MARKING CODES/NASIENKODES	
M	Method/ <i>Metode</i>
A	Accuracy/ <i>Akkuraatheid</i>
AO	Answer only/ <i>Slegs antwoord</i>
CA	Consistent accuracy/ <i>Deurlopende akkuraatheid</i>
F	Formula/ <i>Formule</i>
I	Identity/ <i>Identiteit</i>
R	Rounding/ <i>Afronding</i>
S	Simplification/ <i>Vereenvoudiging</i>
ST	Statement/ <i>Bewering</i>
RE	Reason/ <i>Rede</i>
ST/RE	Statement and correct reason/ <i>Bewering en korrekte rede</i>
SF	Substitution into correctly the correct formula/ <i>Vervanging in die korrekte formule</i>
NPU	No penalty for omitting units/ <i>Geen penalisering vir eenhede weggelaat</i>

QUESTION/VRAAG 1



1.1	$CD = \sqrt{(x_2 - x_1)^2 + (y_2 - y_1)^2}$ $= \sqrt{(4 - 2)^2 + (3 - 0)^2} = \sqrt{(2 - 4)^2 + (0 - 3)^2}$ $= \sqrt{13}$	✓SF A ✓CA	(2)
1.2	$m_{CD} = \frac{y_2 - y_1}{x_2 - x_1}$ $= \frac{3 - 0}{4 - 2} = \frac{0 - 3}{2 - 4}$ $= \frac{3}{2}$ <p>Equation of CD/Vergelyking van CD</p> $y - y_1 = m(x - x_1) \qquad y - y_1 = m(x - x_1)$ $y - 0 = \frac{3}{2}(x - 2) \qquad \text{OR/OF} \qquad y - 3 = \frac{3}{2}(x - 4)$ $y = \frac{3}{2}x - 3 \qquad y = \frac{3}{2}x - 3$	✓SF A ✓CA gradient/gradient ✓SF CA ✓CA	(4)
1.3	$m_{AB} = \frac{3}{2} \left(\begin{array}{l} \text{opp sides of } \parallel^m \\ \text{teenoorst sye van } \parallel^m \end{array} \right)$	✓CA	(1)
1.4	$\tan \alpha = \frac{3}{2}$ $\alpha = 56,31^\circ$	✓M CA ✓CA	(2)

1.5	$M_{AC} = \left(\frac{x_1 + x_2}{2}, \frac{y_1 + y_2}{2} \right)$ $= \left(\frac{-2 + 2}{2}, \frac{2 + 0}{2} \right)$ $= (0;1)$ <p style="text-align: center;">OR/OF</p> $M_{BD} = \left(\frac{x_1 + x_2}{2}, \frac{y_1 + y_2}{2} \right)$ $= \left(\frac{-4 + 4}{2}, \frac{3 - 1}{2} \right)$ $= (0;1)$	<p>✓SF A</p> <p>✓CA</p> <p style="text-align: center;">OR/OF</p> <p>✓SF A</p> <p>✓CA</p> <p>AO: Full marks/Volpunte</p>	(2)
1.6	$m_{\text{new line/nuwe lyn}} = -\frac{1}{m_{CD}}$ $= -\frac{2}{3}$ <p>Equation of new line / Vergelyking van nuwe lyn</p> $y - y_1 = m(x - x_1)$ $y - 1 = -\frac{2}{3}(x - 0)$ $y = -\frac{2}{3}x + 1$	<p>✓M CA</p> <p>✓SF CA</p> <p>✓CA</p>	(3)
			[14]

QUESTION/VRAAG 2



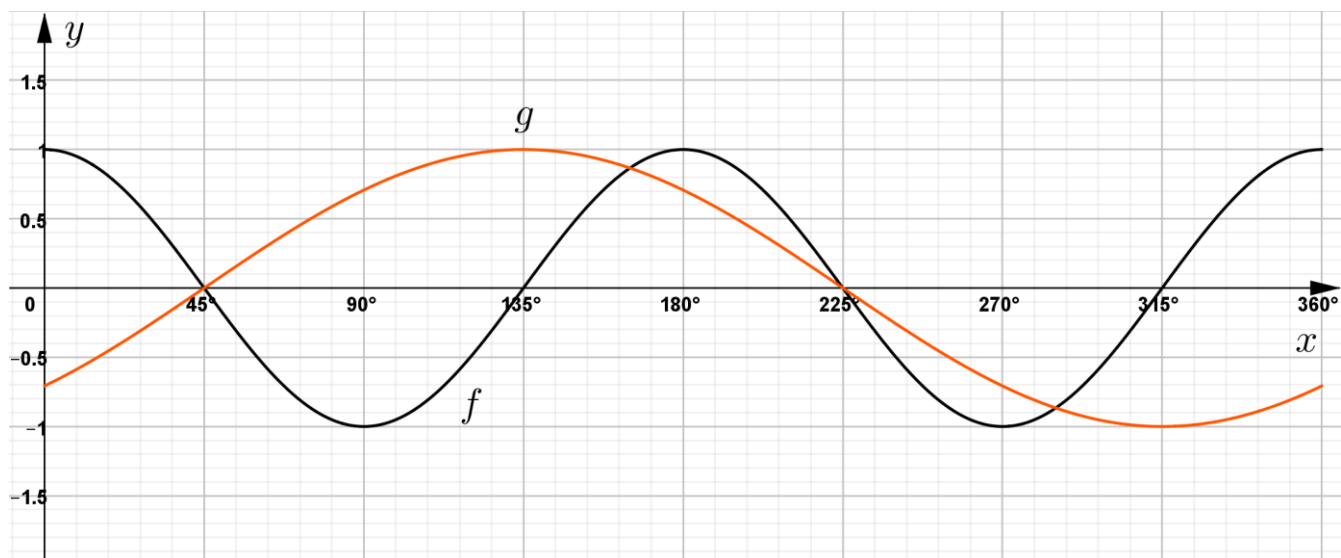
2.1.1	$x^2 + y^2 = r^2 = (2\sqrt{2})^2$ $x^2 + y^2 = 8$	✓SF A ✓CA	(2)
2.1.2	$y = 4 - x \dots \text{eq 1}$ <p>Subst into eq 2:</p> $x^2 + (4 - x)^2 = 8$ $x^2 + 16 - 8x + x^2 = 8$ $2x^2 - 8x + 8 = 0$ $(x - 2)(x - 2) = 0$ $x = 2$ $y = 4 - 2$ $y = 2$ $\therefore Q(2; 2)$ <p>OR/OF</p> $x = 4 - y \dots \text{eq 1}$ <p>Subst into eq 2:</p> $(4 - y)^2 + y^2 = 8$ $16 - 8y + y^2 + y^2 = 8$ $2y^2 - 8y + 8 = 0$ $(y - 2)(y - 2) = 0$ $y = 2$ $x = 4 - 2$ $x = 2$ $\therefore Q(2; 2)$	✓equation/vergelyking 1 M A ✓substitute into/vervang in 2 CA ✓S ✓x-value/waarde CA ✓y-value/waarde CA <p>OR/OF</p> ✓equation/vergelyking 1 M A ✓substitute into/vervang in 2 CA ✓S ✓y-value/waarde CA ✓x-value/waarde CA	(5)

2.1.3	No, $OQ \perp QP$ shortest distance to the centre is Q. Nee, $QA \perp QP$ kortste afstand na die middelpunt is Q.	✓No ✓justification/regverdiging	(2)
2.2.1	$B(0; -\sqrt{64}) = B(0; -8)$	✓A ✓A	(2)
2.2.2	$\frac{e^2}{3600} + \frac{(-4,8)^2}{64} = 1$ $\frac{e^2}{3600} = 1 - \frac{(-4,8)^2}{64}$ $\frac{e^2}{3600} = \frac{89}{100}$ $e^2 = 3204$ $e = 56,60 \quad (4\text{th quadrant}/4\text{de kwadrant})$	✓SF A ✓S CA ✓CA	(3)
			[14]

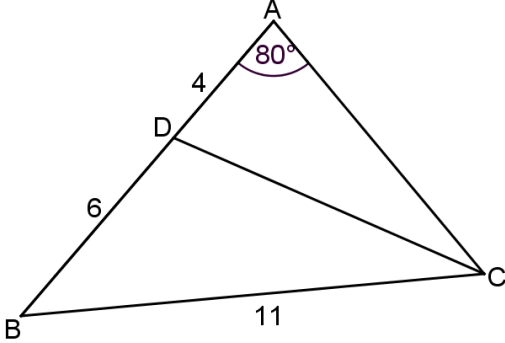
QUESTION/VRAAG 3			
3.1.1	$r^2 = (-5)^2 + (-2)^2$ $r = \sqrt{29}$ $\therefore \sin \beta = \frac{-2}{\sqrt{29}}$	✓quadrant/kwadrant A ✓value of/waarde van r A ✓value of ratio/waarde van verh CA	(3)
3.1.2	$\cot(360^\circ - \beta) + 1 = -\cot \beta + 1$ $= -\frac{5}{2} + 1$ $= -\frac{3}{2}$	✓ $-\cot \beta$ I ✓S CA	(2)
3.2.1	$\sin(\theta + \alpha)$ $= \sin(72^\circ + 96^\circ)$ $= 0,21$	✓S A ✓S CA AO Full marks/Volpunte	(2)
3.2.2	$\sec \alpha + \sin \frac{5\pi}{6} = \frac{1}{\cos \alpha} + \sin 150^\circ$ $= \frac{1}{\cos 96^\circ} + 0,5$ $= -9,07$	✓ $\frac{1}{\cos 96^\circ}$ A ✓ 0,5 A ✓CA AO Full marks/Volpunte	(3)
3.3	$\cos 156^\circ = -\cos 24^\circ$ $= -\sqrt{1 - \sin^2 24^\circ}$ $= -\sqrt{1 - m^2}$ <p style="text-align: center;">OR/OF</p> $\cos 156^\circ = -\cos 24^\circ$ $= -\sqrt{1 - m^2}$	✓ $-\cos 24^\circ$ I ✓ $-\sqrt{1 - \sin^2 24^\circ}$ I ✓S CA <p style="text-align: center;">OR/OF</p> ✓ $-\cos 24^\circ$ I ✓ complete diagram/voltoo diagram A ✓S CA	(3)
			[13]

QUESTION/VRAAG 4			
4.1	1	✓S A	(1)
4.2	$\frac{\tan(180^\circ - x) \cdot \sin(180^\circ + x) + \cos x}{\sec^2 x}$ $= \frac{-\tan x \cdot (-\sin x) + \cos x}{\frac{1}{\cos^2 x}}$ $= \left[\frac{\sin x}{\cos x} (\sin x) + \cos x \right] \times \cos^2 x$ $= \left[\frac{\sin^2 x + \cos^2 x}{\cos x} \right] \times \cos^2 x$ $= \cos x$	$\checkmark -\tan x \quad \text{I} \quad \text{A}$ $\checkmark -\sin x \quad \text{I} \quad \text{A}$ $\checkmark \frac{1}{\cos^2 x} \quad \text{I} \quad \text{A}$ $\checkmark \tan x = \frac{\sin x}{\cos x} \quad \text{I} \quad \text{A}$ $\checkmark \text{S} \quad \text{CA}$ $\checkmark \sin^2 x + \cos^2 x = 1 \quad \text{I} \quad \text{A}$ $\checkmark \text{S} \quad \text{CA}$	(7)
4.3	$\text{LHS/LK} = \cot^2 x \cdot \sin^2 x + \frac{\sin x}{\operatorname{cosec} x}$ $= \frac{\cos^2 x}{\sin^2 x} \cdot \sin^2 x + \frac{\sin x}{\frac{1}{\sin x}}$ $= \cos^2 x + \sin^2 x$ $= 1$ $= \text{RHS} / \text{RK}$	$\checkmark \cot x = \frac{\cos x}{\sin x} \quad \text{I} \quad \text{A}$ $\checkmark \frac{1}{\sin x} \quad \text{I} \quad \text{A}$ $\checkmark \sin^2 x + \cos^2 x = 1 \quad \text{I} \quad \text{A}$	(3)
4.4.1	$\sec 2x = 2$ $\therefore \cos 2x = \frac{1}{2}$ $\text{Ref/Ver } \angle = 60^\circ$ $\therefore 2x = 60^\circ \text{ or / of } 2x = 300^\circ$ $\therefore x = 30^\circ \text{ or / of } x = 150^\circ$	$\checkmark \cos 2x = \frac{1}{2} \quad \text{S} \quad \text{A}$ $\checkmark \text{reference/verwysings } \angle \quad \text{CA}$ $\checkmark 30^\circ \quad \text{CA}$ $\checkmark 150^\circ \quad \text{CA}$	(4)
4.4.2	$2 \tan(x - 30^\circ) = -3$ $\therefore \tan(x - 30^\circ) = -\frac{3}{2}$ $\text{Ref / Verw } \angle = 56,3^\circ$ $\therefore x - 30^\circ = 180^\circ - 56,3^\circ \text{ or/of } x - 30^\circ = 360^\circ - 56,3^\circ$ $\therefore x - 30^\circ = 123,7^\circ \text{ or/of } x - 30^\circ = 303,7^\circ$ $\therefore x = 153,7^\circ \text{ or/of } x = 333,7^\circ$	$\checkmark \tan(x - 30^\circ) = -\frac{3}{2} \quad \text{S} \quad \text{A}$ $\checkmark \text{reference/verwysings } \angle \quad \text{CA}$ $\checkmark 123,7^\circ \quad \text{CA}$ $\checkmark 303,7^\circ \quad \text{CA}$ $\checkmark 153,7^\circ \text{ and/en } 333,7^\circ \quad \text{CA}$	(5)
			[20]

QUESTION/VRAAG 5

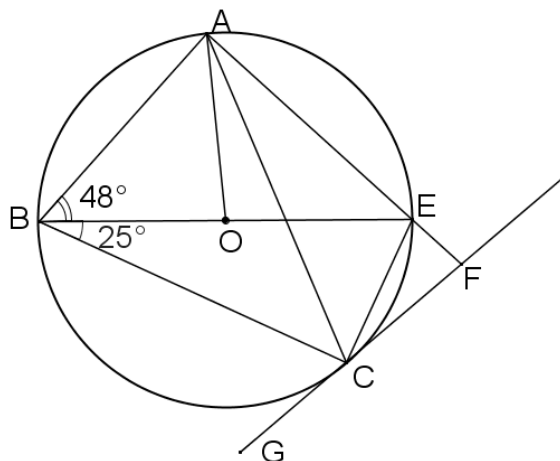


5.1	See diagram above/Sien bostaande diagram	✓x-intercepts/afsnitte A ✓endpoints/eindpunte A ✓turning points/draaipunte A	(3)
5.2.1	Amplitude _g = 1	✓ A	(1)
5.2.2	Period/Periode _f = $\frac{360^\circ}{2} = 180^\circ$	✓ A	(1)
5.2.3	Range/Waardeversameling _g : $y \in [-1;1]$	✓endpoints/eindpunte A ✓notation/notasie A	(2)
5.2.4	$f(135^\circ) - g(135^\circ) = 0 - 1 = -1$	✓function values/funksie waardes A ✓ CA AO Full marks/Volpunte	(2)
5.2.5	$x \in \{0^\circ; 180^\circ; 360^\circ\}$	✓ A	(3)
5.2.6	$x \in (225^\circ; 315^\circ)$ or/of $225^\circ < x < 315^\circ$	✓endpoints/eindpunte A ✓notation/notasie A	(2)
5.2.7	$x \in [135^\circ; 315^\circ]$ or/of $135^\circ \leq x \leq 315^\circ$	✓endpoints/eindpunte A ✓notation/notasie A	(2)
5.2.8	$x \in (90^\circ; 180^\circ)$ or/of $90^\circ < x < 180^\circ$	✓endpoints/eindpunte A ✓notation/notasie A	(2)
			[18]

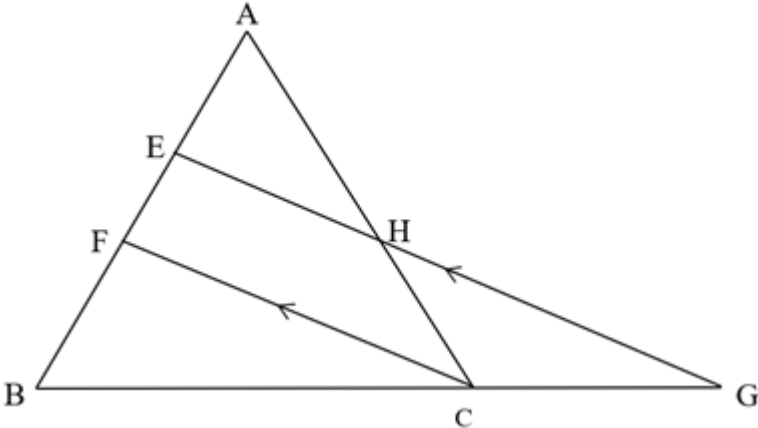
QUESTION/VRAAG 6			
6.1	$\frac{\sin P}{p} = \frac{\sin Q}{q}$	✓ A	(1)
6.2			
6.2.1	In $\triangle ABC$: $\frac{\sin C}{c} = \frac{\sin A}{a}$ $\frac{\sin C}{10} = \frac{\sin 80^\circ}{11}$ $\sin C = 0,8952\dots$ $\hat{C} = 63,534\dots^\circ$ $\hat{B} = 180^\circ - 80^\circ - 63,534\dots^\circ \approx 36$	✓ SF A ✓ S CA ✓ value of/waarde van \hat{C} CA ✓ value of/waarde van \hat{B} R	(4)
6.2.2	In $\triangle DBC$: $DC^2 = DB^2 + BC^2 - 2DB \cdot BC \cos B$ $= 6^2 + 11^2 - 2 \times 6 \times 11 \cos 36^\circ$ $= 950,20975\dots$ $DC = 7,09 \text{ cm}$	✓ F A ✓ SF CA ✓ S CA ✓ CA NPU	(4)
6.2.3	Area $\triangle DBC = \frac{1}{2} DB \cdot BC \sin B$ $= \frac{1}{2} \times 6 \times 11 \times \sin 36^\circ$ $= 19,40 \text{ cm}^2$	✓ F A ✓ SF CA ✓ S CA NPU	(3)

6.2.4	<p>In $\triangle ABC$:</p> $\frac{b}{\sin B} = \frac{a}{\sin A}$ $\therefore \frac{b}{\sin 36^\circ} = \frac{11}{\sin 80^\circ} \quad \text{or/of}$ $\therefore b = \frac{11 \sin 36^\circ}{\sin 80^\circ}$ $\therefore b = 6,57 \text{ units/eenhede}$ <p>In $\triangle ABC$:</p> $\frac{b}{\sin B} = \frac{c}{\sin C}$ $\therefore \frac{b}{\sin 36^\circ} = \frac{10}{\sin 63,53^\circ}$ $\therefore b = \frac{10 \sin 36^\circ}{\sin 63,53^\circ}$ $\therefore b = 6,57 \text{ units/eenhede}$	<p>✓SF CA</p> <p>✓S CA</p> <p>✓ value of/waarde van b CA</p>	<p>(3)</p> <p>[15]</p>
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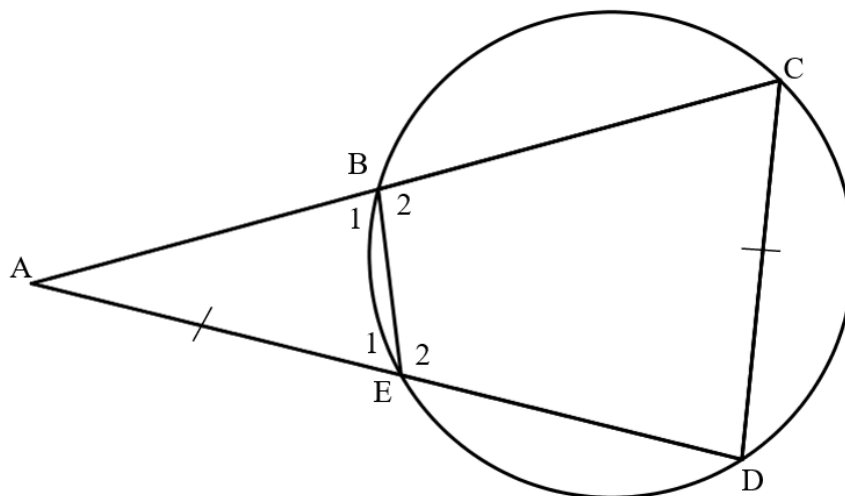
QUESTION/VRAAG 8			
8.1	Same point/ <i>dieselfde punt</i>	✓ A	(1)
8.2.1	$\begin{aligned} \hat{DCE} &= \hat{DEC} && \left(\begin{array}{l} \text{tangents from common pt /} \\ \text{raaklyne vanuit gemene pt} \end{array} \right) \\ \hat{DCE} &= \hat{DEC} \\ &= \frac{180^\circ - 46^\circ}{2} && \left(\begin{array}{l} \angle \text{s of } \Delta \text{ and } \angle \text{s opp} = \text{sides} \\ \angle \text{e van } \Delta \text{ en } \angle \text{e teenoor} = \text{sye} \end{array} \right) \\ &= 67^\circ \\ \hat{F} &= 67^\circ && (\text{tan-chord / raaklyn - koord}) \\ \hat{CGD} &= x = 67^\circ && \left(\begin{array}{l} \text{corresp. } \angle \text{s}; TY \parallel RQ \\ \text{verw. } \angle \text{e}; TY \parallel RQ \end{array} \right) \end{aligned}$	<div>✓ ST ✓ RE</div> <div>✓ ST RE</div> <div>✓ ST ✓ RE</div> <div>✓ ST RE</div>	(6)
8.2.2	From/ <i>Vanaf</i> 8.2.1 $\hat{DCE} = \hat{DGC}$ \therefore CGD is a circle/ <i>sirkel</i> $\left(\begin{array}{l} \text{converse tan-chord th} \\ \text{omgekeerde raaklyn - koord st} \end{array} \right)$	<div>✓ ST</div> <div>✓ RE</div>	(2)
8.2.3	$\begin{aligned} \hat{EGD} &= \hat{ECD} && \left(\begin{array}{l} \angle \text{s in same segment} \\ \angle \text{e in dies. segment} \end{array} \right) \\ \therefore \hat{EGD} &= 67^\circ \\ \therefore \hat{GEF} &= 67^\circ && \left(\begin{array}{l} \text{alt } \angle \text{s}; GD \parallel FE \\ \text{verw } \angle \text{e}; GD \parallel FE \end{array} \right) \end{aligned}$	<div>✓ ST ✓ RE</div> <div>✓ ST RE</div>	(3)
8.2.4	$\begin{aligned} \hat{FGE} &= 46^\circ && \left(\begin{array}{l} \text{Int } \angle \text{s of } \Delta / \angle \text{e van } \Delta \\ \text{or/of} \\ \angle \text{s on str line} / \angle \text{e op reguit lyn} \\ \text{or/of} \\ \text{ext } \angle \text{ of cyclic quad/buite } \angle \text{ kdvh} \end{array} \right) \end{aligned}$	<div>✓ ST</div> <div>✓ RE</div> <div>✓ RE {Any two / <i>Enige twee</i>}</div>	(3)



8.3.1	$\hat{BAE} = 90^\circ$ $\left(\begin{array}{l} \angle \text{ in semi-circle} \\ \angle \text{ in halwe - sirkel} \end{array} \right)$	✓ ST ✓ RE	(2)
8.3.2	$\hat{AOE} = 96^\circ$ $\left(\begin{array}{l} \angle \text{ at centre} = 2 \times \angle \text{ at circumf} \\ \text{midpts } \angle = 2 \times \text{omtreks } \angle \end{array} \right)$	✓ ST ✓ RE	(2)
8.3.3	$\hat{CEF} = 73^\circ$ $\left(\begin{array}{l} \text{ext } \angle \text{ of cyclic quad/} \\ \text{buite } \angle \text{ van kdvk} \end{array} \right)$	✓ ST ✓ RE	(2)
8.3.4	$\hat{ECF} = 25^\circ$ $\left(\begin{array}{l} \text{tan - chord/} \\ \text{raaklyn - koord} \end{array} \right)$	✓ ST ✓ RE	(2)
8.3.5	$\hat{ACE} = 48^\circ$ $\left(\begin{array}{l} \angle \text{ s in same segm} \\ \angle \text{ e in dieselfde segm} \end{array} \right)$	✓ ST ✓ RE	(2)
8.3.6	$\hat{AEC} = 107^\circ$ $\left(\begin{array}{l} \angle \text{ s on straight line OR opp } \angle \text{ s of cyclic quad} \\ \angle \text{ e op reguit lyn OF teenoorst } \angle \text{ e van kdvh} \end{array} \right)$ $\hat{ACG} = 107^\circ$ $\left(\begin{array}{l} \text{tan-chord} \\ \text{raaklyn - koord} \end{array} \right)$	✓ ST RE ✓ ST ✓ RE	(3)
			[28]

QUESTION/VRAAG 9			
9.1	proportionally	✓A	(1)
			
9.2.1	$\frac{AE}{EF} = \frac{AH}{HC}$ $\left(\begin{array}{l} \text{line } \parallel \text{ one side of } \Delta \\ \text{OR / OF} \\ \text{prop th; EH } \parallel \text{ FC} \\ \text{lyn } \parallel \text{ een sy van } \Delta \end{array} \right)$	✓ST ✓RE	(2)
9.2.2	$\frac{AE}{EF} = \frac{AH}{HC}$ $\frac{3}{EF} = \frac{6}{5}$ $EF = \frac{15}{6}$ $= \frac{5}{2}$ (from/vanaf 9.2.1)	✓ST CA ✓ST CA ✓ST CA	(3)
9.2.3	$BF = 7 - EF$ $= 7 - \frac{5}{2}$ $= \frac{9}{2}$	✓ST CA	(1)
9.2.4	In ΔBEG : $\frac{BC}{CG} = \frac{BF}{FE}$ $\left(\begin{array}{l} \text{line } \parallel \text{ one side of } \Delta \\ \text{OR/OF} \\ \text{prop th; EH } \parallel \text{ FC} \\ \text{lyn } \parallel \text{ een sy van } \Delta \end{array} \right)$ $= \frac{4,5}{2,5}$ $= 1,8$ < 3	✓ST/RE ✓ST ✓conclusion/gevolgtrekking	(3)
			[10]

QUESTION/VRAAG 10



10.1	<p>In $\triangle ACD$ and $\triangle AEB$:</p> <p>1) \hat{A} is common /gemeenskaplik</p> <p>2) $\hat{C} = \hat{E}_1$ $\left(\begin{array}{l} \text{ext } \angle \text{ of cyclic quad} \\ \text{buite } \angle \text{ van kdvh} \end{array} \right)$</p> <p>3) $\hat{D} = \hat{B}_1$ $\left(\begin{array}{l} \text{Int } \angle \text{ s of } \Delta / \angle \text{ e van } \Delta \\ \text{or/of} \\ \text{ext } \angle \text{ of cyclic quad} \\ \text{buite } \angle \text{ van kdvh} \end{array} \right)$</p> <p>$\therefore \triangle ACD \equiv \triangle AEB$ (AAA)</p>	<p>✓ ST ✓ ST ✓ RE</p> <p>✓ Mark is either for 3rd angle or for the reason AAA / Punt is vir die 3^{de} hoek of vir die rede AAA</p>	(3)
10.2	<p>$\frac{AC}{AE} = \frac{CD}{EB}$ ($\triangle ACD \equiv \triangle AEB$)</p> <p>$\therefore \frac{AC}{CD} = \frac{CD}{EB}$ ($AE = DC$; given / gegee)</p> <p>$\therefore CD^2 = AC \cdot EB$</p>	<p>✓ ST ✓ RE</p> <p>✓ ST A</p>	(3)
10.3	<p>$CD^2 = 13 \times 3$</p> <p>$CD = \sqrt{39}$</p> <p>$\approx 6,24 \text{ cm}$</p>	<p>✓ ST A</p> <p>✓ M A</p> <p>✓ value of/ waarde van CD CA</p>	(3)
			[10]
		TOTAL / TOTAAL:	150