Province of the
EASTERN CAPE
EDUCATION

NATIONAL
SENIOR CERTIFICATE

GRADE 12

JUNE 2023

TECHNICAL MATHEMATICS P1

MARKS: 150

TIME: 3 hours

This question paper consists of 14 pages, including a 2-page formula sheet and 2 answer sheets.

INSTRUCTIONS AND INFORMATION

Read the following instructions carefully before answering the questions.

1. This question paper consists of NINE questions. Answer ALL the questions.

2. Clearly show ALL calculations, diagrams, graphs, etc. which you have used in determining your answers.

3. Answer QUESTION 4.4 and QUESTION 4.2.5 on the ANSWER SHEETS provided. Write your name in the spaces provided and then hand in the ANSWER SHEETS with your answer book.

4. An approved scientific calculator (non-programmable and non-graphical) may be used, unless stated otherwise.

5. If necessary, answers should be rounded off to TWO decimal places, unless stated otherwise.

6. Number the answers correctly according to the numbering system used in this question paper.

7. Diagrams are NOT necessarily drawn to scale.

8. An information sheet with formulae is included at the end of the question paper.

9. Write neatly and legibly.

QUESTION 1

1.1 Solve for x:

1.1.1 $x(3x-1)=0$ (2)

1.1.2 $2x^2+13=5x$ (correct to TWO decimal places) (4)

1.1.3 $(x-3)(x+4)\geq 0$ (3)

1.2 Solve for x and y if:

$y = x^2 - 11x + 36$ and $y = 2x - 6$ (5)

1.3 The formula to calculate the area of a trapezium is given by $\mathrm{A}=\frac{1}{2}h(a+b)$ where:

h = height
a = first base
b = second base

1.3.1 Use the formula given above to make h the subject of the formula. (1)

1.3.2 The picture below shows a planter in the shape of a trapezium. An artisan is required to make a planter with bases, $a = 15,24$ cm and $b = 20,32$ cm. The area of the planter has to be 1,8064 m^2.

Determine the height of the planter in cm. (3)

1.4 Determine the value of $K = 89 - 16$ in binary form. (3)

[21]

QUESTION 2

2.1 Given the equation: $x^2 - 121 = 0$

2.1.1 Write down the number of roots the equation has. (1)

2.1.2 Determine the numerical value of the discriminant. (2)

2.1.3 Hence, describe the nature of roots of the equation. (1)

2.2 Determine the value(s) of p for which the equation $x^2 + px + 4 = 0$ will have real roots. (4)

[8]

QUESTION 3

3.1 Simplify the following WITHOUT using a calculator:

3.1.1 $\dfrac{m^6n^7}{\left(m^2n\right)^3}$ (2)

3.1.2 $\sqrt{98x^2}+\sqrt{32x^2}$ (3)

3.1.3 $\dfrac{1}{2}\log_2 16+\log_3 27$ (3)

3.2 Solve for x:

3.2.1 $\left(x+1\right)^3=64$ (3)

3.2.2 $1+\log x=\log\left(x+9\right)$ (4)

3.3 Solve for m and n if $m-\left(3-i\right)=ni+5$. (3)

3.4 Write $z=1-5i$ in the form $r\,cis\,\theta$. (5)

[23]

QUESTION 4

4.1 Given the functions f and g defined by $f(x) = -x^2 + 4$ and $g(x) = -2x + 1$

4.1.1 Determine the x and y intercepts of f. (3)

4.1.2 Determine the coordinates of the turning point of f. (2)

4.1.3 Determine the x and y intercepts of g. (3)

4.1.4 Sketch the graph of f and g on the ANSWER SHEET provided.
Clearly show all the intercepts with the axis and the turning points of the graph. (6)

4.1.5 Write down the domain of f. (2)

4.2 Given: $h(x) = 2^x + 1$ and $k(x) = \sqrt{7 - x^2}$

4.2.1 Write down the radius of k. (1)

4.2.2 Write down the equation of the asymptote of h. (1)

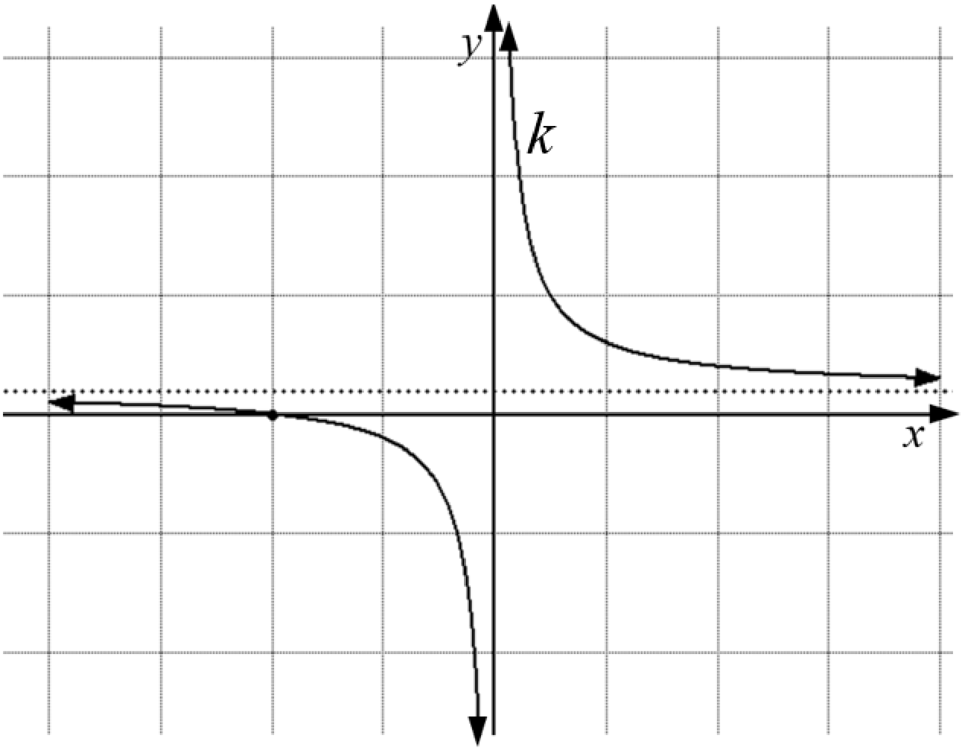
4.2.3 Determine the y-intercept of h. (1)

4.2.4 Write down the domain of k. (2)

4.2.5 On the ANSWER SHEET provided, sketch the graph of h and k on the same set of axes.
Clearly show all the asymptotes and intercepts with the axis. (5)

4.2.6 Show by shading the region from your graphs where $h(x) \leq k(x)$. (1)

4.3 The graph of the function defined by $k(x) = \frac{8}{x} + 2$ is drawn below.



Determine:

4.3.1 The x-intercept of k (3)

4.3.2 The equation of the horizontal asymptote of k (1)

4.3.3 The range of k (1)

[32]

QUESTION 5

5.1 The population of a small-town decreases at a rate of 0,5% per annum on a reducing-balance method over a period of time. Determine the population of the town at the end of 2025 if the population of the town was 300 000 at the beginning of 2015. (3)

5.2 Pauline invested R5 000 into an account that offers 9,5% per annum compound interest. Determine after how many years will the investment be worth R75 000. (5)

5.3 A trading company invested a sum of R200 000 in a savings account at an interest rate of 7,5% per annum compounded monthly. After 36 months, R50 000 was withdrawn and the remaining amount invested at 6% per annum, compounded quarterly for the remaining period. Determine the investment amount at the end of the 5-year period. (5)

[13]

QUESTION 6

6.1 Determine the derivative of $f\left(x\right)=1-3x$ by using FIRST PRINCIPLES. (5)

6.2 Determine:

6.2.1 $\frac{dy}{dx}$ if $y=\frac{2}{x^3}-15x+7m$ (3)

6.2.2 $D_x\left[9+2x^{-1}+\sqrt[3]{x^{27}}\right]$ (3)

6.3 Given: $g\left(x\right)=\frac{x^2-7x}{20}+15$

6.3.1 Write down the gradient function of g. (2)

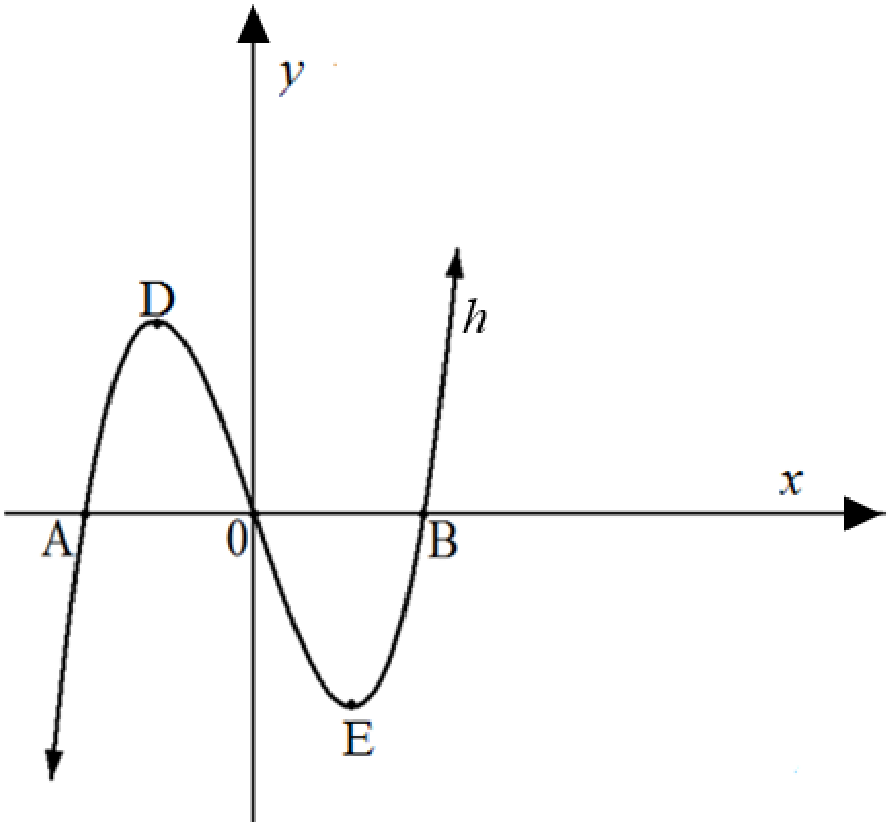
6.3.2 Hence, determine the gradient of g at $x=5$. (2)

[15]

QUESTION 7

The graph of the function *h* defined by $h(x)=x^3-16x$ is drawn below. The graph of *h* intersects the *x*-axis at A, (0;0) and at B. The *y*-axis is at the origin.

D and E are the turning points of *h*.



Determine:

7.1 The coordinates of points A and B (4)

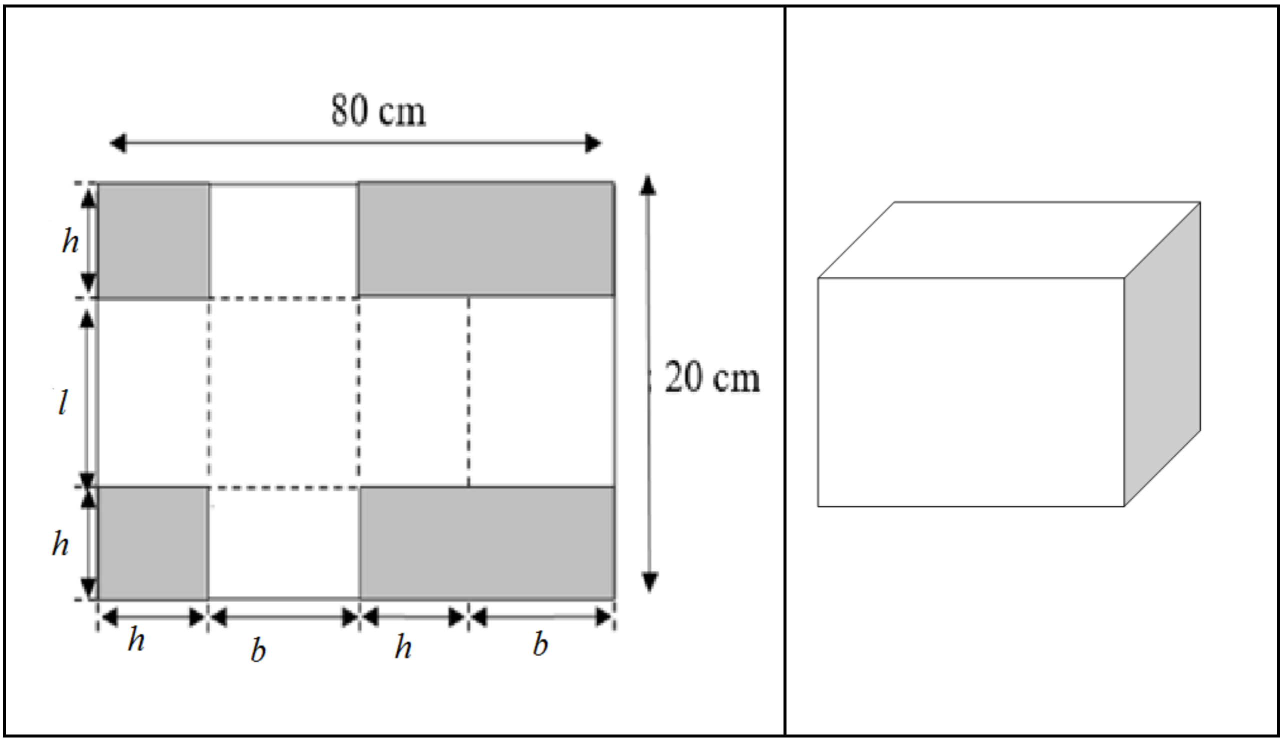
7.2 The coordinates of D and E, the turning points of h (5)

7.3 The value(s) of x for which $h'(x)\geq 0$ (3)

[12]

QUESTION 8

A box is made from a rectangular piece of cardboard, 80 cm by 20 cm, by cutting out the shaded areas and folding along the dotted lines as shown in the diagram below.



8.1 Express the breadth, b, in terms of the height h. (2)

8.2 Hence, show that the volume of the box is given by $V = 2h^3 - 100h^2 + 800h$. (3)

8.3 Determine the numerical value of h that will maximise the volume of the box. (6)

[11]

QUESTION 9

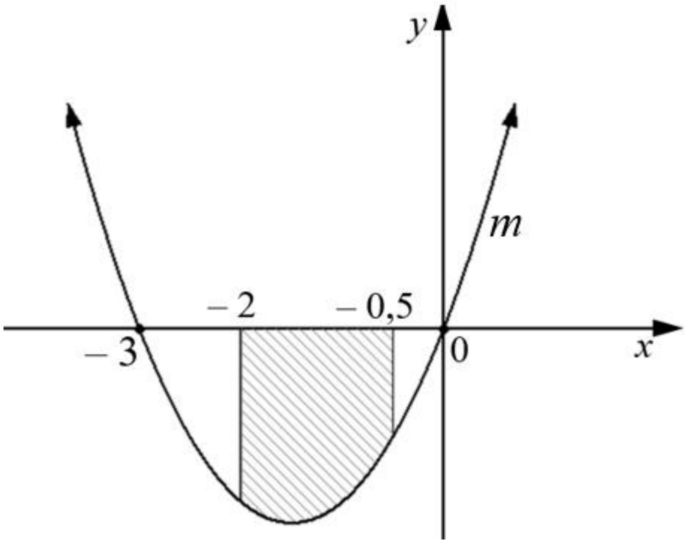
9.1 Determine the following integrals:

9.1.1 $\int\left(2x^2 + x\right)\,dx$ (3)

9.1.2 $\int \dfrac{16x^6 - 4x^2}{2x}\,dx$ (4)

9.1.3 $\int_0^2 x^3\,dx$ (3)

9.2 The sketch below shows the shaded area bounded by the function *m* defined by $m(x) = x^2 + 3x$ and the x-axis between the points where $x = -2$ and $x = -0,5$.



Determine the area of the shaded region of the graph of m bounded by the graph and the x-axis, between $x = -2$ and $x = -0,5$. (5)

[15]

TOTAL: 150

INFORMATION SHEET: TECHNICAL MATHEMATICS

$x = \dfrac{-b \pm \sqrt{b^2 - 4ac}}{2a}$ $\qquad x = -\dfrac{b}{2a}$ $\qquad y = \dfrac{4ac - b^2}{4a}$

$a^x = b \Leftrightarrow x = \log_a b, \quad a > 0, a \neq 1$ and $b > 0$

$A = P(1 + ni)$ $\qquad A = P(1 - ni)$ $\qquad A = P(1 + i)^n$ $\qquad A = P(1 - i)^n$

$i_{eff} = \left(1 + \frac{i}{m}\right)^m - 1$

$f'(x) = \lim\limits_{h \to 0} \dfrac{f(x+h) - f(x)}{h}$

$\int x^n \, dx = \dfrac{x^{n+1}}{n+1} + C, \; n \neq -1$ $\qquad \int k x^n \, dx = k.\dfrac{x^{n+1}}{n+1} + C, \; n \neq -1$

$\int \dfrac{1}{x} dx = \ln x + C, \; x > 0$ $\qquad \int \dfrac{k}{x} dx = k.\ln x + C, \; x > 0$

$\int a^x \, dx = \dfrac{a^x}{\ln a} + C, \; a > 0$ $\qquad \int k a^{nx} \, dx = k.\dfrac{a^{nx}}{n \ln a} + C, \; a > 0$

$d = \sqrt{(x_2 - x_1)^2 + (y_2 - y_1)^2}$ $\qquad M\left(\dfrac{x_2 + x_1}{2} ; \dfrac{y_2 + y_1}{2}\right)$

$y = mx + c$ $\qquad y - y_1 = m(x - x_1)$ $\qquad m = \dfrac{y_2 - y_1}{x_2 - x_1}$ $\qquad \tan\theta = m$

$\dfrac{x^2}{a^2} + \dfrac{y^2}{b^2} = 1$

In ΔABC: $\dfrac{a}{\sin A} = \dfrac{b}{\sin B} = \dfrac{c}{\sin C}$ $\qquad a^2 = b^2 + c^2 - 2bc.\cos A$

$\text{area of } \Delta ABC = \dfrac{1}{2} ab.\sin C$

$\sin^2\theta + \cos^2\theta = 1$ $\qquad 1 + \tan^2\theta = \sec^2\theta$ $\qquad 1 + \cot^2\theta = \operatorname{cosec}^2\theta$

$\pi \, rad = 180°$

Angular velocity = $\omega = 2\pi n$ where n = rotation frequency

Angular velocity = $\omega = 360° n$ where n = rotation frequency

Circumferential velocity $= v = \pi Dn$ where D = diameter and n = rotation frequency

Circumferential velocity $= v = \omega r$ where ω = Angular velocity and r = radius

Arc length $s = r\theta$ where r = radius and θ = central angle in radians

Area of a sector $= \dfrac{rs}{2}$ where r = radius, s = arc length

Area of a sector $= \dfrac{r^2\theta}{2}$ where r = radius and θ = central angle in radians

$4h^2 - 4dh + x^2 = 0$ where h = height of segment, d = diameter of circle and x = length of chord

$A_T = a(m_1 + m_2 + m_3 + ... + m_n)$ where a = equal parts, $m_1 = \dfrac{o_1 + o_2}{2}$
and n = number of ordinates

OR

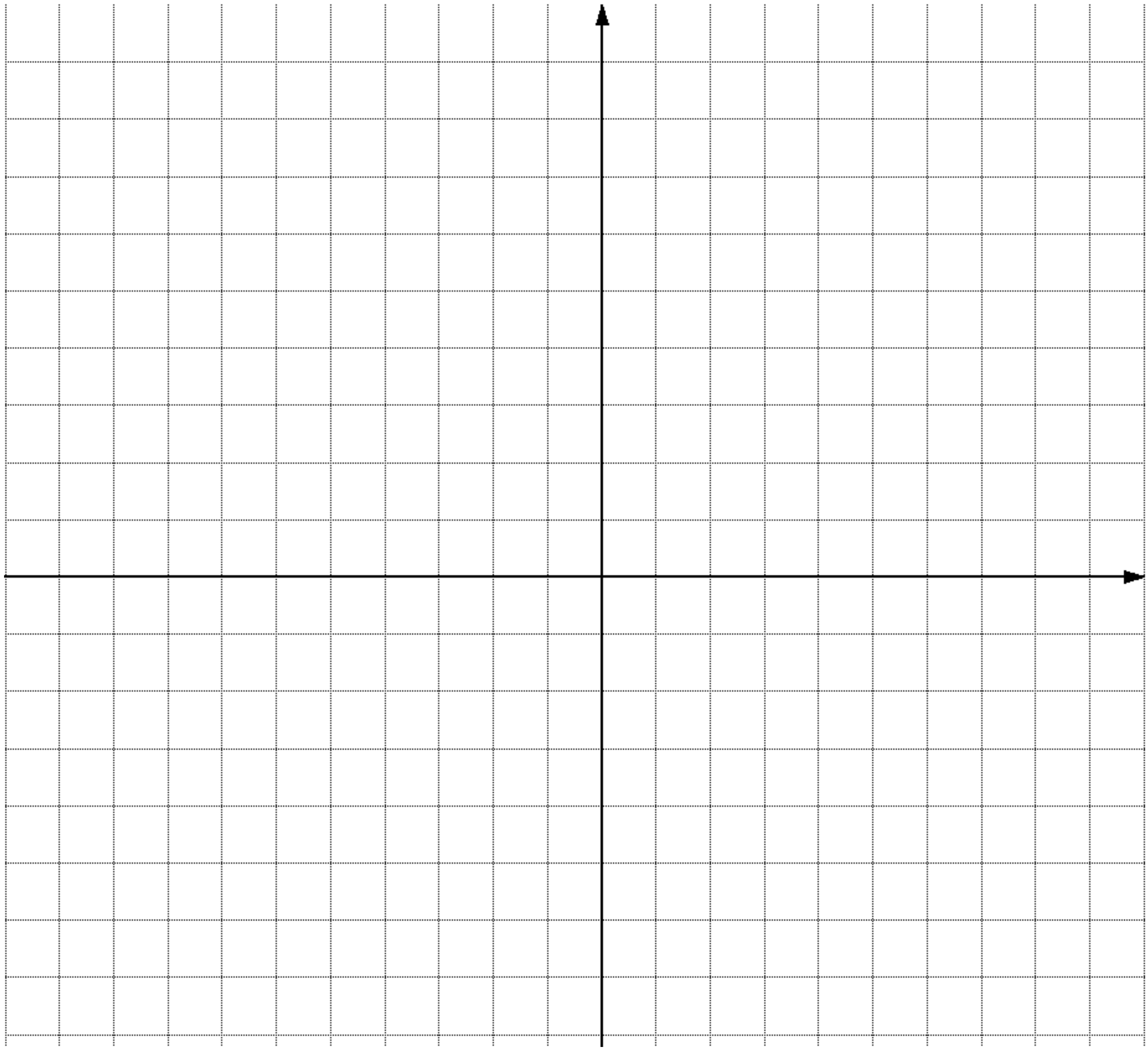
$A_T = a\left(\dfrac{o_1 + o_n}{2} + o_2 + o_3 + ... + o_{n-1}\right)$ where a = equal parts, $o_i = i^{th}$ ordinate
and n = number of ordinates

ANSWER SHEET

Learner's name: ……………………………………………….. Class: ……………………..

School's name: …………………………………………………

QUESTION 4.4



ANSWER SHEET

Learner's name: ……………………………………………….. Class: ……………………..

School's name: …………………………………………………

QUESTION 4.2.5