MARKS: 150
TIME: 3 hours

This question paper consists of 14 pages, including a formula sheet and 4 diagram sheets.
INSTRUCTIONS AND INFORMATION

Read the following instructions carefully before answering the questions.

1. This question paper consists of 12 questions. Answer ALL the questions.

2. Clearly show ALL calculations, diagrams, graphs, et cetera, which you have used in determining the answers.

3. Answers only will not necessarily be awarded full marks.

4. An approved scientific calculator (non-programmable and non-graphical) may be used, unless stated otherwise.

5. Round off your answers to TWO decimal places if necessary, unless stated otherwise.

6. Diagrams are not necessarily drawn to scale.

7. FOUR diagram sheets for answering QUESTION 1.2, QUESTION 2.1, QUESTION 2.2, QUESTION 4.1.1, QUESTION 7.4 and QUESTION 11.2 are attached at the end of this question paper. Write your name and surname in the spaces provided and insert the sheets in your answer book.

8. Number the answers correctly according to the numbering system used in this question paper.

9. Write legibly and present your work neatly.

10. An information sheet, with formulae, is included at the end of this question paper.
QUESTION 1

The graph below shows the number of people and their ages who voted in the Local Government elections in one voting station. Use the graph to answer the questions which follow.

![Ogive for ages of people in one voting station](image)

1.1. How many people voted in that voting station? (1)

1.2. Complete the frequency and cumulative frequency columns of the frequency table on DIAGRAM SHEET 1. (4)

1.3. Determine the median age. (1)

1.4. How many voters were 35 years or older? (1) [7]
QUESTION 2

The ages (in years) of the oldest person in each of 12 homes that were counted by a Statistics South Africa official in Dutywa during the 2011 Census survey are given below:

27  62  75  35  50  44  31  54  40  31  65  39

2.1 Draw a box and whisker diagram for the above data using DIAGRAM SHEET 1. (4)

2.2 Another Statistics South Africa official counted in Alice. The data was summarised as follows: minimum age was 28, lower quartile was 32, the inter-quartile range was 18, the median was 42 and the maximum age was the same as in Dutywa.

Draw, on the same diagram (DIAGRAM SHEET 1), the box and whisker diagram for the above data. (2)

2.3 Which town had more families with older people? Justify your answer. (2)

[8]

QUESTION 3

The following are the pass percentages of 10 subjects done at a high school:

65,3  81,9  70  88,2  56,5  94,8  83  44,1  75  79,4

3.1 Determine the mean percentage. (2)

3.2 Determine the standard deviation of the percentages. (3)

3.3 What is the lowest pass percentage that the subject must get in order to be within one standard deviation of the mean? (2)

[7]

QUESTION 4

In order to determine whether fatigue plays a role in the number of tries scored in a match, the number of tries scored in successive matches is compared. The following table represents the number of tries scored by one rugby team in the IRB rugby world cup in each of their seven matches:

<table>
<thead>
<tr>
<th>Matches</th>
<th>1</th>
<th>2</th>
<th>3</th>
<th>4</th>
<th>5</th>
<th>6</th>
<th>7</th>
</tr>
</thead>
<tbody>
<tr>
<td>No. of tries</td>
<td>11</td>
<td>8</td>
<td>6</td>
<td>6</td>
<td>5</td>
<td>5</td>
<td>4</td>
</tr>
</tbody>
</table>

4.1 Draw the scatter plot to represent the above data using the DIAGRAM SHEET 2. (2)

4.2 Which of the following functions has a graph which will be the best fit for the scatter plot: exponential, quadratic or linear? (1)

4.3 Will the team score more than 5 tries or less than 5 tries in the 8th match? Justify your answer. (1)

[4]
QUESTION 5

In the figure, \( C(-4; 7) \), \( A(1; 4) \) and \( B(s; -1) \) are vertices of \( \triangle CAB \). \( E \) is the \( x \)-intercept of line \( AE \) with equation \( y = -3x + 7 \).

5.1 Determine the length of \( AC \).  
5.2 Determine the values of \( s \) and \( t \) if \( M(-3; t) \) is the midpoint of \( BC \).  
5.3 Prove that \( \triangle CAB \) is a right angled triangle.  
5.4 Determine the equation of a line passing through \( C \) and parallel to \( AB \).  
5.5 Calculate the size of \( \angle CAE \).  
5.6 Determine \( p \) if \( CA \) is extended to \( D(p; 1) \) such that \( C, A \) and \( D \) are collinear.  

\[ \text{[21]} \]
QUESTION 6

6.1 The diagram below shows two circles touching at point E(2 ;1). The smaller circle has its centre at the origin and a bigger circle has centre D(a ;3). The length of the radius of the bigger circle is twice the length of the radius of the smaller circle.

6.1.1 Calculate the length of the radius of the smaller circle. (2)

6.1.2 Determine the value of a if a > 0. (4)

6.1.3 Determine the equation of the bigger circle. (3)

6.1.4 Determine the equation of a tangent to both circles at E. (4)

6.2 The length of the diameter of the circle $x^2 + y^2 - 4x + 5y + k = 0$ is 24. Determine:

6.2.2 the co-ordinates of the centre of the circle. (4)

6.2.3 the value of k. (3)
QUESTION 7

Polygon MATHS has undergone two transformations to obtain M"A"T"H"S". MATHS is first transformed to M'AT'H'S'. M'AT'H'S' is then transformed to M"A"T"H"S".

7.1 Describe, in words, the transformation of MATHS to M'AT'H'S'. (2)

7.2 Write down the rule of transforming M'AT'H'S' to M"A"T"H"S". (2)

7.3 Write down a single rule that transforms MATHS to M"A"T"H"S". (2)


7.5 Hence or otherwise determine a single rule that transforms MATHS to M"A"T"H"S". (2)

7.6 Determine the ratio:
area of MATHS: area of M"A"T"H"S". (2)

7.7 If MATHS is translated to LUCID using the rule (x ; y) → (x − 4 ; y + 3), write down the coordinates of L. (2)
QUESTION 8

\( T' = \left( \frac{5}{2}, -\frac{7}{2} \right) \) is the image of \( T(-3; 2) \) after rotating \( T \) in an anti-clockwise direction about the origin through an angle of \( \theta^\circ \).

8.1 Determine the value of \( \theta \). \hspace{1cm} (6)

8.2 \( T(-3; 2) \) is rotated about the origin through an angle of \( (\theta + \alpha)^\circ \) in an anti-clockwise direction to \( S(3; -2) \). Write down the value of \( \alpha \). \hspace{1cm} (2)

QUESTION 9

9.1 Given: \( 7\cos \beta + 5 = 0 \) and \( \tan \beta > 0 \). Use a suitable diagram to determine:

9.1.1 \( \tan \beta \) \hspace{1cm} (3)

9.1.2 \( \sin(450^\circ + \beta) \) \hspace{1cm} (2)

9.1.3 \( \sin 2\beta \) \hspace{1cm} (2)

9.2 Determine the general solution of: \( \cos 2x - \frac{1}{3} = \frac{1}{3}\sin x \) \hspace{1cm} (9)

QUESTION 10

10.1 Simplify without using a calculator:

\[
\frac{\tan 360^\circ - x \cdot \cos x - 90^\circ + \cos(540^\circ - x)}{\tan x}
\]

\hspace{1cm} (7)

10.2 Consider the identity: \( (\sin x + \cos x)^2 = \sin 2x + 1 \)

10.2.1 Prove the identity. \hspace{1cm} (2)

10.2.2 Hence, or otherwise, determine the maximum value of \( 3 \sin 5\theta + 3 \cos 5\theta \) \hspace{1cm} (4)

10.3 Prove that:

\[
\frac{\sin 2x + 1}{\cos 2x} = \frac{\sin x + \cos x}{\cos x - \sin x}
\]

\hspace{1cm} (4)

\[17\]
QUESTION 11

Given: \( f(x) = \frac{1}{2} \tan x \) and \( g(x) = \sin x + 1 \)

11.1 Write down the equations of the asymptotes of \( f \) for \( x \in [-90^\circ;180^\circ] \). (2)

11.2 Sketch the graphs of \( f \) and \( g \) on the same set of axes provided on DIAGRAM SHEET 4, for \( x \in [-90^\circ;180^\circ] \). (7)

11.3 Use your graphs to write down the values of \( x \) for which \( g(x) - f(x) = 1 \). (2)

11.4 Determine the value of \( f(45^\circ) - g(30^\circ) \). (2)

11.5 If \( h(x) = f(x + m) \), write down two possible values of \( m \) such that one of the asymptotes of \( h \) is the line \( x = 30^\circ \) for \( x \in [-90^\circ;180^\circ] \). (2)

11.6 Write down the period of \( f(2x) \). (1)

TOTAL: 150

QUESTION 12

The diagram below shows the side elevation of a house. The roof of the house is a scalene triangle with \( \angle PQR = \alpha \). QT is a ladder such that UQT is a straight line. The angle of elevation QT R = \( \theta \). PQ = 2t and QR = t.

12.1 Prove that \( \sin UPQ = \sin(\alpha + \theta) \) (2)

12.2 Prove that \( UT = \frac{2t \sin(\theta + \alpha)}{\sin \alpha} + \frac{t}{\sin \theta} \) (5)

12.3 Determine the length of UQ and hence the area of \( \triangle UPQ \) if \( t = 3m \), \( \theta = 42^\circ \) and \( \alpha = 83^\circ \). (4)

TOTAL: 150
INFORMATION SHEET: MATHEMATICS

\[
x = \frac{-b \pm \sqrt{b^2 - 4ac}}{2a}
\]

\[A = P(1 + ni) \quad A = P(1 - ni) \quad A = P(1 - i)^n \quad A = P(1 + i)^n\]

\[\sum_{i=1}^{n} a_i = n \quad \sum_{i=1}^{n} i = \frac{n(n + 1)}{2} \quad T_n = a + (n - 1)d \quad S_n = \frac{n}{2}[a + (n - 1)d] - \]

\[T_n = ar^{n-1} \quad S_n = \frac{a(r^n - 1)}{r - 1}; \quad r \neq 1 \quad S_\infty = \frac{a}{1 - r}; -1 < r < 1\]

\[F = \frac{x}{\sqrt{1 + i^2}} - 1 \quad P = \frac{x^{1-(1+i)^{-n}}}{i}\]

\[f'(x) = \lim_{h \to 0} \frac{f(x + h) - f(x)}{h}\]

\[d = \sqrt{(x_2 - x_1)^2 + (y_2 - y_1)^2} \quad M \left( \frac{x_1 + x_2}{2}; \frac{y_1 + y_2}{2} \right)\]

\[y = mx + c \quad y - y_1 = m(x - x_1) \quad m = \frac{y_2 - y_1}{x_2 - x_1} \quad m = \tan \theta\]

\[\text{In } \triangle ABC: \quad \frac{a}{\sin A} = \frac{b}{\sin B} = \frac{c}{\sin C} \quad a^2 = b^2 + c^2 - 2bc \cos A\]

area \( \triangle ABC = \frac{1}{2} ab \sin C\)

\[\sin \left( \gamma + \beta \right) = \sin \alpha \cos \beta + \cos \alpha \sin \beta \quad \sin \left( \gamma - \beta \right) = \sin \alpha \cos \beta - \cos \alpha \sin \beta\]

\[\cos \left( \gamma + \beta \right) = \cos \alpha \cos \beta - \sin \alpha \sin \beta \quad \cos \left( \gamma - \beta \right) = \cos \alpha \cos \beta + \sin \alpha \sin \beta\]

\[\cos 2\alpha = \begin{cases} 
\cos^2 \alpha - \sin^2 \alpha \\
1 - 2\sin^2 \alpha \\
2\cos^2 \alpha - 1 
\end{cases} \quad \sin 2\alpha = 2\sin \alpha \cos \alpha\]

\[(x; y) \to (x \cos \theta + y \sin \theta; y \cos \theta - x \sin \theta)\]

\[(x; y) \to (x \cos \theta - y \sin \theta; y \cos \theta + x \sin \theta)\]

\[\bar{x} = \frac{\sum fx}{n} \quad \sigma^2 = \frac{\sum (f_i - \bar{x})^2}{n}\]

\[P(A) = \frac{n(A)}{n} \quad P(A \text{ or } B) = P(A) + P(B) - P(A \text{ and } B)\]
**DIAGRAM SHEET 1**

**QUESTION 1.2**

<table>
<thead>
<tr>
<th>AGES</th>
<th>FREQUENCY</th>
<th>CUMULATIVE FREQUENCY</th>
</tr>
</thead>
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<tr>
<td>$18 \leq x &lt; 23$</td>
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<td>$23 \leq x &lt; 28$</td>
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<td>$48 \leq x &lt; 53$</td>
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</tbody>
</table>

**QUESTION 2.2 and 2.2**

![Graph](image-url)
DIAGRAM SHEET 3

QUESTION 7.4
DIAGRAM SHEET 4

QUESTION 11.2