



Province of the
EASTERN CAPE
EDUCATION

**NATIONAL
SENIOR CERTIFICATE**

GRADE 12

SEPTEMBER 2013

MATHEMATICS P1

MARKS: 150

TIME: 3 hours



This question paper consists of 11 pages, including a formula sheet
and 1 diagram sheet.

INSTRUCTIONS AND INFORMATION

Read the following instructions carefully before answering the questions.

1. This question paper consists of 12 questions. Answer ALL questions.
2. Clearly show ALL calculations, diagrams, graphs, et cetera that you have used in determining the answers.
3. Answers only will not necessarily be awarded full marks.
4. An approved scientific calculator (non-programmable and non-graphical may be used), unless stated otherwise.
5. If necessary, answers should be rounded off to TWO decimal places, unless stated otherwise.
6. Number the answers correctly according to the numbering system used in this question paper.
7. Diagrams are NOT drawn to scale.
8. An information sheet with formulae is attached.
9. A diagram sheet is supplied for QUESTION 6.3. Write your name in the space provided and then hand the diagram sheet in with your ANSWER BOOK.
10. Write legibly and present your work neatly.

QUESTION 1

1.1 Solve for x , correct to TWO decimal places, where necessary.

$$1.1.1 \quad (2x + 3)(3 - x) = 4 \quad (3)$$

$$1.1.2 \quad 2x^2 + 3x = 3 \quad (4)$$

$$1.1.3 \quad (2x - 3)^2 < 4 \quad (4)$$

1.2 Solve for x and y simultaneously.

$$\begin{aligned} 3y &= 2x \\ x^2 - y^2 + 2x - y &= 1 \end{aligned} \quad (6)$$

1.3 Evaluate without the use of a calculator:

$$\sqrt[2013]{\frac{2^{2014} - 2^{2013}}{2^{4026}}} \quad (4)$$

[21]

QUESTION 2

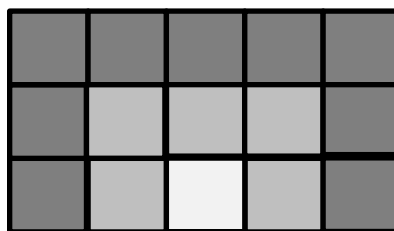
Tiles have been displayed in patterns as shown in the figure below.
The pattern numbers are extended.



Pattern 1



Pattern 2



Pattern 3

2.1 Write the number of tiles required for each pattern as a sequence. (1)

2.2 Extend the sequence to the 5th pattern. (1)

2.3 What type of sequence is listed in QUESTION 2.2? Give a reason for your answer. (2)

2.4 Write down the general term of this sequence. (2)

[6]

QUESTION 3

The sum to n terms of an arithmetic series is $S_n = \frac{n}{2}(7n + 15)$.

3.1 How many terms must be added to give a sum of 425? (5)

3.2 Determine the sixth term of the series. (4)
[9]

QUESTION 4

4.1 $(k - 4); (k + 1); m; 5k$ is a sequence of numbers. The first three numbers form an arithmetic sequence and the last three a geometric sequence. Find k and m if both are positive. (8)

4.2 Calculate p if $\sum_{k=1}^{\infty} 27p^k = \sum_{t=1}^{12} (24 - 3t)$ (6)
[14]

QUESTION 5

5.1 Sizwe's motor car costing R200 000 depreciated at a rate of 8% per annum on the reducing balance method. Calculate how long it took for the car to depreciate to a value of R90 000 under these conditions. (5)

5.2 Pat starts a five year savings plan. Pat begins by depositing R2 000 into the account and makes further deposits of R2 000 at the end of every month immediately thereafter. The interest rate is 6% per annum compounded monthly.

Calculate the value of his investment at the end of the five year period. (5)

5.3 Sally takes out a 20 year loan of R100 000. She repays the loan by means of equal monthly payments starting *three months* after the granting of the loan. The interest rate is 18% per annum compounded monthly.

5.3.1 Calculate the amount Sally would owe, two months after the loan was taken out. (2)

5.3.2 Calculate the value of the monthly repayments. (3)
[15]

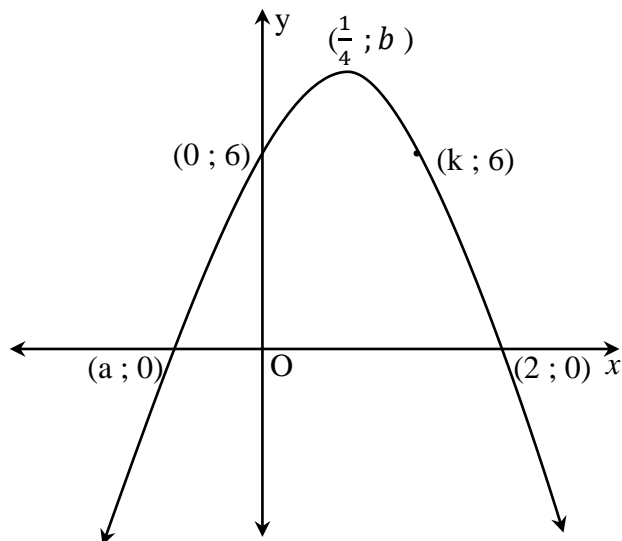
QUESTION 6

Given the function, $f(x) = \frac{3}{x-2} - 3$

- 6.1 Write down the equations of the asymptotes of $f(x)$. (2)
- 6.2 Determine the intercepts of the graph with the axes. (3)
- 6.3 Make a neat sketch of $f(x)$, using the diagram sheet provided. Indicate all intercepts with the axes. (4)
- 6.4 Write down the equation of the axes of symmetry of f with a negative gradient. (2)
- 6.5 Write down the range of $f(x) + 2$. (1)
- [12]**

QUESTION 7

In the figure the graph of the parabola f , intersecting the x-axis at $(a; 0)$ and $(2; 0)$ is given. Furthermore $(\frac{1}{4}; b)$ is the coordinate of the turning point of the curve of f , while its point of intersection with the y-axis is $(0; 6)$. The point $(k; 6)$ lies on the curve of f .



Determine:

- 7.1 the value of k (1)
- 7.2 the value of a (1)
- 7.3 the value of b (show the necessary calculations) (6)
- 7.4 the equation of the tangent to the curve of f at the point $(0; 6)$ (3)
- [11]**

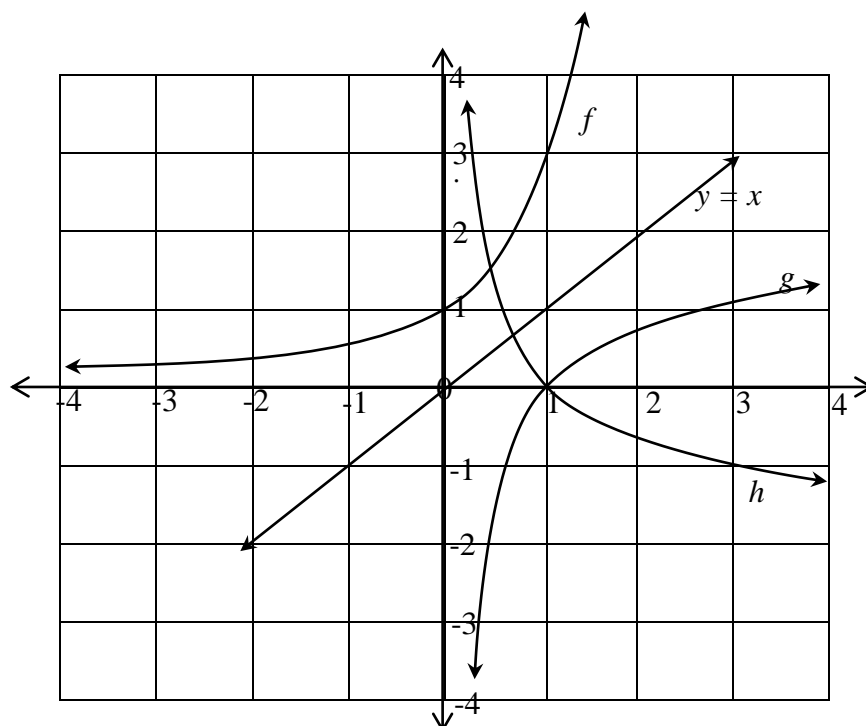
QUESTION 8

8.1 In the sketch, the following functions are represented:

$$f(x) = 3^x$$

$g(x)$, the reflection of f in the line $y = x$.

$h(x)$, the reflection of g in the x -axis



8.1.1 Determine the defining equations of g and h in the form $y = \dots$ (5)

8.1.2 Determine with the aid of the above sketch the value(s) for x for which:

a. $f(x) > g(x)$ (1)

b. $\frac{g(x)}{h(x)} = 1$ (1)

8.2 The following function is given: $f(x) = 2x^2$

8.2.1 Determine $f^{-1}(x)$. (2)

8.2.2 Is f^{-1} a function. Give a reason for your answer. (2)

8.2.3 How can the domain be restricted in order for f^{-1} to be a function? (2)

[13]

QUESTION 9

9.1 If $f(x) = -x^2$, determine $f'(x)$, by using first principles. (5)

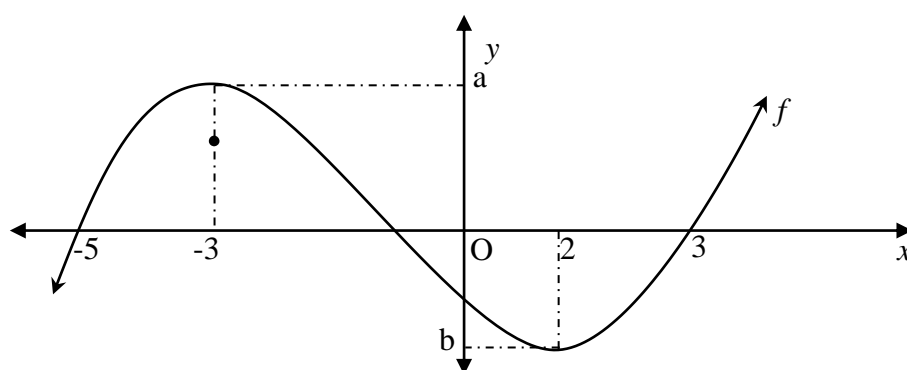
9.2 Determine $f'(x)$ if:

9.2.1 $f(x) = \frac{x^2 - 9}{x - 3}$ (4)

9.2.2 $f(x) = \sqrt{x}(x + \frac{1}{x})^2$ (4)

[13]**QUESTION 10**

10.1 Part of the graph $f(x) = ax^3 + bx^2 + cx + d$ is shown below.



10.1.1 For which values of x is $f'(x) = 0$? (2)

10.1.2 For which values of x is $f'(x) > 0$? (2)

10.1.3 Draw a sketch of the derivative $f'(x)$ indicating only the x -intercepts of the graph. (3)

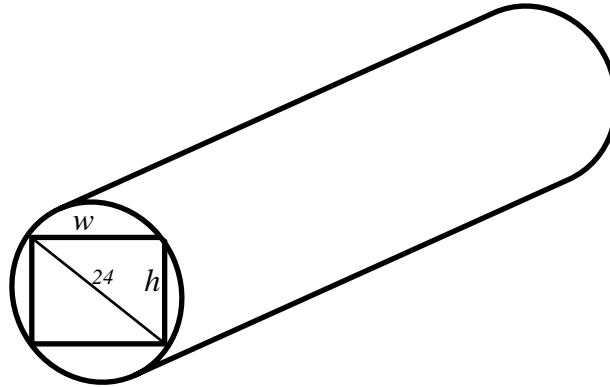
10.1.4 The equation $f(x) - k = 0$ has only one root. What are the possible values of k ? (3)

10.2 Find the average gradient of the graph $f(x) = x^2 - 4$ between the points $x = -1$ and $x = 3$. (4)

[14]

QUESTION 11

A wooden beam with a rectangular cross section of height h and width w is cut from a circular log having a diameter of 24 cm, as shown in the figure. The strength S of the beam is given by $S = h^2w$.



- 11.1 Show that S can be written as $S = 576w - w^3$. (3)
- 11.2 What are the dimensions of the strongest rectangular wooden beam that can be cut from this log? (4)
- [7]

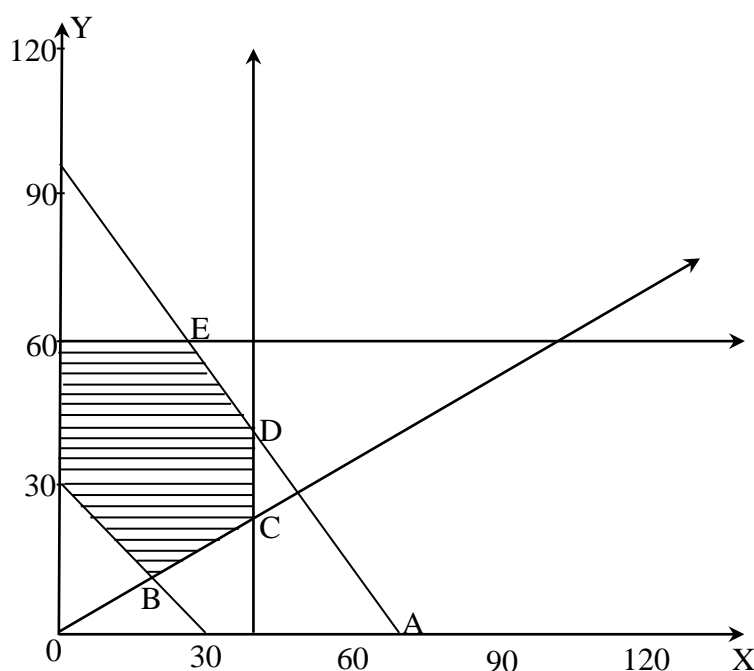
QUESTION 12

A cellphone company makes two types of cellphones 'Easyhear' and 'Longtalk'. Production figures are checked weekly.

- At most 42 Easyhear and 60 Longtalk phones can be manufactured each week.
- At least 30 cellphones must be produced each week to cover cost.
- In order not to flood the market, the number of Easyhear phones cannot be more than twice the number Longtalk phones.
- It takes $\frac{2}{3}$ of an hour to assemble an Easyhear phone and $\frac{1}{2}$ hour to put together a Longtalk phone.
- The trade unions only allow for a 50-hour week.

Let x be the number of Easyhear phones and y be the number of Longtalk phones manufactured each week.

The graph of the feasible region, not drawn to scale, is shown below.



- 12.1 Two of the constraints are: $0 \leq x \leq 42$ and $0 \leq y \leq 60$.
Write down the other constraints. (6)
- 12.2 Write down the coordinates of point A. (2)
- 12.3 If the profit on an Easyhear phone is R225 and the profit on a Longtalk is R75:
- 12.3.1 Write down an expression for the total profit. (2)
- 12.3.2 Determine the maximum profit per week. (5)

[15]

TOTAL: 150

INFORMATION SHEET: MATHEMATICS

$$x = \frac{-b \pm \sqrt{b^2 - 4ac}}{2a}$$

$$A = P(1 + ni)$$

$$A = P(1 - ni)$$

$$A = P(1 - i)^n$$

$$A = P(1 + i)^n$$

$$\sum_{i=1}^n 1 = n$$

$$\sum_{i=1}^n i = \frac{n(n+1)}{2}$$

$$\sum_{i=1}^n (a + (i-1)d) = \frac{n}{2}(2a + (n-1)d)$$

$$\sum_{i=1}^n ar^{i-1} = \frac{a(r^n - 1)}{r - 1} ; \quad r \neq 1$$

$$\sum_{i=1}^{\infty} ar^{i-1} = \frac{a}{1-r} ; \quad -1 < r < 1$$

$$F = \frac{x[(1+i)^n - 1]}{i}$$

$$P = \frac{x[1 - (1+i)^{-n}]}{i} \quad f'(x) = \lim_{h \rightarrow 0} \frac{f(x+h) - f(x)}{h}$$

$$d = \sqrt{(x_2 - x_1)^2 + (y_2 - y_1)^2}$$

$$M\left(\frac{x_1 + x_2}{2}; \frac{y_1 + y_2}{2}\right)$$

$$y = mx + c$$

$$y - y_1 = m(x - x_1)$$

$$m = \frac{y_2 - y_1}{x_2 - x_1}$$

$$m = \tan \theta \quad (x-a)^2 + (y-b)^2 = r^2$$

$$\text{In } \triangle ABC: \quad \frac{a}{\sin A} = \frac{b}{\sin B} = \frac{c}{\sin C} \quad a^2 = b^2 + c^2 - 2bc \cos A \quad \text{area } \triangle ABC = \frac{1}{2} ab \sin C$$

$$\sin(\alpha + \beta) = \sin \alpha \cos \beta + \cos \alpha \sin \beta$$

$$\sin(\alpha - \beta) = \sin \alpha \cos \beta - \cos \alpha \sin \beta$$

$$\cos(\alpha + \beta) = \cos \alpha \cos \beta - \sin \alpha \sin \beta$$

$$\cos 2\alpha = \begin{cases} \cos^2 \alpha - \sin^2 \alpha \\ 1 - 2\sin^2 \alpha \\ 2\cos^2 \alpha - 1 \end{cases}$$

$$\cos(\alpha - \beta) = \cos \alpha \cos \beta + \sin \alpha \sin \beta$$

$$\sin 2\alpha = 2 \sin \alpha \cos \alpha$$

$$\bar{x} = \frac{\sum fx}{n} \quad \hat{\sigma}^2 = \frac{\sum_{i=1}^n (x_i - \bar{x})^2}{n}$$

$$P(A) = \frac{n(A)}{n(S)}$$

$$(A \text{ or } B) = P(A) + P(B) - P(A \text{ and } B)$$

DIAGRAM SHEET

NAME:	
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QUESTION 6.3

