



Province of the  
**EASTERN CAPE**  
EDUCATION

**NATIONAL  
SENIOR CERTIFICATE/  
NASIONALE  
SENIOR SERTIFIKAAT**

**GRADE/GRAAD 12**

**SEPTEMBER 2014**

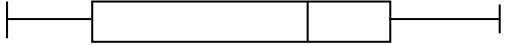
**MATHEMATICS P2/WISKUNDE V2  
MEMORANDUM**

**MARKS/PUNTE: 150**

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This memorandum consists of 12 pages./  
*Hierdie memorandum bestaan uit 12 bladsye*

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<b>QUESTION/VRAAG 1</b>			
1.1	16	✓ answer/antwoord	(1)
1.2	<p>Semi-IQR = <math>\frac{30-18,5}{2} = 5,75</math></p>	✓ correct values <i>korrekte waardes</i> ✓ substitution into correct formula / <i>vervanging in korrekte formule</i> ✓ answer/antwoord	(3)
1.3	 10    18,5              25    30    40	✓ Min and/en Max/Maks ✓ $Q_1$ & $Q_3$ ✓ $Q_2$	(3)
1.4	The distribution is skewed to the left/ <i>Die verspreiding is skeef na links.</i>	✓ answer/antwoord	(1)
			[8]

QUESTION/VRAAG 2			
2.1		✓✓ plotting points/ afsteek van punte  ✓ regression line/ regressielyn (2.3)	(2)
2.2	$a = 29,22 \quad (29,21542....)$ $b = 0,89$ $\therefore y = 29,22 + 0,89x$	✓ calculating value of $a$ and $b$ bereken waardes van $a$ en $b$ ✓ equation for the line/vergelyking vir die lyn	(2)
2.3	On graph/Op grafiek		(1)
2.4	$r = 0,66$ OR/OF $S_y = \sqrt{\frac{\sum(y - \bar{y})^2}{n}}$ $= \sqrt{\frac{1290,9}{10}} = 11,36$  $S_x = \sqrt{\frac{\sum(x - \bar{x})^2}{n}}$ $= \sqrt{\frac{720,9}{10}} = 8,49$  $b = r \frac{S_y}{S_x}$ then/dan $0,89 = r \frac{11,36}{8,49}; r = 0,66$	✓✓ answer/antwoord  ✓ $S_y, S_x$  ✓ answer/antwoord	(2)
2.5	Mean/Gemiddelde, $\bar{x} = \frac{559}{10} = 55,90$ (Answer only: Full marks) (Slegs antwoord: Volpunte)  Standard deviation/Standaardafwyking, $\sigma = 11,36$	✓✓ answer/antwoord  ✓✓ answer/antwoord	(4)
2.6	$55,9 - 11,36$ and $55,9 + 11,36$ [44,54; 67,26] 6 scores/tellings	✓ interval/interval  ✓ answer/antwoord	(2)
			[13]

<b>QUESTION/VRAAG 3</b>			
3.1	$y = 9$	✓ answer/antwoord	(1)
3.2	<p>AB = 11 units/eenhede  <b>OR/OF</b></p> $AB = \sqrt{(x_2 - x_1)^2 + (y_2 - y_1)^2}$ $AB = \sqrt{(11 - 0)^2 + (9 - 9)^2}$ <p>AB = 11 units/eenhede</p>	✓ answer/antwoord	(1)
3.3	D(13; 9)	✓ answer/antwoord	(1)
3.4	<p>Area of <math>\Delta ABC = \frac{1}{2} AB \cdot CD</math> (<i>Oppervlakte van ...</i>)</p> $= \frac{1}{2}(11 \times 8)$ $= 44$ square units/vierkante eenhede	✓ length of CD / lengte van CD ✓ Substitute into correct formula / <i>Vervanging in korrekte formule</i> ✓ answer/antwoord	(3)
3.5	$M = \left[ \frac{x_2+x_1}{2}; \frac{y_2+y_1}{2} \right]$ $= \left[ \frac{13}{2}; \frac{10}{2} \right]$ $= [6,5; 5]$	✓ substitution into midpoint formula/ <i>vervanging in middelpunt formule</i> ✓ answer/antwoord	(2)
3.6	$m_{AC} = \frac{y_2 - y_1}{x_2 - x_1}$ $= \frac{9-1}{0-13}$ $m_{AC} = -\frac{8}{13}$ $y - y_1 = m(x - x_1)$ $y - 5 = \frac{13}{8}(x - 6,5)$ $y = \frac{13}{8}x - 5\frac{9}{16}$	✓ $m_{AC}$ ✓ correct gradient of perpendicular bisector/korrekte gradient van middelloodlyn ✓ Substitution into correct formula/ <i>Vervanging in korrekte formule</i> ✓ answer/antwoord	(4)
3.7	$y = \frac{13}{8}(11) - \frac{89}{16}$ $y = \frac{143}{8} - \frac{89}{16}$ $y = \frac{197}{16}$ <p>No, it does not pass through B/Nee, dit gaan nie deur B nie.</p>	✓ substitution into correct equation / <i>vervanging in korrekte vergelyking</i> ✓ answer/antwoord (justification/regverdiging)	(2)

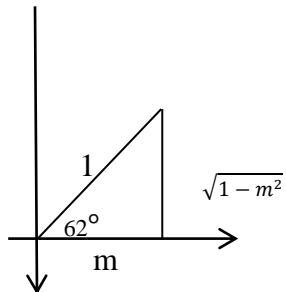
3.8	$m_{BC} = \frac{y_2 - y_1}{x_2 - x_1}$ $= \frac{9 - 1}{11 - 13}$ $= \frac{8}{-2} = -4$ $\tan \theta = -4$ $\theta = 104,04^\circ$ $A\hat{B}C = 104,04^\circ$	✓ $m_{BC}$ ✓ $\tan \theta$ ✓ answer/antwoord (3)	
3.9	$y - y_1 = m(x - x_1)$ $y - 9 = -\frac{8}{13}(x - 13)$ $y = -\frac{8}{13}x + 8 + 9$ $y = -\frac{8}{13}x + 17$	✓ substitution/vervanging ✓ answer/antwoord (2)	
			[19]

## QUESTION/VRAAG 4

4.1	$x^2 - 2x + y^2 + 4y = a$ $(x - 1)^2 + (y + 2)^2 = a + 1 + 4$ $(x - 1)^2 + (y + 2)^2 = a + 5$ $\therefore a + 5 = r^2$ $a + 5 = 25$ $a = 20$	✓ ✓ completing the square/voltooiing van vierkant ✓ equating $r^2$ /gelyk stel aan $r^2$ ✓ $r^2 = 25$ (4)
4.2	M (1; -2)	✓ answer in coordinate form/antwoord in koördinaat vorm (1)
4.3	$x^2 - 2x + y^2 + 4y = 20$ A(x; y) $\therefore (4)^2 - 2(4) + y^2 + 4y = 20$ $16 - 8 + y^2 + 4y = 20$ $y^2 + 4y - 12 = 0$ $(y + 6)(y - 2) = 0$ $\therefore y = -6$ or/of $y = 2$ $y = 2$ $A(4; 2)$ $y > 0$	✓ substituting into correct formula/vervanging in korrekte formule ✓ standard form/standaardvorm ✓ factors/faktore ✓ correct answer/korrekte antwoord (4)
4.4	$m_{MA} = \frac{2 + 2}{4 - 1} = \frac{4}{3}$ $m_{tangent} = -\frac{3}{4}$ $y - y_1 = m(x - x_1)$ $\therefore y - 2 = -\frac{3}{4}(x - 4)$ $y = -\frac{3}{4}x + 3 + 2$ $\therefore y = -\frac{3}{4}x + 5$	✓ $m_{MA} = \frac{4}{3}$ ✓ $m_{tan.} = -\frac{3}{4}$ ✓ substituting into correct formula/vervanging in korrekte formule ✓ answer/antwoord (4)

4.5	$M(1; -2), T(-1; -2)$ $MT = \sqrt{(-1 - 1)^2 + (-2 + 2)^2}$ $= 2$ $2 < \sqrt{20}$  <b>OR/OF</b>  $x^2 - 2x + y^2 + 4y = 20 \quad T(-1; -2)$ $LHS = (-1)^2 - 2(-1) + (-2)^2 + 4(-2)$ $LHS = 1 + 2 + 4 - 8$ $LHS = -1$ $LHS \neq RHS$ $-1 < 20$ $\therefore T \text{ lies inside the circle}/T \text{ lê binne die sirkel}$	✓ substitution/vervanging ✓ simplification/vereenvoudiging ✓ answer/antwoord	
4.6	$(x - 1)^2 + (y + 2)^2 = 20$ $(x - 1 + 3)^2 + (y + 2 - 1)^2 = 20$ $(x + 2)^2 + (y + 1)^2 = 20$	✓ substitution into correct formula/vervanging in korrekte formule ✓ ✓ simplification and answer $(x + 2)$ and $(y + 1)$ / vereenvoudiging en antwoord $(x + 2)$ en $(y + 1)$	(3)
			[19]

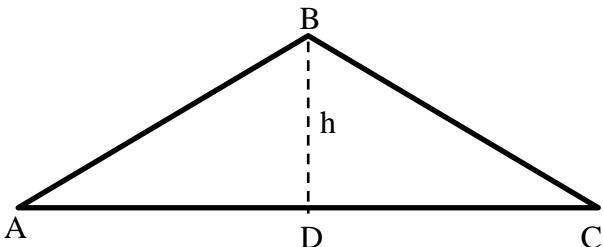
**QUESTION/VRAAG 5**

5.1.1	$\sin 28^\circ = \sin(90^\circ - 62^\circ)$ $= \cos 62^\circ$ $= m$		✓ equation/vergelyking ✓ answer/antwoord	
5.1.2	$\cos 362^\circ = \cos(2^\circ)$ $= \cos(62^\circ - 60^\circ)$ $= \cos 62^\circ \cos 60^\circ + \sin 62^\circ \sin 60^\circ$ $= \frac{1}{2}m + \frac{\sqrt{3}}{2}(\sqrt{1 - m^2})$	✓ $\cos 2^\circ$ ✓ $62^\circ - 60^\circ$ ✓ expansion/uitbreiding ✓ substitution/vervanging	(4)	
5.2.1	$\frac{\tan(360^\circ - x) \cdot \sin(90^\circ + x)}{\sin(-x)}$ $= \frac{-\tan x \cdot \cos x}{-\sin x}$ $= \frac{\sin x \times \cos x}{\cos x}$ $= 1$	✓ - $\tan x$ ✓ $\cos x$ ✓ - $\sin x$ ✓ $\frac{\sin x}{\cos x}$ ✓ answer/antwoord		(5)

<p>5.3</p> $\begin{aligned} 4 \sin^2 \theta &= 3 \\ \sin^2 \theta &= \frac{3}{4} \\ \therefore \sin \theta &= \pm \frac{\sqrt{3}}{2} \\ \therefore \theta &= 120^\circ; 90^\circ < \theta < 180^\circ \\ \\ &= \cos \frac{1}{4} \theta \cdot \sin \frac{1}{2} \theta - \tan(3\theta - 45^\circ) \\ &= \cos 30^\circ \cdot \sin 60^\circ - \tan(360^\circ - 45^\circ) \\ &= \cos 30^\circ \cdot \sin 60^\circ + \tan 45^\circ \\ &= \frac{\sqrt{3}}{2} \times \frac{\sqrt{3}}{2} + 1 \\ &= 1\frac{3}{4} \end{aligned}$	<ul style="list-style-type: none"> <li>✓ simplification/vereenvoudiging</li> <li>✓ value of/waarde van <math>\theta</math></li> <li>✓ substitution of/vervanging van <math>\theta</math></li> <li>✓ <math>\cos 30^\circ = \frac{\sqrt{3}}{2}</math></li> <li>✓ <math>\sin 60^\circ = \frac{\sqrt{3}}{2}</math></li> <li>✓ <math>\tan 45^\circ = 1</math></li> </ul>	(6) [17]
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**QUESTION/VRAAG 6**

$\begin{aligned} \text{LHS} &= \frac{\cos 2x}{1 + \sin 2x} \\ &= \frac{\cos^2 x - \sin^2 x}{1 + 2 \sin x \cos x} \\ &= \frac{\cos^2 x - \sin^2 x}{\sin^2 x + 2 \sin x \cos x + \cos^2 x} \\ &= \frac{(\cos x + \sin x)(\cos x - \sin x)}{(\sin x + \cos x)(\sin x + \cos x)} \\ &= \frac{\cos x - \sin x}{\sin x + \cos x} \end{aligned}$	<ul style="list-style-type: none"> <li>✓ expansion in numerator/uitbreiding van teller</li> <li>✓ expansion in denominator/uitbreiding van noemer</li> <li>✓ <math>\sin^2 x + \cos^2 x</math></li> <li>✓ factorisation/faktorisering</li> </ul>	[4]
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QUESTION/VRAAG 7			
7.1	 <p>Let/Laat <math>BD = h</math> (height of triangle/hoogte van driehoek ABC)      Let/Laat <math>DC = x</math>  <math>\therefore AD = (b - x)</math>      Proof/Bewys:  <math>c^2 = h^2 + (b - x)^2</math>  <math>= h^2 + b^2 - 2bx + x^2</math>  <math>= b^2 + (h^2 + x^2) - 2bx</math>  <math>= b^2 + a^2 - 2bx</math>  <math>a^2 = h^2 + x^2</math>  <math>\frac{x}{a} = \cos C</math>  <math>x = a \cos C</math>  <math>\therefore b^2 + a^2 - 2ab \cos C</math></p>	<ul style="list-style-type: none"> <li>✓ AD in terms of <math>x</math>  <math>AD</math> in terme van <math>x</math></li> <li>✓ <math>c^2</math> (using/gebruik van Pythagoras)</li> <li>✓ <math>(h^2 + x^2) = a^2</math></li> <li>✓ substitution of <math>x</math>/vervanging van <math>x</math></li> </ul>	(4)
7.2.1	$PSQ = 90^\circ$ $Q_2 = 90^\circ + \alpha$  In $\Delta QSR$ : $SR^2 = SQ^2 + QR^2 - 2SQ.QR.\cos SQR$ $= x^2 + x^2 - 2x.x.\cos(90^\circ + \alpha)$ $= 2x^2 - 2x^2(-\sin \alpha)$ $= 2x^2 + 2x^2(\sin \alpha)$ $= x^2(2 + 2\sin \alpha)$ $\therefore SR = x\sqrt{2(1 + \sin \alpha)}$	<ul style="list-style-type: none"> <li>✓ angle in semi-circle/hoek in semi-sirkel</li> <li>✓ use of cos rule/gebruik van cos-reël</li> <li>✓ correct substitution/korrekte vervanging</li> <li>✓ <math>x^2(2 + 2\sin \alpha)</math></li> </ul>	(4)
7.2.2	$5\sqrt{3} = x\sqrt{2(1 + \sin \alpha)}$ $5\sqrt{3} = 5\sqrt{2(1 + \sin \alpha)}$ $\sqrt{3} = \sqrt{2(1 + \sin \alpha)}$ $3 = 2(1 + \sin \alpha)$ $\frac{3}{2} = 1 + \sin \alpha$ $\frac{1}{2} = \sin \alpha$ $\alpha = 30^\circ$ In $\Delta APSQ$ $\frac{QS}{PQ} = \sin \alpha$ $PQ = \frac{QS}{\sin \alpha}$ $= \frac{5}{\sin 30^\circ}$ $= 10$ units/eenhede	<ul style="list-style-type: none"> <li>✓ Substitution/Vervanging</li> <li>✓ <math>\sin \alpha = 0,5</math></li> <li>✓ <math>\alpha = 30^\circ</math></li> <li>✓ answer/antwoord</li> </ul>	(4)
			[12]

<b>QUESTION/VRAAG 8</b>			
8.1	2	✓ answer / antwoord	(1)
8.2		$f$ : ✓ $x$ - int/ $x$ -afsnit ✓ shape/vorm  $g$ : ✓ $x$ -ints/ $x$ -afsnitte ✓ $y$ -int/ $y$ -afsnit ✓ shape/vorm	(5)
8.3	$-120^\circ \leq x \leq 60^\circ$	✓ critical values / kritisiese waardes ✓ notation/notasie	(2)
8.4	Translate $f$ $30^\circ$ to the right/Skuif $f$ $30^\circ$ na regs  <b>OR/OF</b>  Translate $f$ down by $\frac{1}{2}$ / Skuif $f$ 'n $\frac{1}{2}$ eenheid af	✓ correct translation/korrekte translasie  ✓ correct translation/korrekte translasie	(2)
			[10]

**QUESTION/VRAAG 9**

9.1	<p>Const: Join O to C Konstr: Verbind O met C</p>	<ul style="list-style-type: none"> <li>✓ Construction/ konstruksie</li> <li>✓ A and D (in terms of <math>x</math> and <math>y</math>) with reasons <i>A en D in terme van x en y met redes</i></li> <li>✓ AOC (in terms of <math>x</math>) with a reason <i>AOC in terme van x met rede</i></li> <li>✓ equation for AOD/ <i>gelykstel vir AOD</i></li> <li>✓ <math>\angle AOD = 2y</math></li> </ul>	(5)
9.2	<p>9.2.1    <math>MBC = MCB</math> [MB = MC; radii]  <math>\angle B_3 = C_2 = x</math> [BO bisects/halveer MBC and/en OC bisects/halveer MCB]  <math>\angle O_2 = 180^\circ - 2x</math> [angles of/hoeke van <math>\Delta</math>]</p>	<ul style="list-style-type: none"> <li>✓ reason/rede</li> <li>✓ <math>B_3</math> and/en <math>C_2 = x</math> and reason/en rede</li> <li>✓ answer/antwoord</li> </ul>	(4)
9.2.2	<p><math>\angle A = \frac{1}{2}BMC</math> [angle at centre = <math>2 \times</math> angle at circ /middelpuntshoek = <math>2 \times</math> omtrekshoek]  <math>= \frac{1}{2}(180^\circ - 4x)</math> [angles of/hoeke van <math>\Delta</math>]  <math>\angle A = 90^\circ - 2x</math></p> <p>Therefore/Daarom:  <math>\angle ABO = O_2 - A</math> [ext. angle of/buitehoek van <math>\Delta OAB</math>]  <math>= (180^\circ - 2x) - (90^\circ - 2x)</math>  <math>= 90^\circ</math></p> <p>AO subtends a right angle/AO onderspan 'n reghoek  <math>\therefore</math> AO is a diameter/AO is 'n middellyn</p>	<ul style="list-style-type: none"> <li>✓ <math>\angle A = \frac{1}{2}BMC</math> (statement) and reason. (stelling) en rede</li> <li>✓ substitution and answer/vervanging en antwoord</li> <li>✓ <math>\angle ABO = O_2 - A</math> (statement) and reason (stelling) en rede</li> <li>✓ substitution/ vervanging</li> <li><math>ABO = 90^\circ</math></li> <li>✓ answer/antwoord</li> </ul>	(5)

9.3	<p>In <math>\Delta EHO</math>, <math>OE = x</math> and <math>OH = x - 2</math></p> $EH^2 = x^2 - (x - 2)^2 \quad [\text{Theorem of Pythagoras}]$ $EH^2 = x^2 - x^2 + 4x - 4$ $= 4(x - 1)$ $\therefore EH = 2\sqrt{x - 1}$ $\therefore EF = 4\sqrt{x - 1} \text{ line from centre of circle .../lyn van middelpunt van sirkel ...]$	<p>✓ <math>OH = x - 2</math>  ✓ Theorem of/Stelling van Pythagoras  ✓ <math>EH</math>  ✓ EF [with reason/met rede]</p>	(4)
<b>[19]</b>			

**QUESTION/VRAAG 10**

10.1	<p><math>\angle ATB = 90^\circ</math> (subtended by diameter/onderspan deur middellyn)  <math>\therefore \angle ATB = \angle EPB = 90^\circ</math>  <math>\therefore \Delta EPB</math> is a cyclic quad/is 'n koordevierhoek (ext/buite <math>\angle = \text{int opp}/\text{teenoorst. binne } \angle</math>)</p>	<p>✓ <math>\angle ATB = 90^\circ</math>  ✓ <math>\angle ATB = \angle EPB = 90^\circ</math>  ✓ conclusion with reason/gevolgtrekking met rede</p>	(3)
10.2	<p>In <math>\Delta ATB</math> and/en <math>\Delta APE</math>  <math>\angle A = \angle A</math> [common/gemeen]  <math>\angle ATB = \angle APE = 90^\circ</math>  <math>\therefore \Delta ATB \parallel\!\!/\! \Delta APE</math> (<math>\angle, \angle, \angle</math>)</p>	<p>✓ statement with reason / stelling met rede  ✓ <math>\angle ATB = \angle APE = 90^\circ</math>  ✓ <math>\Delta ATB \parallel\!\!/\! \Delta APE</math> with reason /met rede</p>	(3)
10.3	<p><math>\angle RTA = \angle TBA</math> (tangent chord/raaklyn-koord)  <math>\angle RTA = \angle ETP</math> (vertically opposite/regoorst)  <math>\angle TBA = \angle TEP</math> (ext/buite <math>\angle</math> of cyclic quad/van koordevierhoek)  <math>\Delta TPE</math> is Isosceles/gelykbenig. (<math>\angle ETP = \angle TEP</math>)  <math>\therefore TP = PE</math></p>	<p>✓ <math>\angle RTA = \angle TBA</math> (statement) with reason. (stelling) met rede  ✓ <math>\angle RTA = \angle ETP</math> (statement) with reason (stelling) met rede  ✓ statement with reason (stelling) met rede  ✓ <math>\Delta TPE</math> is Isosceles/gelykbenig  ✓ reason/rede  ✓ conclusion/slotsom</p>	(6)
10.4	<p><math>\angle ATB = \angle EPB = 90^\circ</math>  <math>\angle PTB = \angle A</math> [tan chord/raaklyn-koord]  <math>\angle PTB = \angle PEB</math> [subtended by BP/onderspan deur BP]  <math>\therefore \angle A = \angle PEB</math>  Hence/Vervolgens <math>\Delta ATB \parallel\!\!/\! \Delta EPB</math> [equiangular/gelykhoekig]</p>	<p>✓ <math>\angle ATB = \angle EPB = 90^\circ</math>  ✓ <math>\angle PTB = \angle A</math> (statement) with reason (stelling) met rede  ✓ <math>\angle PTB = \angle PEB</math> (statement) with reason (stelling) met rede  ✓ <math>\angle A = \angle PEB</math>  ✓ conclusion/slotsom</p>	(5)
10.5	$\frac{BP}{TB} = \frac{BE}{AB} \quad (\Delta ATB \parallel\!\!/\! \Delta EPB)$ $AB \cdot BP = BE \cdot TB$ $2BP \cdot BP = BE \cdot TB \quad [AO=OB=BP]$ $2BP^2 = BE \cdot TB$	<p>✓ statement with reason stelling met rede  ✓ <math>AB \cdot BP = BE \cdot TB</math>  ✓ <math>2BP \cdot BP = BE \cdot TB</math> (statement) with reason (stelling) met rede  ✓ answer/antwoord</p>	(4)
<b>[21]</b>			

**TOTAL/TOTAAL:** 150