Province of the
EASTERN CAPE
EDUCATION

NATIONAL/*NASIONALE*
SENIOR
CERTIFICATE/*SERTIFIKAAT*

GRADE/*GRAAD* 12

JUNE/*JUNIE* 2017

MATHEMATICS P2/*WISKUNDE V2*
MEMORANDUM

MARKS/*PUNTE*: 150

This memorandum consists of 12 pages.
Hierdie memorandum bestaan uit 12 bladsye.

QUESTION 1 / *VRAAG 1*

1.1

Percentages / *Persentasies*	Frequency / *Frekwensie*	Cumulative Frequency / *Kumulatiewe Frekwensie*
$30 \leq x < 40$	1	1
$40 \leq x < 50$	2	3
$50 \leq x < 60$	9	12
$60 \leq x < 70$	12	24
$70 \leq x < 80$	11	35
$80 \leq x < 90$	9	44
$90 \leq x < 100$	6	50

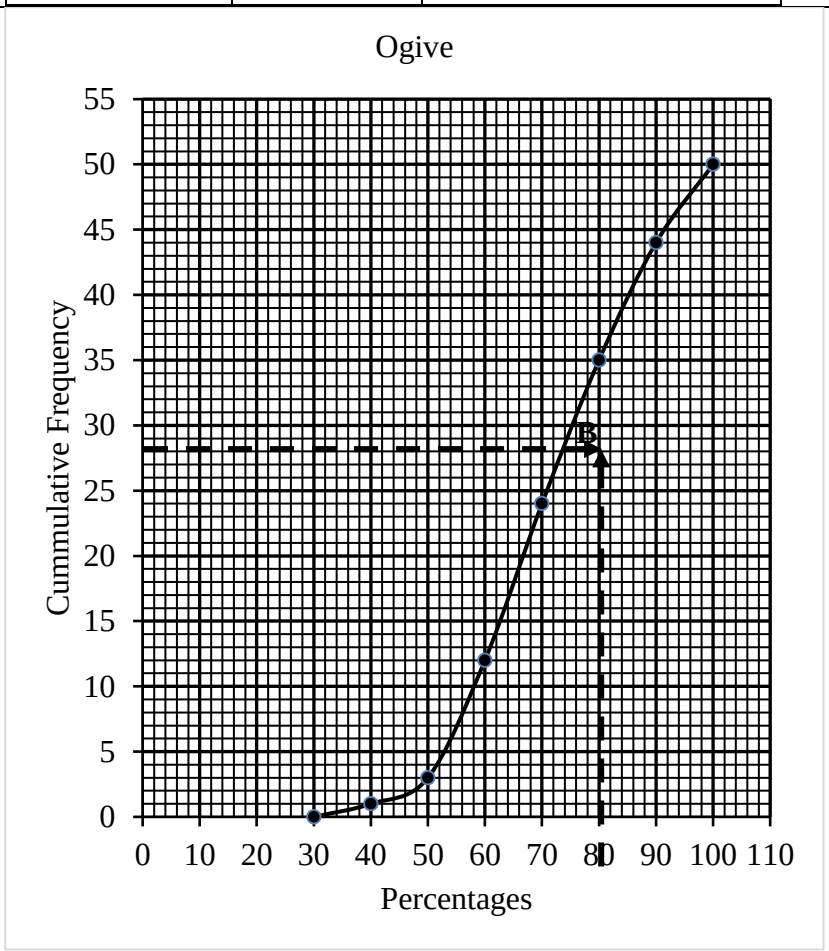
✓ 3, 12

✓ 24, 35, 44

✓ 50

(3)

1.2



✓ upper limits / *bo-limiete*

✓cum *f* / *kum. f*

✓shape / *vorm*

✓grounded / *ge-anker*

(4)

1.3 Approx. 30 [accept between 28 – 32]
Ongeveer 30 [aanvaar tussen 28 – 32]

✓✓ answer/ indicated on graph.
antwoord / op grafiek aangedui (2)

[9]

QUESTION 2 / *VRAAG 2*

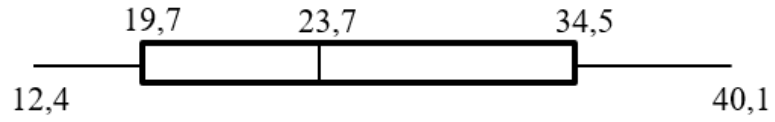
	12,4	15,1	18,9	19,7	19,7	20,0
	20,9	23,7	23,8	31,1	33,6	34,5
	34,9	36,5	40,1			

2.1

Minimum / *Minimum* = 12.4
Lower quartile / *Onderste kwartiel* (Q_1) = 19.7
Median / *Mediaan* (Q_2) = 23.7
Upper quartile / *Boonste kwartiel* (Q_3) = 34.5
Maximum / *Maksimum* = 40.1

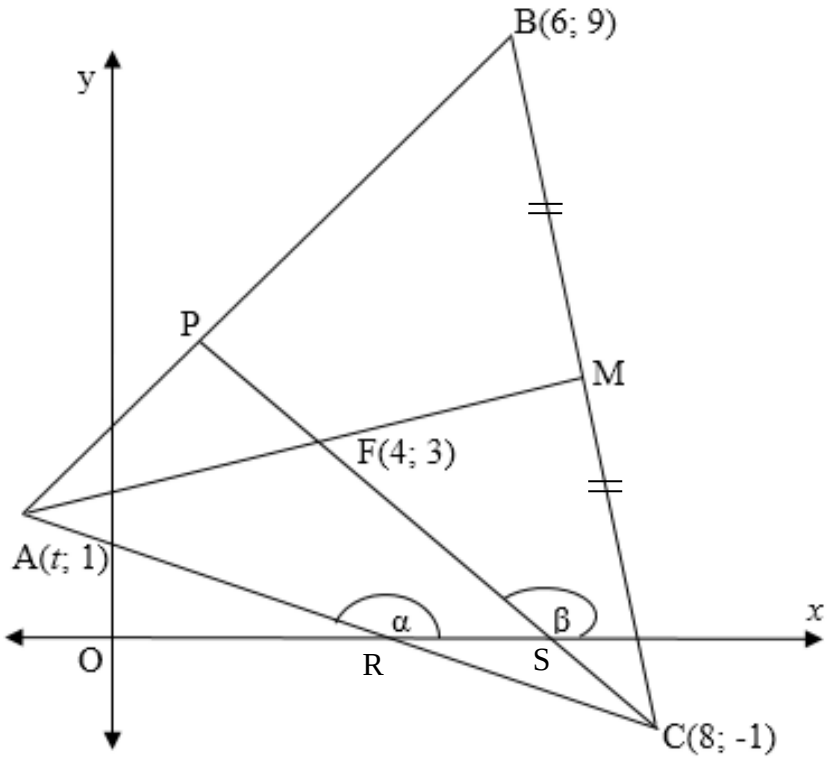
✓ min & max
✓ Q_1
✓ Q_2
✓ Q_3

(4)

2.2		✓ min / max ✓ Q_1 / Q_3 ✓ Q_2 (3)
2.3	Skewed positively to the right. Skeef positief na regs	✓positively skewed / *positief skeef* (1)
2.4	SD/SA = 8,36	✓✓ answer / *antwoord* (2)
2.5	A small standard deviation indicates that the data is clustered around the mean. **OR/OF** A large standard deviation indicates that the data is more spread out. *'n Klein standaardafwyking dui aan dat die data rondom die gemiddelde gegroepeer is. 'n Groot standaardafwyking dui aan dat die data meer versprei is.*	✓ answer / *antwoord* (1)

[11]

QUESTION 3 / *VRAAG 3*

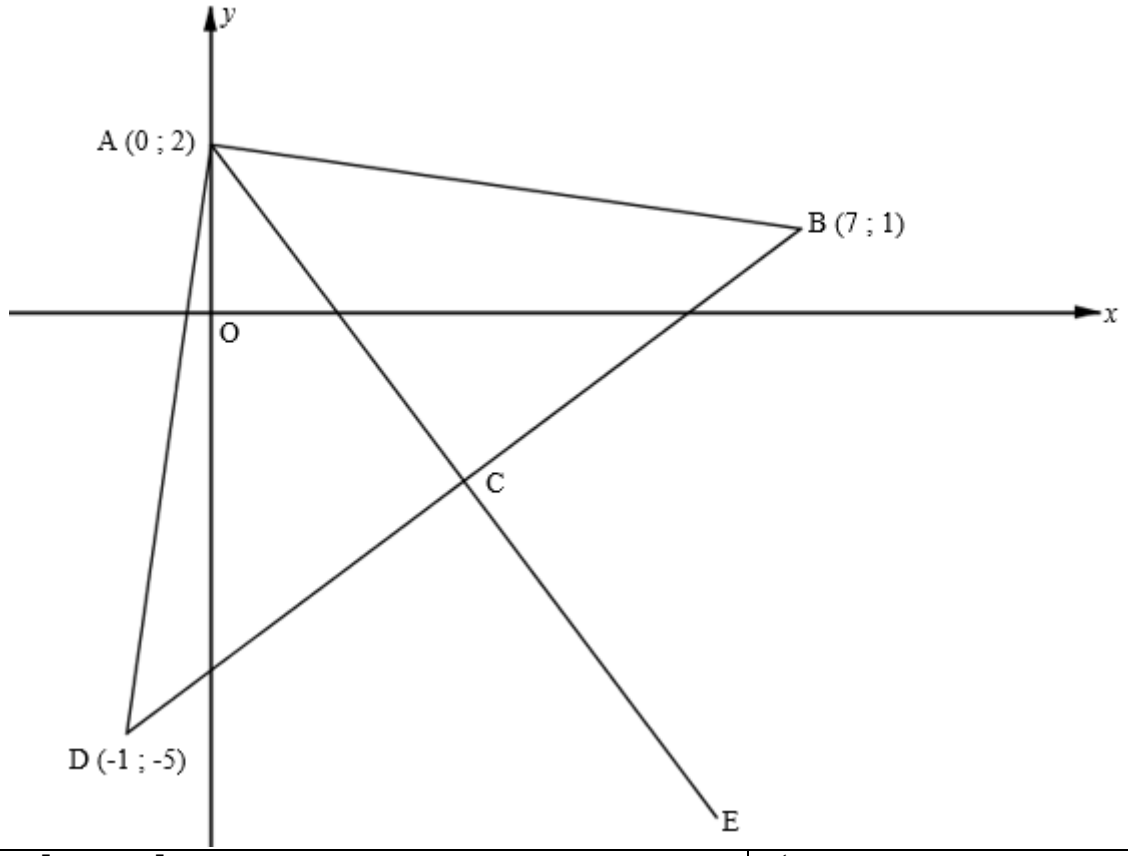


3.1	M = $\left[\frac{6+8}{2} ; \frac{9-1}{2}\right]$ M = (7 ; 4)	✓*x*- value of M / *x-waarde van M* ✓*y*- value of M / *y-waarde van M* (2)
3.2	$m_{FM} = \frac{4-3}{7-4} = \frac{1}{3}$ $y - y_1 = \frac{1}{3}(x - x_1)$ m = $\frac{1}{3}$ $y - 4 = \frac{1}{3}(x - 7)$ M = (7; 4) $\therefore y = \frac{1}{3}x + \frac{5}{3}$	✓substituting / *vervanging* ✓ value of m_{FM} / *waarde van* m_{FM} ✓ substituting M(7; 4) / *vervanging* M(7; 4) ✓ answer / *antwoord* (4)

3.3	$1 = \frac{1}{3}t + \frac{5}{3}$ $t = -2$ **OR / OF** $m_{AF} = m_{FM}$ $\frac{3-1}{4-t} = \frac{1}{3}$ $4 - t = 6$ $t = -2$	✓ substitution into line equation / *vervanging in lyn vergelyking* ✓ answer (as negative) / *antwoord (as negatief)* **OR / OF** ✓ substitution into grad eqn / *vervanging in gradiënt vergelyking* ✓answer as negative / *antwoord as negatief* (2)
3.4	$m_{PC} = \frac{3-(-1)}{4-8} = -1$ ANSWER ONLY FULL MARKS/ *SLEGS ANTWOORD*	✓ substitution / *vervanging* ✓ answer / *antwoord* (2)
3.5	$\tan\beta = -1$ $\beta = 135°$	✓ $\tan\beta = -1$ ✓ $\beta = 135°$ (2)
3.6	$\tan\alpha = \frac{-2}{10} = -\frac{1}{5}$ $\therefore \alpha = 180° - 11.31^0$ $= 168.69°$ $A\hat{C}P = \alpha - \beta$ $= 33.69°$	✓ $\tan\alpha = -\frac{1}{5}$ ✓ $\alpha = 168.69°$ ✓$A\hat{C}P = \alpha - \beta$ ✓ answer / *antwoord* (4)

[16]

QUESTION 4 / *VRAAG 4*

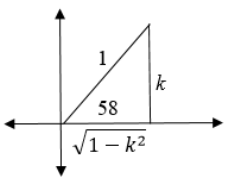


4.1	$C = \left[\frac{7-1}{2}; \frac{1-5}{2}\right]$ $= [3; -2]$	ANSWER ONLY / *SLEGS ANTWOORD*	✓*x*-value /*waarde* ✓ *y*-value /*waarde* (2)

4.2	$CA^2 = (3-0)^2 + (2+2)^2$ $CA^2 = 25$ $CA = 5$ $CB^2 = (7-3)^2 + (1+2)^2$ $CB^2 = 25$ $CB = 5$ $\therefore CA = CB$	✓ substitution / *substitusie* ✓ answer for CA *antwoord vir* CA ✓ answer for CB *antwoord vir* CB (3)
4.3	$m_{AD} = \frac{2+5}{0+1}$ $= 7$ $m_{AB} = \frac{2-1}{0-7}$ $= -\frac{1}{7}$ $m_{AD} \times m_{AB} = 7 \times \left(-\frac{1}{7}\right) = -1$ $\therefore AD \perp AB \quad [m_{AD} \times m_{AB} = -1]$ $\therefore D\hat{A}B = 90^0$	✓ substitution ✓ $m_{AD} = 7$ ✓ substitution ✓ m_{AB} ✓ $m_{AD} \times m_{AB} = -1$ (5)
4.4	$(x-3)^2 + (y+2)^2 = 25$	✓correct centre / *korrek middelpunt* ✓ correct / *korrekte* r^2 (2)
4.5	$m_{BC} = \frac{1-(-2)}{7-3}$ $= \frac{3}{4}$	✓ substitution ✓ m_{BC} (2)
4.6	$m_{tan} = -\frac{4}{3}$ $y - 1 = -\frac{4}{3}(x-7)$ $y = -\frac{4}{3}x + \frac{31}{3}$	✓ m_{tan} ✓ subst $m=-\frac{4}{3}$ and B(7;1) verv. $m=-\frac{4}{3}$ en B(7;1) ✓ answer / *antwoord* (3)
4.7	AE = DB [diametersof same circle] $\therefore$ ABED is a rectangle [diagonals =]	✓ AE = DB ✓ reason ✓ reason (3)

[20]

QUESTION 5 / *VRAAG 5*

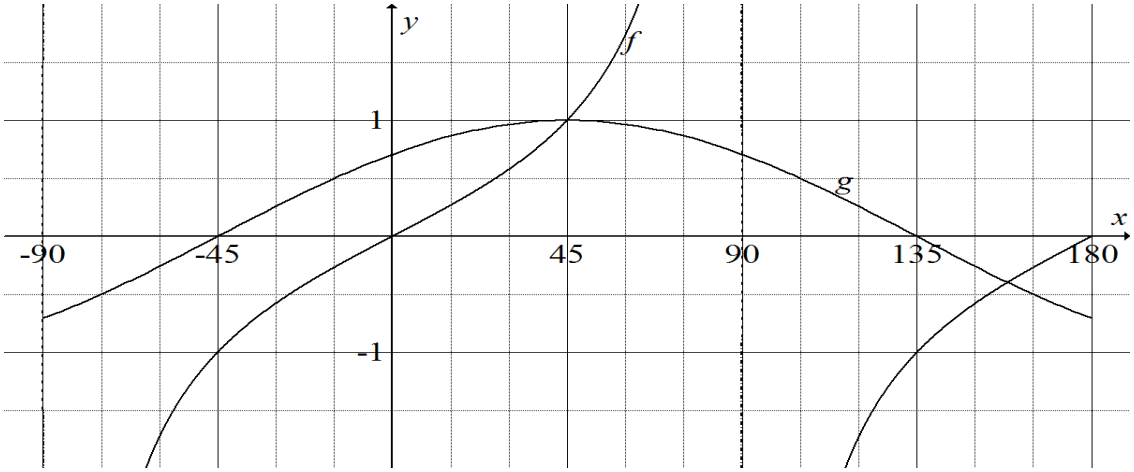
5.1.1	$\sin 238° = -\sin 58°$ $= -k$	✓reduction / *reduksie* ✓ answer / *antwoord* (2)
5.1.2	$\cos 58° = \sin 32°$ $= \sqrt{1-k^2}$  	✓ sin 32° ✓ answer / *antwoord* (2)

5.2	$= \frac{\tan(180°-30°).\sin(360°-60°).\sin 10°}{\cos(180°+45°).\sin(180°-45°).\cos(90°-10°)}$ $= \frac{(-\tan 30°)(-\sin 60°)\sin 10°}{(-\cos 45°)(\sin 45°)\sin 10°}$ $= \frac{\frac{1}{\sqrt{3}}.\frac{\sqrt{3}}{2}}{-\frac{1}{\sqrt{2}}.\frac{1}{\sqrt{2}}}$ $= -1$	✓ - tan 30° ✓ - sin 60° ✓ - cos 45° ✓ sin 45° ✓ sin 10° ✓ simplification / *vereenvoudiging* ✓ answer / *antwoord* (7)
5.3	$\sin(\alpha+\beta) = \cos[90° - (\alpha+\beta)]$ $= \cos[(90 - \alpha) - \beta]$ $= \cos(90° - \alpha)\cos\beta - \sin(90° - \alpha)\cos\beta$ $= \sin\alpha\cos\beta - \cos\alpha\sin\beta$	✓ $\cos[90° - (\alpha+\beta)]$ ✓ $\cos[(90 - \alpha) - \beta]$ ✓ $\cos(90° - \alpha)\cos\beta - \sin(90° - \alpha)\cos\beta$ ✓ $\sin\alpha\cos\beta - \cos\alpha\sin\beta$ (4)
5.4	$\frac{\cos 2x + 1}{\sin 2x.\tan x} = \frac{2\cos^2 x - 1 + 1}{2\sin x\cos x.\frac{\sin x}{\cos x}}$ $= \frac{2\cos^2 x}{2\sin^2 x}$ $= \frac{1}{\tan^2 x}$	✓ identity numerator *identiteit teller* ✓ identity denominator *identiteit noemer* ✓ $\frac{\sin x}{\cos x}$ ✓ $\frac{\cos^2 x}{\sin^2 x}$ simplification / *vereenvoudiging* (4)
5.5.1	$\frac{\sin x}{\cos x} = 2\sin x$ $\sin x = 2\sin x\cos x$ $\sin x - 2\sin x\cos x = 0$ $\sin x(1 - 2\cos x) = 0$ $\sin x = 0$ or/*of* $\cos x = \frac{1}{2}$	✓ identity / *identiteit* ($\frac{\sin x}{\cos x}$) ✓ simplification / *vereenvoudiging* ✓ factors / *faktore* (3)
5.5.2	$\sin x = 0$ or $\cos x = \frac{1}{2}$ $x = 0° + 360°k, k \in Z$ OR $x = \pm 60° + 360°k$ $x = 180° + 360°k$ $k \in Z$	✓ $x = 0°$ ✓ $x = 180°$ ✓ $x = \pm 60°$ ✓ $360°k, k \in Z$ (4)

[26]

QUESTION 6 / *VRAAG 6*

6.1

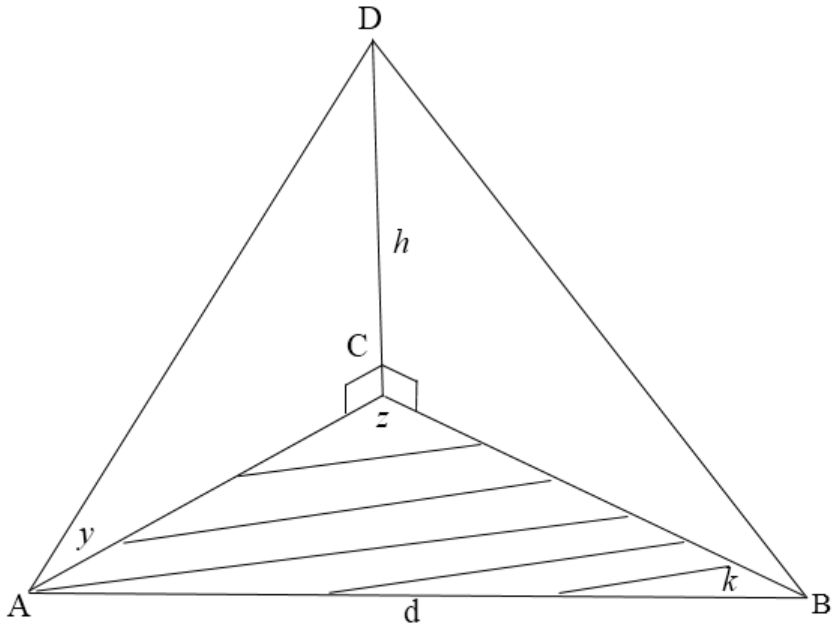


✓ Asymptotes / Asimptote (*f*)
✓ Shape / *Vorm* (*g*)
✓ (45° ; 1) (*f*)
✓ (-45° ; 0) / (135° ; 0) / *x*-intercepts (*g*)
✓ Endpoints / Eindpunte (*f*)
✓ Endpoints / *Eindpunte* (*g*)
(6)

6.2.1	$x = -45°$	✓✓ -45° (2)
6.2.2	(-90°;45°] **OR/OF** $-90^0 < x \leq 45^0$	✓ -90^0 and 45^0 ✓correct inequalities (2)
6.3	90°	✓ answer / *antwoord* (1)

[11]

QUESTION 7 / *VRAAG 7*



7.1 In ΔABC

$$\frac{AC}{\sin k} = \frac{d}{\sin z}$$

$$\therefore AC = \frac{d.\sin k}{sinz}$$

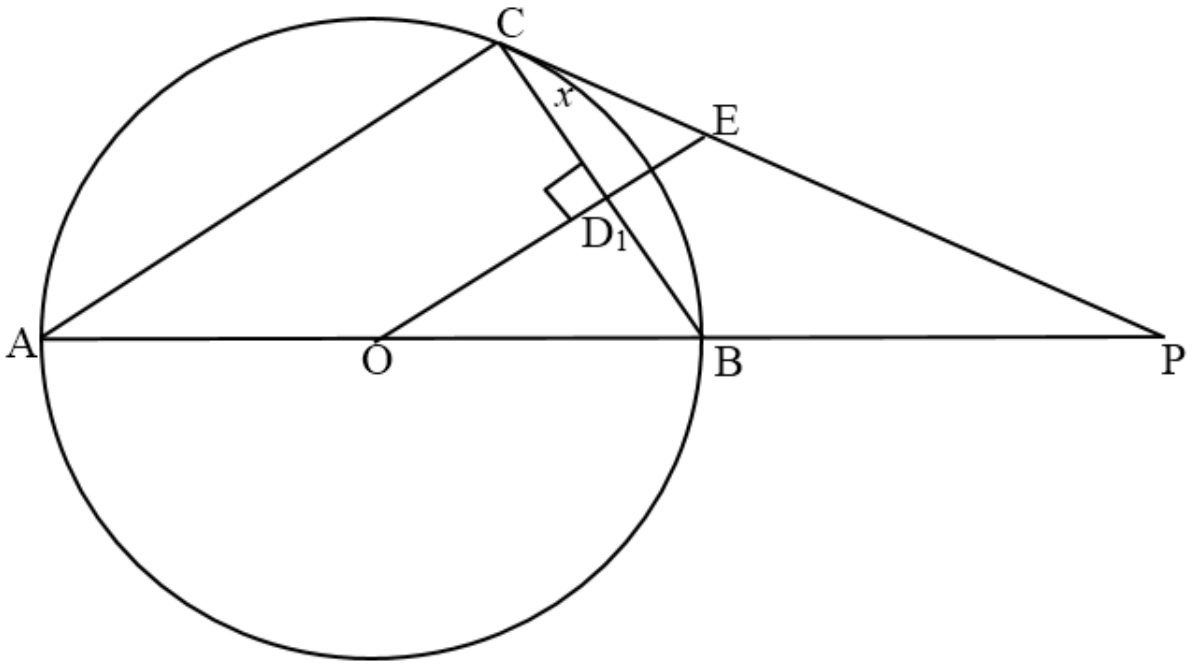
ANSWER ONLY/ *SLEGS ANTWOORD*

✓ proportion / *verhouding*
✓ answer / *antwoord*
(2)

7.2	In ΔADC $\frac{\text{AC}}{\sin(90°-y)} = \frac{h}{\sin y}$ $\text{AC} = \frac{h.\cos y}{\sin y}$ $\text{AC} = \frac{h}{tany}$ **OR/OF** $\frac{AC}{h} = \frac{1}{\tan y}$ $AC = \frac{h}{\tan y}$	✓ proportion / *verhouding* ✓ answer / *antwoord* ✓ $\frac{AC}{h} = \frac{1}{\tan y}$ ✓ $AC = \frac{h}{\tan y}$ (2)
7.3	$h = \frac{\text{AC}.\sin y}{\cos y}$ $h = \frac{d\sin k.\sin y}{\cos y.\sin z}$ $h = \frac{\text{d}.\tan y.\sin k}{\sin z}$ **OR/OF** $AC = \frac{h}{\tan y}$ $AC = \frac{d.\sin k}{\sin z}$ $\therefore \frac{h}{\tan y} = \frac{d.\sin k}{\sin z}$ $\therefore h = \frac{d\sin k\,.tany}{sinz}$	✓ subst/*verv*. $\text{AC} = \frac{d.\sin k}{sinz}$ **OR/OF** ✓ equating AC / *gelykstel aan* AC (1)
7.4	$\therefore h = \frac{d\sin k\,.tany}{sinz}$ $h = \frac{80\,.\sin 38°.\tan 40°}{\sin 125}$ =50,45m	✓ substitution / *vervanging* ✓ answer / *antwoord* (2)

[7]

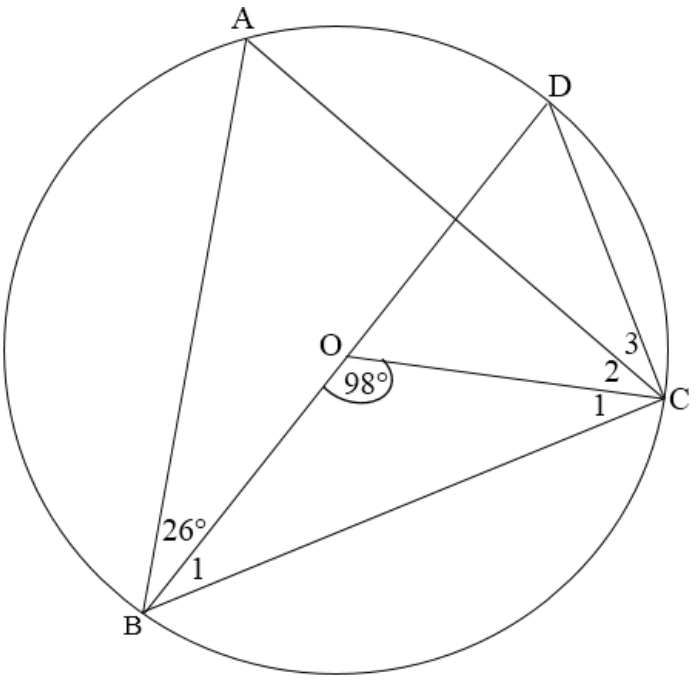
QUESTION 8 / *VRAAG 8*



8.1	Line from centre perpendicular to chord, bisects the chord. / *Lyn vanaf die middelpunt loodreg op die koord, halveer die koord.*	✓ answer / *antwoord* (1)
8.2.	$A\hat{C}B = 90°$ [angle in semi-circle] / [*hoek in semi-sirkel*] $A\hat{C}B = \hat{D}_1$ [both = 90º] / [*beide* = 90º] $\therefore$ OE ‖ AC [corresp $\angle$'*s* equal] / [*ooreenkomstige* $\angle^e$ *is gelyk*]	✓ S ✓R ✓ R (3)
8.3	$\hat{A} = x$ [tan chord] / [*raaklyn koord*] $E\hat{O}B = x$ [corresp $\angle$'*s* ; AC ‖ OE] / [*ooreenkomstige* $\angle^e$; AC ‖ OE]	✓S ✓R ✓S ✓R (4)
8.4	$E\hat{O}B = E\hat{C}B$ [both = *x*] / [*beide* = *x*] ∴ OBEC is cyclic quad [converse angles in same segment] *OBEC is 'n koordevierhoek [hoeke in dieselfde segment]*	✓ S ✓R (2)

[10]

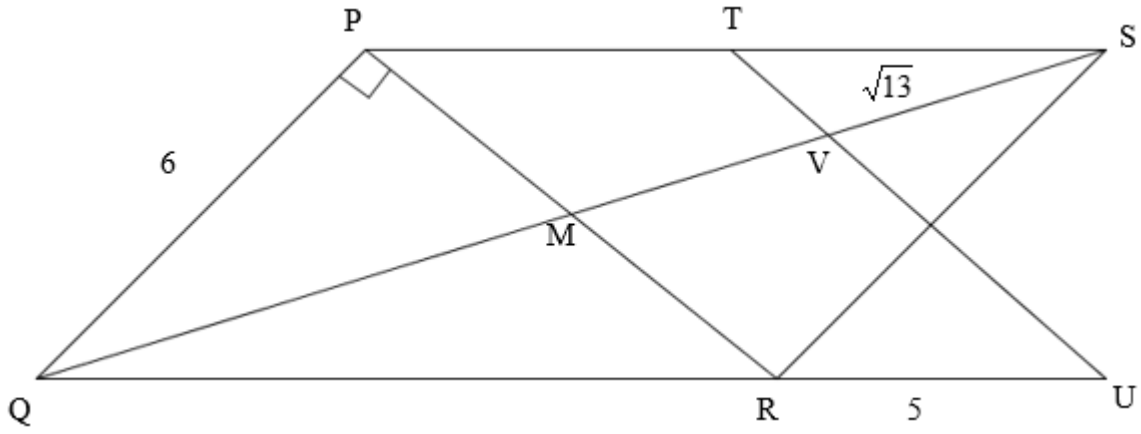
QUESTION 9 / *VRAAG 9*



9.1	$\hat{A} = 49°$ [∠at centre=2∠at circumf.] / [*Middelpunts* ∠]	✓S ✓R	(2)
9.2	$\hat{C}_1 = \hat{B}_1$ [angles opp equal sides] / [*hoeke teenoor gelyke sye*] $\hat{B}_1 = \frac{180°-98°}{2}$ [angles of Δ] / [*hoeke van* Δ] $\hat{B}_1 = 41°$	✓R ✓R ✓ S	(3)
9.3	$B\hat{C}D = 90°$ [∠'s in semi-circle] / [$\angle^e$ *in 'n semi-sirkel*] $\hat{B}_2 = \hat{C}_3 = 26°$ [∠'s in same segment] / [$\angle^e$ *in dieselfde segment*] $\hat{C}_2 = 23°$	✓ S/R ✓ S/R ✓ S	(3)

[8]

QUESTION 10 / *VRAAG 10*

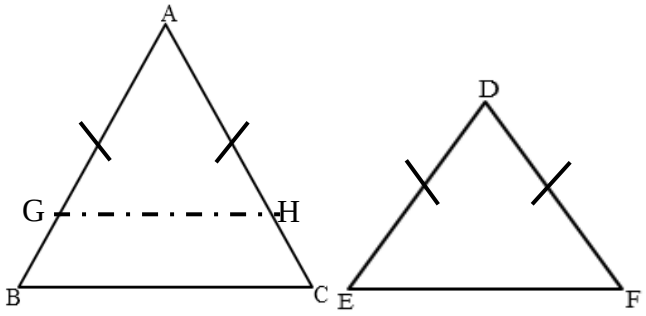


10.1.1	$QR^2 = PQ^2 + PR^2$ Pyth.Theo $= 6^2 + 8^2$ $\therefore QR = 10$ $\therefore \frac{UR}{RQ} = \frac{5}{10}$ $= \frac{1}{2}$	✓ subst. in Pyth ✓ QR = 10 ✓ $\frac{UR}{RQ} = \frac{1}{2}$	(3)

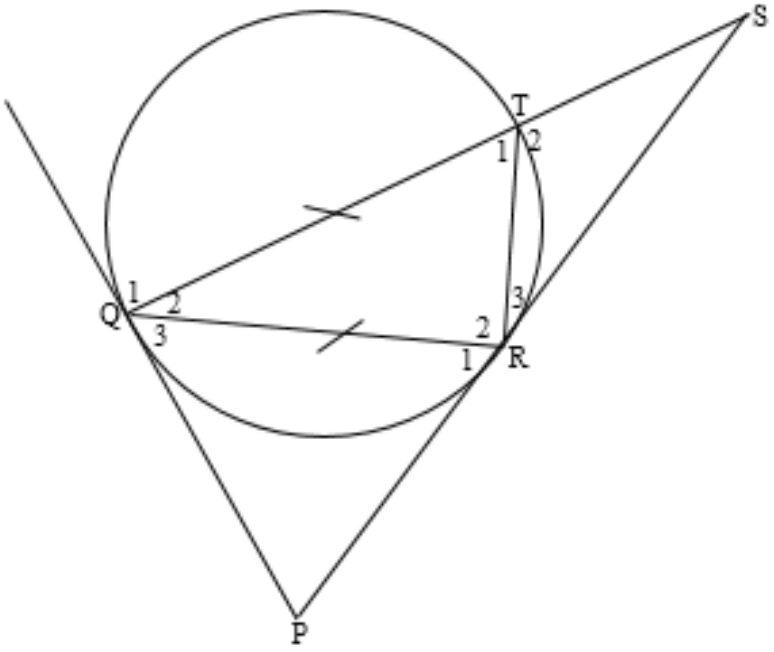
10.1.2	$PM = 4$ [diagonals bisect each other] $QM^2 = 6^2 + 4^2$ [Pyth. Theo] $QM = 2\sqrt{13}$ $MS = QM = 2\sqrt{13}$ [Diagonals bisect each other] $\therefore MV = \sqrt{13}$ $\therefore \frac{VM}{MQ} = \frac{\sqrt{13}}{2\sqrt{13}}$ $= \frac{1}{2}$	✓ R ✓ $QM = 2\sqrt{13}$ ✓ $MV = \sqrt{13}$ ✓ $\frac{VM}{MQ} = \frac{1}{2}$ (4)
10.2	$\frac{UR}{RQ} = \frac{VM}{MQ}$ $\left[\text{both} = \frac{1}{2}\right]$ $\therefore MR \parallel VU$ [line divides two sides of Δ in prop]	✓ **S** ✓ R (2)

[9]

QUESTION 11 / *VRAAG 11*



11.1	Constr/*Konstr*: On AB mark off AG = DE / *Merk* AG = DE *af op* AB On AC mark off AH = DF / *Merk* AH = DF *af op* AC Join GH. / *Verbind* GH Proof / *Bewys*: In ΔAGH & ΔDEF: i) AG = DE (constr) / (*konstr.*) ii) $\hat{A} = \hat{D}$ (given) / (*gegee*) iii) AH = DF (constr) / (*konstr.*) $\therefore$ $\Delta AGH \,|||\, \Delta DEF$ (SAS) / (*SHS*) $\therefore$ $\hat{G}_1 = \hat{E}$ But / *Maar* $\hat{B} = \hat{E}$ given/gegee $\therefore$ $\hat{G}_1 = \hat{B}$ $\therefore$ GH || BC (corresp angles equal) / (*ooreenk. hoeke gelyk*) $\therefore \frac{AB}{AG} = \frac{AC}{AH}$ $\therefore \frac{AB}{DE} = \frac{AC}{DF}$ (AG= DE, AH = DF) $\therefore \frac{AB}{DE} = \frac{AC}{DF} = \frac{BC}{EF}$	✓ constr / *konstr.* ✓S ✓ S/R ✓S ✓R ✓S ✓R (7)

11.2		
11.2.1	$R_2 = x$ [tan chord] : [*raaklyn koord*] $T_1 = x$ [∠'s opp equal sides] : [∠'e *teenoor gelyke sye*] $Q_3 = x$ [tan chord] : [*raaklyn koord*] $R_1 = x$ [tan from same point] : [*raaklyne vanaf dieselfde punt*]	✓ S/R ✓ S/R ✓ S/R ✓ S/R (any three) / (*enige drie*) (3)
11.2.2	$Q_2 = 180° - 2x$ [angles of Δ] : [*hoeke van* Δ]	✓S ✓R (2)
11.2.3	$\hat{P} = 180° - 2x$ [sum of angles of Δ PQR] $R_3 = Q_2 = 180° - 2x$ [tan chord] : [*raaklyn koord*] ∴ TR || QP [corresp ∠'s =] : [*ooreenkomstige* ∠'e =]	✓ S ✓ S/R ✓ R (3)
11.2.4	In Δ STR & Δ SRQ $\hat{S} = \hat{S}$ common / *gemeen* $\hat{R}_3 = \hat{Q}_2$ tan chord / *raaklyn koord* ∴ ΔSTR |||ΔSRQ [AAA] / [*HHH*]	✓ S ✓ S ✓ R (3)
11.2.5	$\frac{ST}{SR} = \frac{SR}{SQ}$ ΔSTR |||ΔSRQ $RS^2 = ST.SQ$	✓ S ✓R (2)
11.2.6	$\frac{SP}{PR} = \frac{SQ}{TQ}$ [line ∥ to one side of a Δ] $= \frac{5}{3}$ $PQ = PR$ [tan from same point] $\frac{SP}{PQ} = \frac{5}{3}$	✓ S/R ✓ R ✓ *value of* $\frac{SP}{PQ} = \frac{5}{3}$ (3)

[23]

TOTAL/*TOTAAL*: 150