Province of the
EASTERN CAPE
EDUCATION

NATIONAL
SENIOR CERTIFICATE

GRADE 12

JUNE 2018

TECHNICAL MATHEMATICS P1
MARKING GUIDELINE

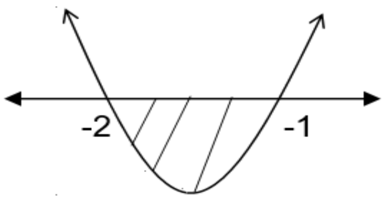
MARKS: 150

This marking guideline consists of 13 pages.

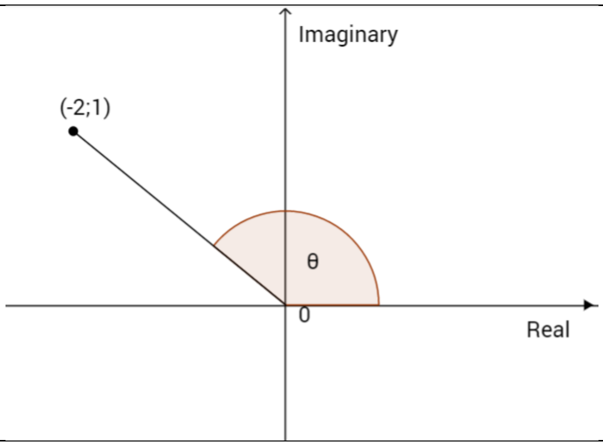
NOTE:

- If a candidate answers a question TWICE, only mark the FIRST attempt.
- If a candidate has crossed out an attempt of a question and not redone the question, mark the crossed-out version.
- Consistent accuracy (CA) applies to ALL aspects of the marking guideline.
- Assuming answers/values to solve a problem is NOT acceptable.

QUESTION 1				
1.1	$\begin{array}{r} 111010 \\ -\underline{\ 10101} \\ \underline{100001_2} \end{array}$		✓✓Accurate value	(2)
1.2	1.2.1	$x(x-3)=0$ $x=0$ or $x=3$	✓✓Each correct x-value	(2)
	1.2.2	$x^2+3x+1=0$ (correct to ONE decimal) $x=\dfrac{-b\pm\sqrt{b^2-4ac}}{2a}$ $x=\dfrac{-3\pm\sqrt{(3)^2-4(1)(1)}}{2(1)}$ **-1 Mark for incorrect rounding** $x\approx-0,4$ or $x\approx-2,6$ **OR** $x^2+3x+\frac{9}{4}=\frac{9}{4}-1$ $\left(x+\frac{3}{2}\right)^2=\frac{5}{4}$ $x=\dfrac{-3\pm\sqrt{5}}{2}$ $x=-0,4$ or $-2,6$	✓ Formula ✓Substitution ✓ $x\approx-0,4$ ✓ $x\approx-2,6$ ✓ Each *x value* ✓ Expansion ✓Quadratic factors ✓ $x\approx-0,4$ ✓ $x\approx-2,6$	(4)

	1.2.3	$x^2+3x+2<0$ $(x+2)(x+1)<0$ Critical values : $x=-1$ or $x=-2$  => $-2<x<-1$	✓Standard form ✓Critical values ✓Notation ✓Both values **OR** ✓ ✓ ✓ $-2<x<-1$ Accurate answer ✓ $-2<x$ $x<-1$	(4)
1.3	$y=x^2-1$ …………. (1) and $y=x+1$……………(2) $x^2-1=x+1$ $x^2-x-2=0$ $(x+1)(x-2)=0$ $x=2$ or $x=-1$ $y=3$ or $y=0$		✓Equating (1) and (2) ✓Standard form ✓Factors ✓Both x-values ✓Both y-values	(5)
1.4	$b^2-4ac=0$ $b^2-4.1.4=0$ $b=4$ or $b=-4$		✓Discriminant = 0 ✓Substitution ✓✓Each value of b	(4)
				[21]

QUESTION 2			
2.1	$\dfrac{2^x.2^1-2^x.2^{-1}}{3.2^x}$ $=\dfrac{2^x(2-2^{-1})}{3.2^x}$ $=\dfrac{3}{6}$ $=\dfrac{1}{2}$ ✓	✓Prime bases ✓ Factor 2^x ✓ Factor $2-2^{-1}$ ✓ $\dfrac{1}{2}$	(4)
2.2	L.H.S = $\dfrac{log_a\left(\frac{25}{125}\right)}{2log_a\left(\frac{5^4}{5^6}\right)}$ L.H.S = $\dfrac{log_a\left(\frac{1}{5}\right)}{2log_a\left(\frac{1}{5^2}\right)}$ L.H.S = $\dfrac{log_a 5^{-1}}{2log_a 5^{-2}}$ L.H.S = $\dfrac{-log_a 5}{-2.2log_a 5}$ L.H.S = $\dfrac{1}{4}$ = R.H.S **OR** L.H.S = $\dfrac{log_a(5)^2-log_a(5)^3}{2\left[log_a(5)^4-log_a(5)^6\right]}$ L.H.S = $\dfrac{2log_a 5-3og_a 5}{2\left[4log_a(5)^4-6log_a 6\right]}$ L.H.S = $\dfrac{-log_a 5}{2.-2log_a 5}$ L.H.S = $\dfrac{1}{4}$ = R.H.S	✓Log Rule(numerator) ✓Log Rule (denominator) ✓Simplification ✓ $\dfrac{log_a 5^{-1}}{2log_a 5^{-2}}$ ✓Power rule ✓Prime factors of 25 ✓Prime factors of 125 ✓Power rule (numerator) ✓Power rule (denominator) ✓Simplification	(5)

2.3	2.3.1	Rabbits $= 1000 \times 2^{0,05(30)}$ Rabbits = 2828	✓Substitution ✓Answer	(2)
	2.3.2	$8000 = 1000 \times 2^{0,05t}$ $8 = 2^{0,05t}$ $0,05t = log_2 8$ $t = 60$ days	✓Substitution ✓*log* form ✓$t = 60$ days	(3)
				[14]
QUESTION 3				
3.1	3.1.1	$\mid Z \mid = \sqrt{(-2)^2 + (1)^2}$ $\mid Z \mid = \sqrt{5}$	✓Substitution ✓Answer	(2)
	3.1.2		✓Quadrant ✓Point/Coordinates	(2)
	3.1.3	$tan\theta = -\frac{1}{2}$ $\theta = -26,57^\circ$ $\theta = 180^\circ - 26,57^\circ = 153,43^\circ$ **Accept angles in radians**	✓*tan* ratio ✓Ref Angle ✓Argument	(3)
	3.1.4	**Accept angles in radians** $\mid Z \mid = \sqrt{5}$ $\theta = 153,43^\circ$ $z = \sqrt{5}\left[cos\left(153,43^\circ\right) + isin\left(153,43^\circ\right)\right]$ **OR** $z = \sqrt{3}cis\left(153,43^\circ\right)$	✓✓Accurate polar form	(2)

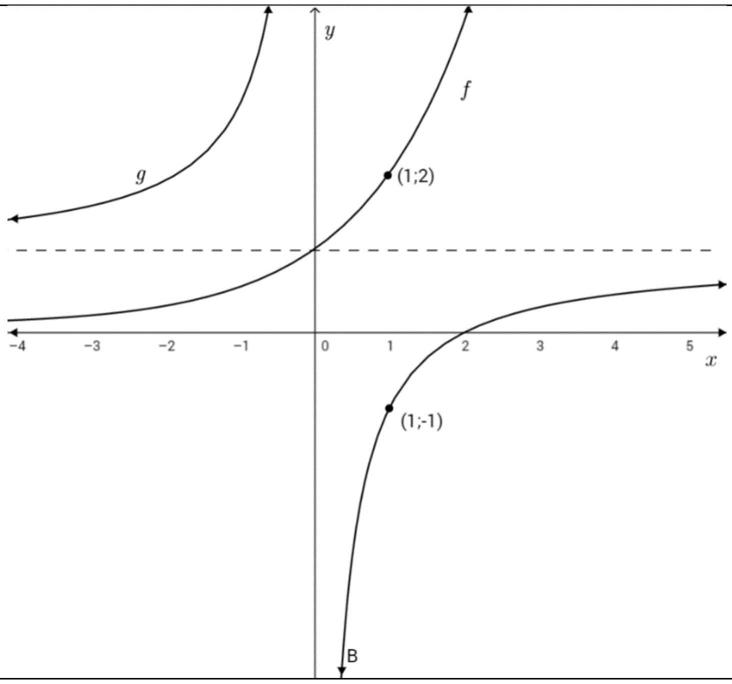
3.2	$(x - yi) = \frac{-2+i}{1+i}$ $x - yi = \frac{-2+i}{1+i} \times \frac{1-i}{1-i}$ $x - yi = \frac{-2+2i+i-i^2}{1-i^2}$ $x - yi = -\frac{1}{2} + \frac{3}{2}i$ $\therefore x = -\frac{1}{2}$ and $y = -\frac{3}{2}$ **OR** $1(x - yi) + i(x - yi) = -2 + i$ $x - yi + ix - y(i)^2 = -2 + i$ $x - yi + ix + y = -2 + i$ $x + y + (x - y)i = -2 + i$ $x + y = -2$...............(1) $x - y = 1$.................(2) (1)+(2) : $x = -\frac{1}{2}$ and $y = -\frac{3}{2}$	✓Simplification ✓Conjugate product ✓Simplification ✓ x-value ✓ y-value ✓ Multiplication ✓Simplification ✓Comparing real values and imaginary values ✓ x-value ✓ y-value	(5)
			[14]

QUESTION 4				
4.1	4.1.1	$i_{norm} = \frac{14\%}{4} = 0,035$ $= 3,5\%$quarterly	✓ Answer	(1)
	4.1.2	$i_{eff} + 1 = \left(1 + \frac{i^m}{m}\right)^m$ $i_{eff} = (1 + 0,035)^4 - 1$ $i_{eff} = 0,15 = 15\%$	✓Formula ✓Substitution ✓Interest	(3)
	4.1.3	$A = 2500(1 + 0,035)^{7\times 4}$ $A = R6550,43$	✓Substitution ✓Correct i = 0,035 and n = 21 ✓ Value of A	(3)
4.2	$A_1 = R250000\left(1 + \frac{0,08}{12}\right)^{2\times 12} + R250000\left(1 + \frac{0,08}{12}\right)^{2\times 12}(1 + 0,025)^{4\times 3}$ $A_1 = R687\,572,9508$ $A_2 = R80000(1 + 0,025)^{2\times 4} = R97472,2318$ Final Amount $= A_1 + A_2 = R785\,045,18$		In A_1 ✓ $i = \frac{0,08}{12}$ ✓$n = 24$ ✓$i = \frac{0,1}{4} = 0,025$ ✓$n = 12$ ✓ $A_1 = R687\,572,9508$ In A_2 ✓$n = 8$ ✓ $A_2 = R97\,472,2318$ ✓ Final Amount R785 045,18	(8)
				[15]

QUESTION 5				
5.1	5.1.1	$0=-(x-3)^2+4$ $(x-3)^2=4$ $x-3=\pm 2$ $x=5$ or $x=1$ $A(1;0)$ or $B(5;0)$ **OR** $0=-^2+6x-5$ $0=(-x+1)(x-5)$ $x=5$ or $x=1$ $A(1;0)$ or $B(5;0)$	✓*h(x)* =0 ✓Transposition ✓A co-ordinates ✓B co-ordinates ✓*h(x)* =0 ✓Factors ✓A coordinates ✓B coordinates	(4)
	5.1.2	$h(x)=-x^2+6x-5$ $\frac{dy}{dx}=-2x+5$ $0=-2x+6$ $x=3$ $h(3)=(3)^2+6(3)-5$ $=4$ $\therefore D(3;4)$	✓ $\frac{dy}{dx}$ ✓Coordinates	(2)
	5.1.3	$x\in[0;6]$ **OR** $0\leq x\leq 6$	✓0 ✓6 ✓Correct notation	(3)
	5.1.4	Maximum height = 4 units	✓Answer	(1)
	5.1.5	*y*-intercept of *h* = –5 Beams have height of 5 units	✓*y*-intercept ✓ 5 units	(2)
	5.1.6	$y\leq 4$ **OR** $y\in(-\infty;4]$ **OR** $-\infty<y\leq 4$	✓ Notation ✓ Value(s)	(2)
	5.1.7	$x\in[3;5]$ **OR** $3\leq x\leq 5$	✓3 ✓5 ✓Correct notation	(3)
5.2	No. The truck height (4,5) is greater than the bridge height (4 units) and the bridge has cross bars on top.		✓ No ✓ Bridge less than truck height **OR** Truck height greater than the bridge height ✓Cross bar	(3)

5.3	At F, $x = 3$ $y = -3+5 = 2$ FD = D – F FD = 4 – 2 FD = 2 units	✓y-value at F ✓Subtracting y-values ✓ FD	(3)
			[23]

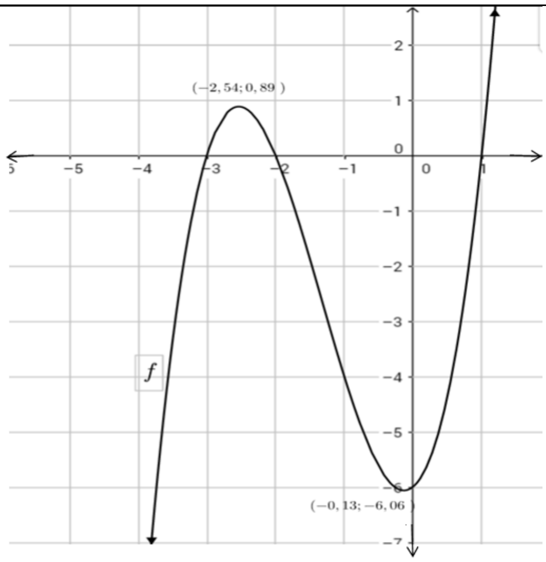
QUESTION 6

6.1	6.1.1	$0 = \frac{-2}{x} + 1$ $x = 2$ $(2;0)$	✓$y = 0$ ✓ Coordinates	(2)
	6.1.2	$f(x) = 2^0$ $y = 1$	✓ Value of y	(1)
	6.1.3	y=0 for $f(x)$ x = 0 and y =1 for $g(x)$ **–1 Mark for 1 omitted asymptote**	✓y =0 ✓x =0 and $y=1$	(2)
6.2			✓Shape of f ✓ y-intercept of f ✓ y=1 Asymptote of g ✓ 1 more point on f ✓Shape of g ✓ 1 more point on g ✓ x- intercept of g	(7)
6.3	6.3.1	$x \in \mathbb{R}, x \neq 0$	✓Restriction ✓Domain value	(2)
	6.3.2	$x \in (0;+\infty)$ **OR** $x > 0$	✓Correct inequality	(1)
				[15]

QUESTION 7

7.1	$f'(x)=\lim_{h\to 0}\frac{f(x+h)-f(x)}{h}$ $f'(x)=\lim_{x\to h}\frac{-2(x+h)^2-(-2x^2)}{h}$ $f'(x)=\lim_{x\to h}\frac{-2x^2-4xh-2h^2+2x^2}{h}$ $f'(x)=\lim_{x\to h}\frac{h(-4x-2h)}{h}$ $f'(x)=-4x$	✓Formula ✓Substitution ✓Expansion ✓Factors ✓ $f'(x)=-4x$	(5)
	-1 Mark for incorrect notation in 7.1 or 7.2		
7.2	$y=2\sqrt{x}-\frac{1}{x}$ $y=2x^{\frac{1}{2}}-x^{-1}$ $\frac{dy}{dx}=x^{-\frac{1}{2}}+x^{-2}$ OR $\frac{dy}{dx}=\frac{1}{x^{\frac{1}{2}}}+\frac{1}{x^2}$	✓$2x^{\frac{1}{2}}$ ✓ x^{-1} ✓ $x^{-\frac{1}{2}}$ ✓ x^{-2}	(4)
7.3	$g'(x)=2x-2$ $m_{tangent}=2(2)-2=2$ $y=2^2-2.2=0$ $(2;0)$ $y=mx+c$ $0=2.2+c$ $c=-4$ $y=2x-4$	✓ $g'(x)$ ✓ $m_{tangent}$ ✓(2;0) ✓ $c=-4$ ✓ $y=2x-4$	(5)
			[14]

QUESTION 8			
8.1	$f(-1) = (-1)^3 + 4(-1)^2 + (-1) - 6$ $f(-1) = -4 \neq 0$ So $x+1$ is not a factor of $f(x)$ because $f(-1)$ is not equal to 0.	✓$f(-1) = -4 \neq 0$	(1)
8.2	$f(1) = (1)^3 + 4(1)^2 + (1) - 6 = 0$ $(x-1)$ is a factor of f $\begin{array}{c\|cccc} & 1 & 4 & 1 & -6 \\ 1 & 0 & 1 & 5 & 7 \\ \hline & 1 & 5 & 6 & 0 \end{array}$ $f(x) = (x-1)(x^2+5x+6)$ $f(x) = (x-1)(x+3)(x+2)$ $x = 1$ or $x = -3$ or $x = -2$ (1;0), (–2;0), (–3;0)	✓ $f(x) = 0$ ✓ First linear factor ✓Quadratic factor ✓Factors of x^2+5x+6 ✓All coordinates	(5)
8.3	y-intercept = –6	✓Answer	(1)
8.4	$f'(x) = 3x^2 + 8x + 1$ $0 = 3x^2 + 8x + 1$ $x = \dfrac{-8 \pm \sqrt{64 - 4.3.1}}{2.3}$ $x = -0,13$ or $x = -2,54$ $(-0,13; -6,06)$ or $(-2,54; 0,89)$	✓ $f'(x) = 0$ ✓x- values (-0,13; -6,06) (-2,54; 0,89) **OR** Y – coordinates of TP ✓ $y = -6,06$ ✓ $y = 0,89$	(4)

<table>
<tr><td>8.5</td><td colspan="2">
</td><td>✓Shape

✓x-intercepts

✓Max. Turning point
✓Min. Turning point

✓ y-intercepts</td><td>(5)</td></tr>
<tr><td></td><td colspan="2"></td><td></td><td>[16]</td></tr>
<tr><td colspan="5">QUESTION 9</td></tr>
<tr><td>9.1</td><td>9.1.1</td><td>Surface Area = 2(2x.x +2x.h + x.h)

$4x^2 + 4xh + 2xh = 120$

$6x.h = 120 - 4x^2$

$\therefore h = \frac{120 - 4x^2}{6x}$

$h = \frac{20}{x} - \frac{2x}{3}$</td><td>✓ Formula

✓Substitution

✓ Simplification

✓ h</td><td>(3)</td></tr>
<tr><td></td><td>9.1.2</td><td>$V = l.b.h$
$V = 2x.x\left(\frac{20}{x} - \frac{2x}{3}\right)$
$V = 40x - \frac{4x^3}{3}$</td><td>✓$V = l.b.h$

✓Substitution</td><td>(2)</td></tr>
<tr><td></td><td>9.1.3</td><td>$\frac{dV}{dx} = 40 - 4x^2$

$0 = 10 - x^2$
$x = \sqrt{10}$ or $x \neq -\sqrt{10}$</td><td>✓ $\frac{dV}{dx}$
✓ $\frac{dV}{dx} = 0$
✓
$x = \sqrt{10} \approx 3{,}16\, cm^3$</td><td>(3)</td></tr>
</table>

9.2	$T = t^3 - 9t^2 + 50t - 66$ $\frac{dT}{dt} = 3t^2 - 18t + 50$ $\frac{dT}{dt} = 3(5)^2 - 18(5) + 50$ $\frac{dT}{dt} = 35^\circ \text{C.s}^{-1}$	✓ $3t^2 - 18t + 50$ ✓Substitution by 5 ✓ $\frac{dT}{dt} = 35^\circ \text{C.s}^{-1}$	(3)
			[11]
QUESTION 10			
10.1	$\int (3x^2 - x)\, dx = \frac{3x^3}{3} - \frac{x^2}{2} + c$ $= x^3 - \frac{x^2}{2} + c$	✓ x^3 ✓ $-\frac{x^2}{2}$ ✓ c	(3)
10.2	$\int_0^1 (-x^2 + x)\, dx = \left[-\frac{x^3}{3} + \frac{x^2}{2} \right]_0^1$ $= \left(-\frac{1}{3} + \frac{1}{2} \right) - (0)$ $= \frac{1}{6}$ square units	✓Integration expression ✓Simplification ✓Substitution by 1 and 0 ✓ $\frac{1}{6}$ square units	(4)
			[7]
		TOTAL:	**150**