



Province of the
EASTERN CAPE
EDUCATION

**NATIONAL SENIOR
CERTIFICATE/*NASIONALE
SENIOR SERTIFIKAAT***

GRADE/*GRAAD* 12

JUNE/*JUNIE* 2018

**TECHNICAL MATHEMATICS P2/*TEGNIESE WISKUNDE V2*
MARKING GUIDELINE/*NASIENRIGLYN***

MARKS/*PUNTE*: 150

This marking guideline consists of 19 pages./*Hierdie nasienriglyn
bestaan uit 19 bladsye.*

NOTE:

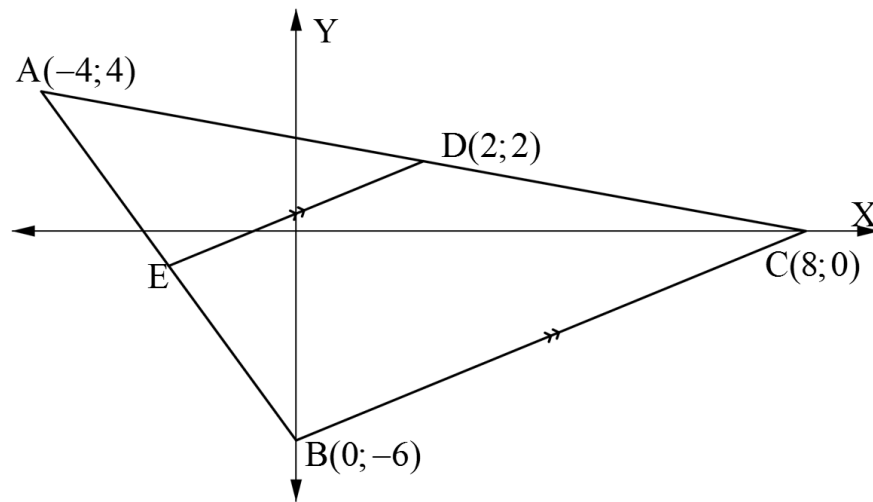
- Continuous accuracy (CA) applies in ALL aspects of the marking guideline.
- After two mistakes, do not apply CA marking.
- Assuming values/answers in order to solve a problem is unacceptable.

LET WEL:

- *Volgehoue akkuraatheid (CA) is deurgaans in ALLE aspekte van die nasienriglyn van toepassing.*
- *Na twee foute word CA nie toegepas nie.*
- *Aanvaarding van waardes/antwoorde om 'n problem op te los, is onaanvaarbaar.*

Symbol/Simbool	Explanation/Verduideling
M	Method/ <i>Metode</i>
MA	Method with accuracy/ <i>Metode met akkuraatheid</i>
A	Accuracy/ <i>Akkuraat</i>
CA	Consistent accuracy/ <i>Deurlopende akkuraatheid</i>
S	Simplification or Statement <i>Vereenvoudiging of bewering</i>
R	Reason/ <i>Rede</i>
SR	Statement and correct reason/ <i>Bewering en korrekte rede</i>

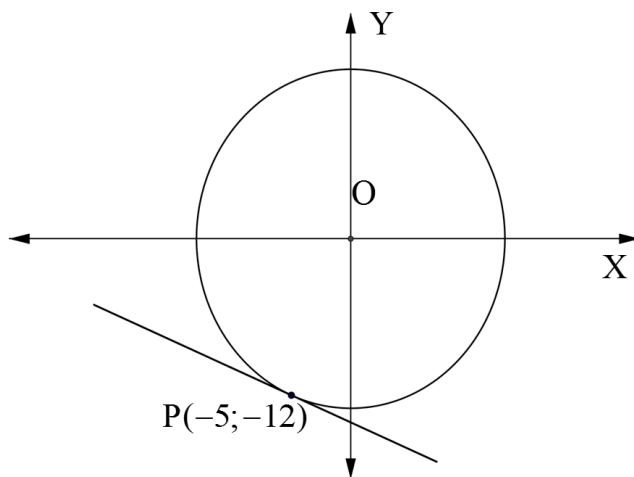
QUESTION/VRAAG 1



<p>1.1</p>	$BC = \sqrt{(x_2 - x_1)^2 + (y_2 - y_1)^2}$ $= \sqrt{(8 - 0)^2 + (0 + 6)^2}$ $= \sqrt{100}$ $= 10$	<p>✓MA formula/formule</p> <p>✓ substitution/vervanging</p> <p>✓CA Answer/antwoord</p>	<p>(3)</p>
<p>1.2</p>	$M\left(\frac{x_1 + x_2}{2}; \frac{y_1 + y_2}{2}\right)$ $M\left(\frac{-4 + 0}{2}; \frac{4 - 6}{2}\right)$ $M(-2; -1)$	<p>✓A x-coordinate/koördinaat</p> <p>✓A y-coordinate/koördinaat</p>	<p>(2)</p>
<p>1.3</p>	$m_{BC} = \frac{y_2 - y_1}{x_2 - x_1}$ $= \frac{0 + 6}{8 - 0}$ $= \frac{6}{8}$ $= \frac{3}{4}$ $= 0,75$	<p>✓MA formula and substitution/formule en vervanging</p> <p>✓CA Simplification/vereenvoudiging</p>	<p>(2)</p>

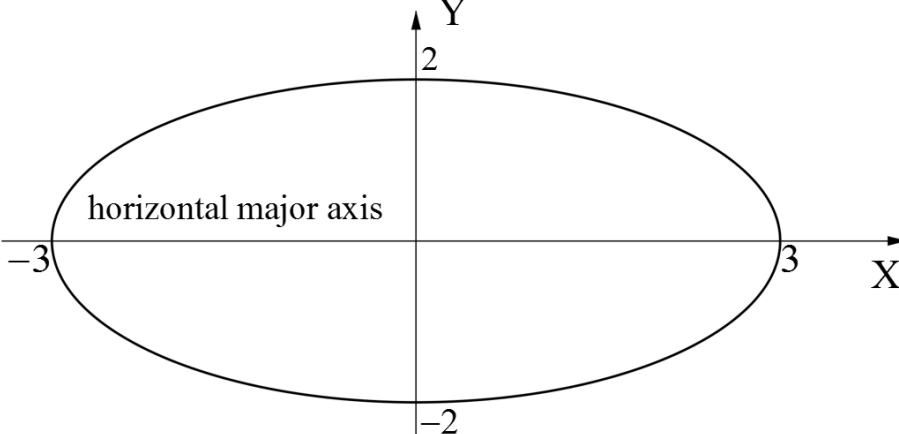
1.4	$y - y_1 = m(x - x_1)$ $y - 2 = 0,75(x - 2)$ $y = 0,75x + 0,5$	<p>✓M Correct formula/korrekte formule</p> <p>✓A Correct substitution pt D or E/Korrekte vervanging pt D of E</p> <p>✓CA Simplification/vereenvoudiging</p> <p>✓CA Standard form/standaardvorm</p>	(4)
1.5	<p>inclination of DE = inclination of BC (□ lines; same gradient)</p> $m_{DE} = m_{BC}$ <p>tan incl DE = 0,75</p> <p>incl DE = 36,87°</p> $m_{AE} = \frac{4+6}{-4-0} = -\frac{5}{2}$ <p>tan incl AE = $-\frac{5}{2}$</p> <p>incl AE = 180° - 68,198...°</p> <p style="padding-left: 40px;">= 111,8°</p> <p>AÊD = 111,8° - 36,87°</p> <p style="padding-left: 40px;">= 74,93°</p>	<p>✓M $m_{DE} = m_{BC}$</p> <p>✓A tan incl DE = 0,75</p> <p>✓CA incl DE = 36,87°</p> <p>✓MA $m_{AE} = -\frac{5}{2}$</p> <p>✓CA incl AE = 111,8°</p> <p>✓CA AÊD = 74,93°</p>	(6)
			[17]

QUESTION/VRAAG 2



2.1.1	$r^2 = x^2 + y^2$ $= (-5)^2 + (-12)^2$ $= 169$ \therefore Equation of circle/Vergelyking van sirkel: $x^2 + y^2 = 169$	✓M Correct formula/ korrekte formule ✓A Correct substitution/korrekte vervanging ✓CA Value of r^2 /waarde van r^2	(3)
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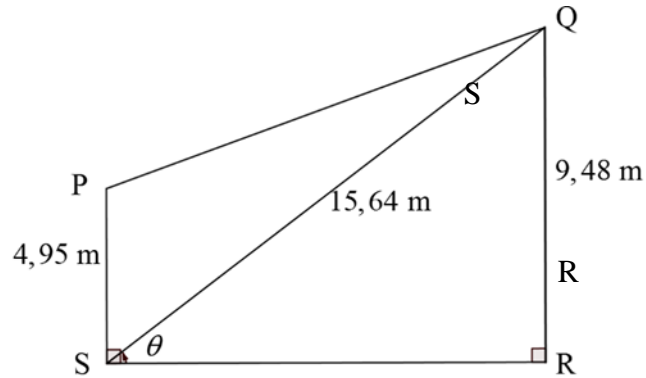
<p>2.1.2</p> $m_{\text{radius}} = \frac{0+12}{0+5} = \frac{12}{5}$ $m_{\text{tangent}} = -\frac{5}{12} \approx -0,42$ <p>\therefore Equation of tangent: \therefore <i>Vergelyking van raaklyn</i></p> $y - y_1 = m(x - x_1)$ $y + 12 = -0,42(x + 5)$ $y = -0,42x - 14,1$ <p>OR/OF</p> $m_{\text{radius}} = \frac{0+12}{0+5} = \frac{12}{5}$ $m_{\text{tangent/raaklyn}} = \frac{5}{12}$ $y - y_1 = m(x - x_1)$ $y + 12 = -\frac{5}{12}(x + 5)$ $y + 12 = -\frac{5}{12}x - \frac{25}{12}$ $y = -\frac{5}{12}x - \frac{169}{12}$ <p>OR/OF</p> $x_1x + y_1y = r^2$ $-5x - 12y = 169$ $y = -\frac{5}{12}x - \frac{169}{12}$		<p>✓MA Gradient of radius/<i>Gradiënt van radius</i></p> <p>✓CA Gradient of tangent/<i>Gradiënt van raaklyn</i></p> <p>✓A Subst. P into equation/<i>vervanging van P in vergelyking</i></p> <p>✓CA Equation of line/<i>vergelyking van lyn</i></p> <p>✓CA Gradient of tangent/<i>Gradiënt van raaklyn</i></p> <p>✓MA Gradient of radius / <i>Gradiënt van radius</i></p> <p>✓CA Gradient of tangent/<i>raaklyn</i></p> <p>✓A Subst. P into equation/<i>vervanging van P in vergelyking</i></p> <p>✓M Correct formula / <i>Korrekte formule</i></p> <p>✓A Correct substitution of P/ <i>Korrekte vervanging van P</i></p> <p>✓A Correct substitution of grad./ <i>Korrekte vervanging van grad.</i></p> <p>✓CA Equation of line/<i>vergelyking van lyn</i></p>	(4)
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2.2	$4x^2 + 9y^2 = 36$ $\frac{x^2}{9} + \frac{y^2}{4} = 1$ $\frac{x^2}{3^2} + \frac{y^2}{2^2} = 1$	<p>✓M Rewrite the equation/ <i>Herskryf die vergelyking</i>, RHS = 1</p> <p>✓M Rewrite into/<i>Herskryf as</i> a^2 & b^2</p> <p>✓CA <i>x</i>-intercepts/<i>afsnitte</i></p> <p>✓CA <i>y</i>-intercepts/<i>afsnitte</i></p> <p>✓CA shape – clearly showing the horizontal major axis/<i>vorm – wys duidelik die horisontale hoofas</i></p>	
		(5)	
		[12]	

QUESTION/VRAAG 3			
3.1			
3.1.1	$13 \sin \theta = 12$ $\sin \theta = \frac{12}{13}$ $x = -\sqrt{13^2 - 12^2} \text{ (Pyth)}$ $x = -5$ $y = 12$ $P(-5; 12)$	<ul style="list-style-type: none"> ✓M sin θ the subject/die onderwerp ✓M Pythagoras ✓A x-value/waarde ✓A y-value/waarde 	(4)
3.1.2	$\tan \theta + \sec \theta = \frac{12}{-5} + \frac{13}{-5}$ $= \frac{25}{-5}$ $= -5$	<ul style="list-style-type: none"> ✓CA correct value/korrekte waarde: tan ✓CA correct value/korrekte waarde: sec ✓CA answer/antwoord 	(3)
3.1.3	$\sin \theta = \frac{12}{13}$ $\therefore \text{Ref } \angle = \sin^{-1} \left(\frac{12}{13} \right) = 67,38\dots^\circ$ $\therefore \theta = 180^\circ - 67,38\dots^\circ$ $\therefore \theta = 112,6^\circ$	<ul style="list-style-type: none"> ✓M $180^\circ -$ ✓CA $112,6^\circ$ 	(2)

<p>3.2</p>	$\frac{\tan(180^\circ - \theta) \cdot \sqrt{1 - \sin^2 \theta}}{\cos^2(180^\circ + \theta) + \sin^2(360^\circ - \theta)}$ $= \frac{(-\tan \theta)(\cos \theta)}{(-\cos \theta)^2 + (-\sin \theta)^2}$ $= \frac{-\frac{\sin \theta}{\cos \theta} \times \cos \theta}{\cos^2 \theta + \sin^2 \theta}$ $= \frac{-\sin \theta}{1}$ $= -\sin \theta$	<p>✓A $-\tan \theta$</p> <p>✓A $\cos \theta$</p> <p>✓A $(-\cos \theta)^2$</p> <p>✓A $(-\sin \theta)^2$</p> <p>✓A $\frac{\sin \theta}{\cos \theta}$</p> <p>✓A $\cos^2 \theta + \sin^2 \theta = 1$</p> <p>✓CA $-\sin \theta$</p>	<p>(7)</p>
<p>3.3</p>	<p>RHS = $\frac{2}{\sin \alpha}$</p> <p>LHS = $\frac{1 + \cos \alpha}{\sin \alpha} + \frac{\sin \alpha}{1 + \cos \alpha}$</p> $= \frac{(1 + \cos \alpha)^2 + \sin^2 \alpha}{\sin \alpha (1 + \cos \alpha)}$ $= \frac{1 + 2 \cos \alpha + \cos^2 \alpha + \sin^2 \alpha}{\sin \alpha (1 + \cos \alpha)}$ $= \frac{1 + 2 \cos \alpha + 1}{\sin \alpha (1 + \cos \alpha)}$ $= \frac{2 + 2 \cos \alpha}{\sin \alpha (1 + \cos \alpha)}$ $= \frac{2(1 + \cos \alpha)}{\sin \alpha (1 + \cos \alpha)}$ $= \frac{2}{\sin \alpha}$ <p>= RHS</p>	<p>✓A RHS</p> <p>✓A denominator/noemer</p> <p>✓A numerator/teller</p> <p>✓CA expansion/ uitbreiding</p> <p>✓A $\cos^2 \alpha + \sin^2 \alpha = 1$</p> <p>✓CA factorising/ faktorisering</p>	<p>(6)</p>
			<p>[22]</p>

QUESTION/VRAAG 4



4.1	$\sin \theta = \frac{9,48}{15,64} = 0,606\dots$ $\theta = 37,3^\circ$	✓A trig ratio/verhouding ✓CA value for θ /waarde vir θ	(2)
4.2	$\text{Area } \Delta PSR = \frac{1}{2} PS \times SQ \sin \hat{P}SQ$ $= 0,5 \times 4,95 \times 15,64 \times \sin 52,69^\circ$ $= 30,79 \text{ cm}^2$	✓A area-rule/area-reël ✓CA value for $\hat{P}SQ$ /waarde vir $\hat{P}SQ$ ✓A correct substitution/vervanging ✓CA Area of ΔDEF	(4)
4.3	$\hat{P}SQ = 90^\circ - 37,31^\circ = 52,69^\circ$ $PQ^2 = PS^2 + SQ^2 - 2PS \times SQ \cos \hat{P}SQ$ $= 4,95^2 + 15,64^2 - 2 \times 4,95 \times 15,64 \cos 52,69^\circ$ $= 175,261\dots$ $PQ = \sqrt{175,261\dots}$ $\approx 13,24 \text{ m}^2$	✓S ✓A cos-rule/reël ✓A correct substitution/vervanging ✓CA simplification/vereenvoudiging ✓CA value of/waarde van PQ	(5)
			[11]

QUESTION/VRAAG 5			
5.1.		<p>✓A x-int's 120° and/en 300°</p> <p>✓A TP $(30^\circ; 1)$</p> <p>✓A TP $(210^\circ; -1)$</p> <p>✓A shape/vorm</p>	(4)
5.2.1	Amplitude of $f = 2$	<p>✓A</p> <p>amplitude = 2</p>	(1)
5.2.2	Range $g: -1 \leq y \leq 1$	<p>✓A y-values/waardes</p> <p>✓CA interval</p>	(2)
5.2.3	$x = 90^\circ$	✓CA	(1)
5.2.4	<p>$f(x) \leq g(x) \Leftrightarrow x \in [210^\circ; 360^\circ]$</p> <p>OR/OF</p> <p>$210^\circ \leq x \leq 360^\circ$</p>	<p>✓CA $x = 210^\circ$</p> <p>✓CA $x = 360^\circ$</p> <p>✓A notation/notasie</p>	(3)
			[11]

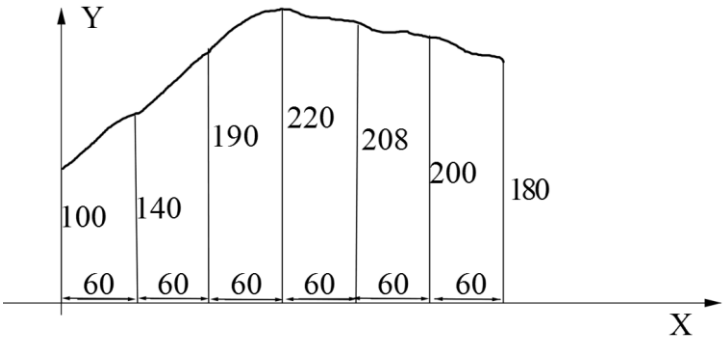
QUESTION/VRAAG 6			
6.1	Proportionally/Eweredig ✓	(1)	
6.2			
	<p>Let $QS = x$</p> <p>$\therefore SC = 4 - x$</p> $\frac{CS}{SA} = \frac{CR}{RB} \quad (\text{line } \parallel \text{ to one side of } \Delta)$ $\frac{4 - x}{x + 2} = \frac{1}{3}$ $12 - 3x = x + 2$ $10 = 4x$ $x = 2,5$ <p>$\therefore QS = 2,5 \text{ units/eenhede}$</p>	<p>✓S $SC = 4 - x$</p> <p>✓S ✓R</p> <p>✓S $SA = x + 2$</p> <p>✓A Setup equation/stel vergelyking op</p> <p>✓CA Simplify equation/Ver-eenvoudig vergelyking</p> <p>✓CA value / waarde x</p>	(7)
		[8]	

QUESTION/VRAAG 7			
7.1			
7.1.1	AB = 8,36 cm (sides opp = ∠s) BD = 5,91 cm	✓S ✓R ✓S	(3)
7.1.2	$\frac{AB}{BD} = \frac{8,36}{5,91} \approx 1,41$ $\frac{BC}{DC} = \frac{8,36}{5,91} \approx 1,41$ $\frac{AC}{BC} = \frac{11,82}{8,36} \approx 1,41$	✓S ✓S ✓S	(3)
7.1.3	triangles are similar/ <i>driehoek is gelykvorming</i>	✓S similar triangles / <i>gelykvormige driehoek</i>	(1)
7.2			
7.2.1(a)	$\hat{R}_3 = \hat{T}_2 = 56^\circ$ (∠s in same segment)	✓S ✓R	(2)

7.2.1(b)	$\hat{R}_4 = \hat{T}_1 = 56^\circ$ (tan-chord)/(tan-koord)	✓S ✓R	(2)
7.2.2	$\hat{A} = \hat{M}$ (\angle s in same segment) $\hat{A} = \hat{R}_1$ (tan-chord)	✓S ✓S	(2)
7.2.3	$\frac{AP}{MP} = \frac{PR}{PT}$ (Δ s $///$) $\frac{8,29}{MP} = \frac{11,11}{12,01 - 8,29}$ $MP = \frac{8,29 \times 3,72}{11,11}$ $= 2,78 \text{ cm}$	✓S ✓S ✓S PT = 12,01 – 8,29 = 3,72 ✓S ✓S <u>value of /</u> <u>waarde van MP</u>	(5)
			[18]

QUESTION/VRAAG 8			
8.1			
	$\hat{P}TR = 90^\circ$ (line from centre to midpt of chord) $TR^2 = PR^2 - PT^2$ (Pythagoras) $= 9,47^2 - 8,43^2$ $= 18,616$ $TR = 4,31\dots$ $WR \approx 8,63$ units	✓S ✓R ✓S Pythagoras ✓A Simplification/ Vereenvoudiging ✓CA Value of/waarde van TR ✓CA Value of/waarde van WR	(6)
8.2			
	$\hat{A}BO = 40^\circ$ (radii; equal \angle s opp equal sides) $\hat{A}OB = 100^\circ$ (int \angle s of Δ) $\hat{D} = 50^\circ$ (\angle at centre = $2 \times \angle$ at circumf.) $x = 130^\circ$ (opp \angle s of cyclic quad)	✓S ✓S ✓S ✓R ✓S ✓R	(6)
			[12]

QUESTION/VRAAG 9			
9.1.1	$108,5^\circ = 108,5^\circ \times \frac{\pi}{180^\circ}$ $= \frac{217}{360} \pi$ $= 1,89 \text{ radians}$	✓A Multiply with factor/ <i>maal met faktor</i> ✓CA Answer/ <i>antwoord</i>	(2)
9.1.2	$\text{Radius} = \frac{10,84}{2} = 5,42 \text{ units /eenhede}$	✓A Answer/ <i>antwoord</i>	(1)
9.1.3	$s = r\theta$ $d = 5,42 \times 1,89$ $= 10,24 \text{ units}$	✓A correct/ <i>korrekte</i> formula ✓CA Substitution/ <i>vervanging</i> ✓CA answer/ <i>antwoord</i>	(3)
9.1.4	$\text{Area} = \frac{r^2\theta}{2}$ $= \frac{5,42^2 \times 1,89}{2}$ $= 27,76 \text{ units}^2$	✓A Correct formula / <i>korrekte formule</i> ✓CA Substitution/ <i>vervanging</i> ✓CA answer/ <i>antwoord</i>	(3)
9.2	$4h^2 - 4dh + x^2 = 0$ $d = \frac{x^2}{4h} + h$ $= \frac{8,76^2}{4 \times 3,15} + 3,15$ $= 9,24 \text{ units}$	✓A Correct formula / <i>korrekte formule</i> ✓A Making d the subject/ <i>maak d die onderwerp</i> ✓A Substitution/ <i>vervanging</i> ✓CA answer/ <i>antwoord</i>	(4)

9.3			
	$A_T = a \left(\frac{o_1 + o_n}{2} + o_2 + o_3 + o_4 + \dots + o_{n-1} \right)$ $= 6 \left(\frac{10 + 18}{2} + 14 + 19 + 22 + 20,8 + 20 \right)$ $= 6(109,8)$ $= 658,8 \text{ cm}^2$	<p>✓A Correct formula / <i>korrekte formule</i></p> <p>✓A Convert to cm /<i>herlei na cm</i></p> <p>✓A Substitution/ <i>vervanging</i></p> <p>✓CA 109,8</p> <p>✓CA 658,8</p>	(5)
			[18]

QUESTION/VRAAG 10			
10.1.1	$n = 15 \text{ rev/s}$ $\omega = 2\pi n$ $= 2\pi(15)$ $= 30\pi$ $= 94,25 \text{ rad/s}$	✓A Correct formula / <i>korrekte formule</i> ✓A Substitution/ <i>vervanging</i> ✓CA value of/ <i>waarde</i> <i>van</i> ω	(3)
10.1.2	$d = 570\text{mm} = 0,57\text{m}$ $v = \pi Dn$ $= \pi(0,57)(15)$ $= 26,86 \text{ m/s}$	✓A Convert diameter to metre/ <i>herlei middelyn</i> <i>na meter</i> ✓A Correct formula / <i>korrekte formule</i> ✓A Substitution/ <i>vervanging</i> ✓CA value of/ <i>waarde</i> <i>van</i> v	(4)
10.1.3	$s = vt$ $= 26,86... \times (2 \times 60)$ $\approx 3223 \text{ m}$	✓A Correct formula / <i>korrekte formule</i> ✓A Multiply $t = 2$, with 60 / <i>Vermenigvuldig t</i> $= 2$, met 60 ✓CA value of/ <i>waarde</i> <i>van</i> s ✓CA rounded answer <i>/afgeronde antwoord</i>	(4)
10.1.4	$\theta = \omega t$ $= (30\pi)(0,3)$ $= 9\pi \text{ rad}$	✓A Correct/ <i>korrekte</i> <i>formula</i> ✓A Substitution/ <i>vervanging</i> ✓CA value of/ <i>waarde</i> <i>van</i> θ	(3)

10.2	<p>Vol of cylinder = $\pi r^2 h$</p> $= \pi \left(\frac{50}{2} \right)^2 (100)$ $= 62500\pi$ $\approx 196350 \text{ mm}^3$ <p>Vol of hemisphere = $\frac{1}{2} \times \text{Vol of sphere}$</p> $= \frac{1}{2} \times \frac{4}{3} \pi r^3$ $= \frac{2}{3} \pi \times 25^3$ $= \frac{31250\pi}{3}$ $\approx 32725 \text{ mm}^3$ <p>\therefore Total/Totale volume = $229\,075 \text{ mm}^3$</p>	<p>✓A Formula of cylinder/formule vir silinder</p> <p>✓A Divide diameter by 2 / deel middellyn deur 2</p> <p>✓A Substitution/vervanging</p> <p>✓CA Vol of cylinder/silinder</p> <p>✓A Formula of hemisphere/formule vir hemisfeer</p> <p>✓A Substitution/vervanging</p> <p>✓CA Vol of hemisphere/hemisfeer</p> <p>✓CA Adding the Volumes/tel die volumes bymekaar</p>	(7)
			[21]
		TOTAL/TOTAAL:	150