Province of the
EASTERN CAPE
EDUCATION

NATIONAL SENIOR CERTIFICATE
NASIONALE SENIOR SERTIFIKAAT

GRADE/*GRAAD* 12

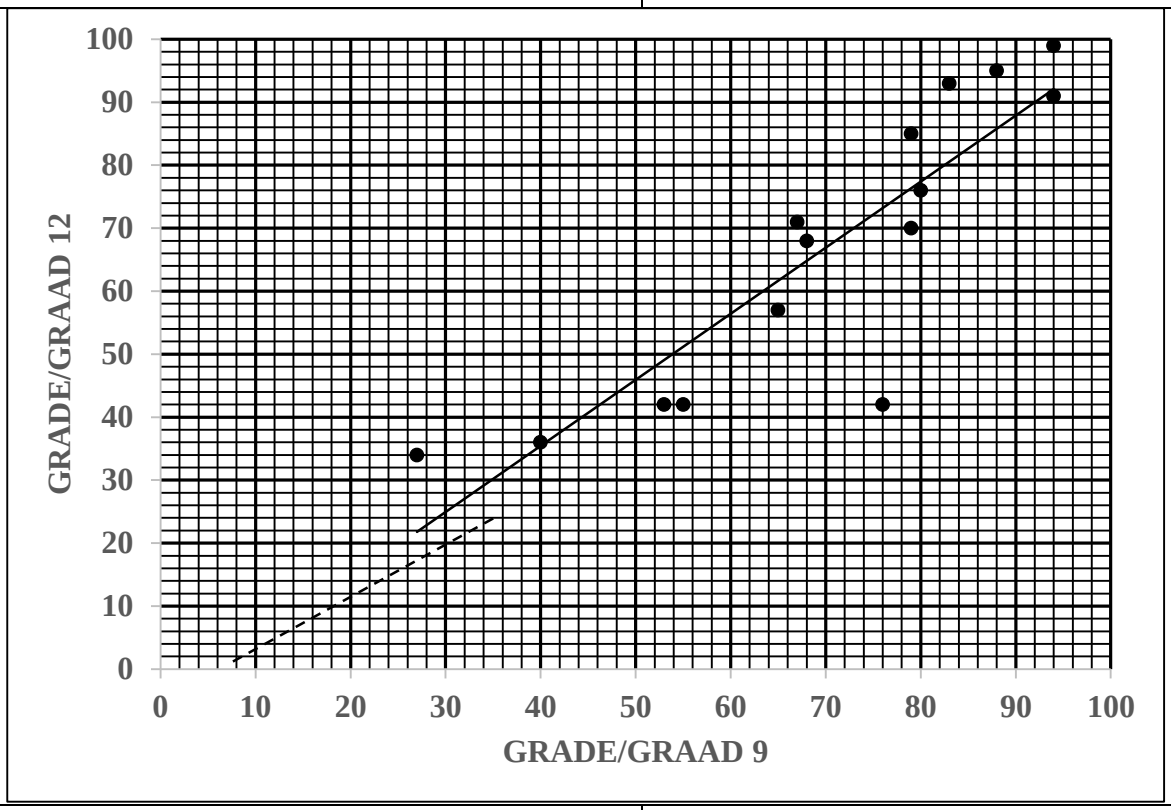
SEPTEMBER 2018

MATHEMATICS P2/*WISKUNDE V2*
MARKING GUIDELINE/*NASIENRIGLYN*

MARKS/*PUNTE*: 150

This marking guideline consists of 15 pages.
Hierdie nasien riglyn bestaan uit 15 bladsye.

QUESTION 1/*VRAAG 1*



1.1	$a = -6,54$ $b = 1,05$ $\hat{y} = 1,05\,x - 6,54$	✓ value of *a / waarde van a* ✓ value of *b / waarde van b* ✓ equation / *vergelyking*	(3)
1.2	$y = -6,54 + 1,05\,(41)$ $= 36,51 \approx 37$	✓ substitution / *vervanging* ✓ answer / *antwoord*	(2)
1.3	On the scatter plot / *Op spreidiagram*	✓✓ *x*-intercept / *x-afsnit* $6 < x < 8$ and / *en* (45;41) both correct / *beide korrek* **OR/*OF*** ✓✓ (69,87;66,73) and/*en* (45;41) both correct / *beide korrek*	(2)
1.4	$r = 0,88$	✓ ✓ answer / *antwoord*	(2)
1.5	Yes. The strong positive correlation *Ja. Die sterk positiewe korrelasie*	✓ Yes / *Ja* ✓ strong positive / *sterk positief*	(2)
			[11]

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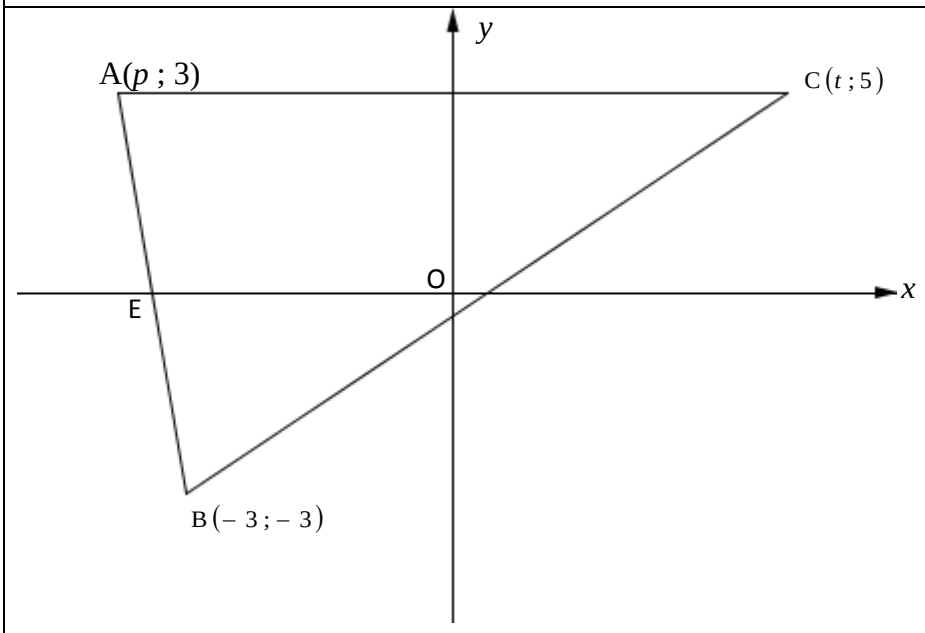
QUESTION 2/*VRAAG 2*

2.1	$\text{Range/Omvang} = 29 - 10$ $= 19$	✓ answer / *antwoord*	(1)
2.2	$\bar{x} = \frac{15 + 23 + 17 + 24 + 26 + 18 + 28 + 13 + 10 + 28 + 29}{11}$ $= \frac{231}{11}$ $= 21$ Answer ONLY full marks *Slegs antwoord - volpunte*	✓✓ answer / *antwoord*	(2)
2.3	$\sigma = 6{,}37$	✓ min ✓ max/*maks* ✓ answer / *antwoord*	(3)
2.4	$(21 - 6{,}37 ; 21 + 6{,}37) = (14{,}63 ; 27{,}37)$ 5 weeks/weke	✓ 231 ✓ answer / *antwoord*	(2)
			[8]

QUESTION 3 / *VRAAG 3*



3.1	$\left(\sqrt{89}\right)^2 = (-3 - t)^2 + (-3 - 5)^2$ $89 = 9 + 6t + t^2 + 64$ $t^2 + 6t - 16 = 0$ $(t - 2)(t + 8) = 0$ $\therefore t = 2 \text{ or / of } t \neq -8$	✓substitution / *vervaning* ✓ simplification / *vereenvoudiging* ✓ standard form / *standaardvorm* ✓ factors / *faktore* ✓ value of *t* / *waarde van t*	(5)

3.2	$m_{AB} = -3$ $\frac{3+3}{p+3} = -3$ $-3p - 9 = 6$ $p = -5$	✓ m_{AB} ✓ m_{AB} in terms of *p / in terme van p* ✓ equating / *gelykstelling* ✓ value of *p / waarde van p*	(4)
3.3	$-3x - 12 = 0$ $x = -4$ $E(-4;0)$	✓ $y = 0$ ✓ $x = 4$	(2)
3.4	$M = \left(\frac{-5+2}{2}; \frac{3+5}{2}\right)$ $= \left(-\frac{3}{2}; 4\right)$	✓ x – coordinate/*koördinaat* ✓ y – coordinate/*koördinaat*	(2)
3.5	$m_{EM} = \frac{0-4}{-4+\frac{3}{2}}$ $= \frac{8}{5}$ $m_{BC} = \frac{5+3}{2+3}$ $= \frac{8}{5}$ $\therefore$ EM $\|$ BC [= gradients/*gradiënte*]	✓ correct substitution / *korrekte vervanging* ✓ m_{EM} ✓ m_{BC} ✓ = gradients / = *gradiënte*	(4)
3.6	$\tan\theta = -3$ $\theta = 108,4349488^0$ $\tan\beta = \frac{8}{5}$ $\beta = 57,99461679^0$ $\therefore A\hat{B}C = 50,44^0$	✓ size of θ / *grootte van* θ ✓ size of α / *grootte van* β ✓ size of $A\hat{B}C$ / *grootte van* $A\hat{B}C$	(4)
			[21]

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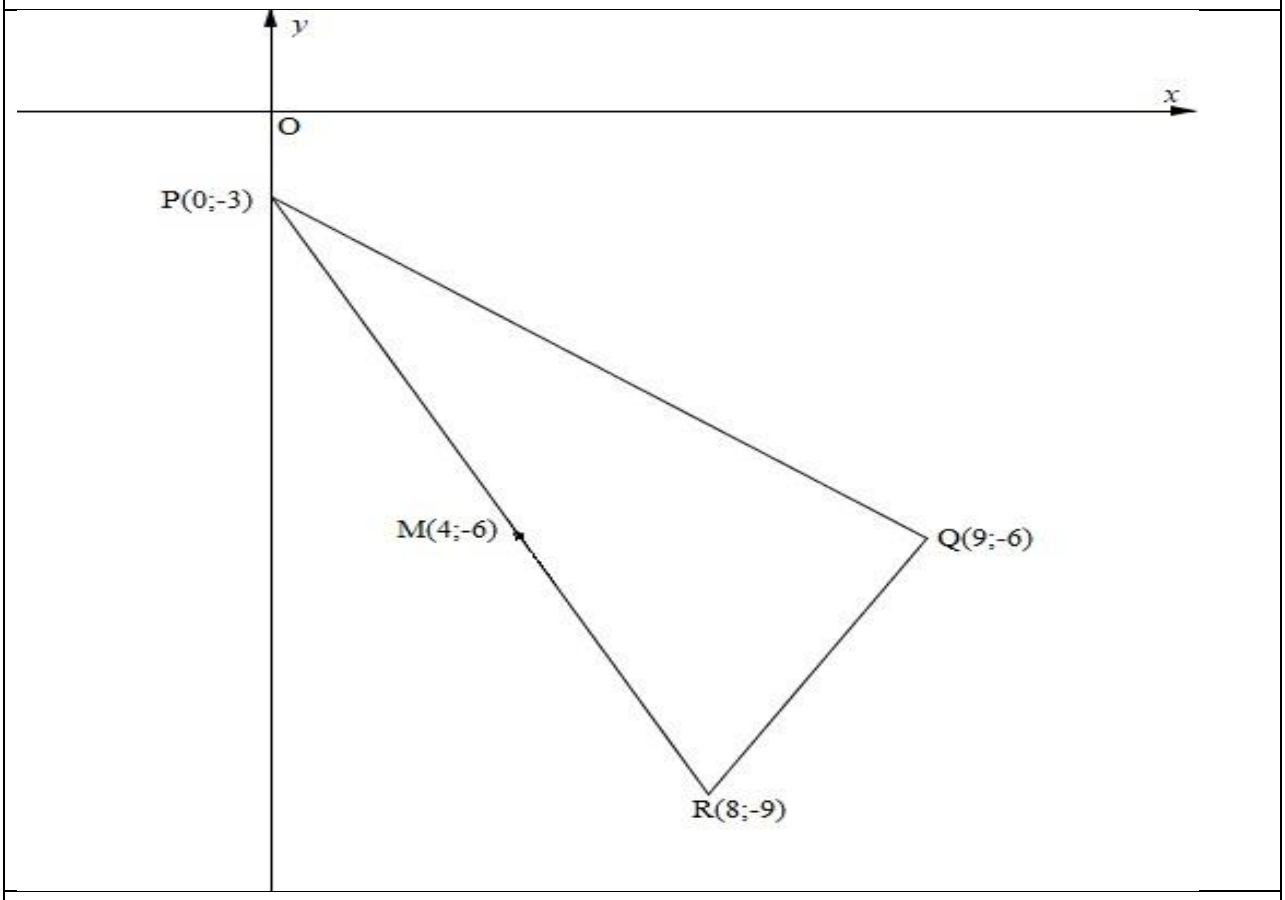
QUESTION 4 / *VRAAG 4*



4.1

$PR = \sqrt{(0-8)^2 + (-3+9)^2}$

$= 10$

✓ correct substitution / *korrekte vervanging*

✓ answer / *antwoord* (2)

4.2

$M = \left(\frac{0+8}{2}; \frac{-3-9}{2}\right)$

$= (4;-6)$

✓ *x*-coordinate/*koördinaat*

✓ *y*-coordinate/*koördinaat* (2)

4.3

$m_{PQ} = \frac{-3+6}{0-9}$

$= -\frac{1}{3}$

$m_{QR} = \frac{-6+9}{9-8}$

$= 3$

$\therefore P\hat{Q}R = 90^0 \quad [m_{PQ} \times m_{QR} = -\frac{1}{3} \times 3 = -1]$

✓ correct substitution
korrekte vervanging

✓ m_{PQ}

✓ m_{QR}

✓ $m_{PQ} \times m_{QR} = -\frac{1}{2} \times 2$ (4)

4.4

$(x-4)^2 + (y+6)^2 = 25$

✓ $r^2 = 25$

✓ equation / *vergelyking* (2)

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4.5	$m_{rad} = \frac{-6+3}{4-0}$ $= -\frac{3}{4}$ $m_{\tan} = \frac{4}{3}$ $y = \frac{4}{3}x - 3$	✓ correct subst. / *korrekte verv.* ✓ $m_{rad} = -\frac{3}{4}$ ✓ $m_{\tan} = \frac{4}{3}$ ✓ Subst / *Verv.* (0 ; -3) & m ✓ equation / *vergelyking* (5)
4.6	$\left(\sqrt{146}\right)^2 = (\cos\theta - 8)^2 + (\sin\theta + 9)^2$ $146 = \cos^2\theta - 16\cos\theta + 64 + \sin^2\theta + 18\sin\theta + 81$ $0 = -16\cos\theta + 18\sin\theta$ $0 = -16\cos\theta + 18\sin\theta$ $\frac{\sin\theta}{\cos\theta} = \frac{16}{18}$ $\tan\theta = \frac{8}{9}$	✓ correct substitution *korrekte vervanging* ✓ simplification/*vereenvoudiging* ✓ $\sin^2\theta + \cos^2\theta = 1$ ✓ equation/*vergelyking* ✓ $\tan\theta = \frac{8}{9}$ (5)
		[20]

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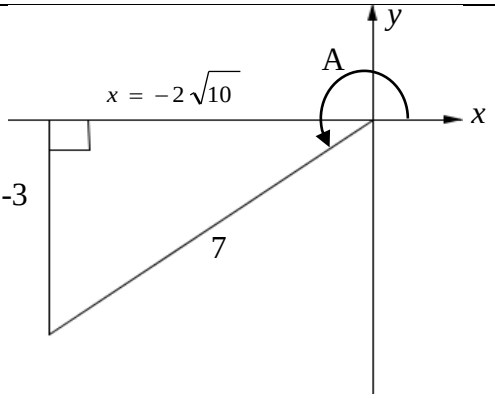
QUESTION 5/*VRAAG 5*

5.1



$\sin\left(A + 30^0\right) = \sin A\cos 30^0 + \cos A\sin 30^0$

$= \left(-\frac{3}{7}\right)\left(\frac{\sqrt{3}}{2}\right) + \left(\frac{-2\sqrt{10}}{7}\right)\left(\frac{1}{2}\right)$

$= -\frac{3\sqrt{3} - 2\sqrt{10}}{14}$

✓ $x = -2\sqrt{10}$

✓ Expansion/*Uitbreiding*

✓ Both/*Beide* $\frac{\sqrt{3}}{2}$ & $\frac{1}{2}$

✓ $-\frac{2\sqrt{10}}{7}$ (4)

5.2

$-\sin^2\left(90^0 - x\right) - \tan\cos\left(-x\right).\sin\left(-x - 360^0\right)$

$= -\left(\cos x\right)^2 - \frac{\sin x}{\cos x}\left(\cos x\right)\left(-\sin x\right)$

$= -\cos^2 x + \sin^2 x$

$= -\left(\cos^2 x - \sin^2 x\right)$

$= -\cos 2x$

✓ $(\cos x)^2$

✓ $\frac{\sin x}{\cos x}$

✓ $\cos x$

✓ $\left(-\sin x\right)$

✓ $-\left(\cos^2 x - \sin^2 x\right)$

✓ $-\cos 2x$ (6)

5.3

$x^2 - 2x\sin A = \cos^2 A$

$\cos^2 A + 2x\sin A - x^2 = 0$

$\Delta = \left(2\sin A\right)^2 - 4\left(-1\right)\left(\cos^2 A\right)$

$= 4\left(\sin^2 A + \cos^2 A\right)$

$= 4$

✓ standard form/*standaardvorm*

✓ correct substitution/*korrekte vervanging*

✓ $\Delta = 4$ (3)

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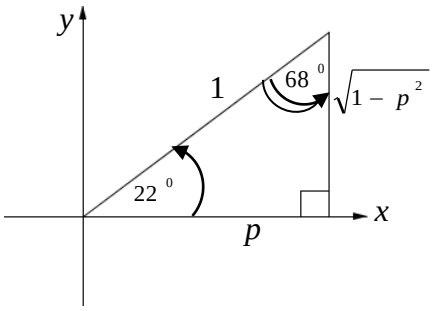
5.4	$\text{LHS/LK} = \frac{\cos 3x}{\sin x} + \frac{\sin 3x}{\cos x}$ $= \frac{\cos 3x \cos x + \sin 3x \sin x}{\sin x \cos x}$ $= \frac{\cos(3x - x)}{\sin x \cos x}$ $= \frac{\cos 2x}{\frac{1}{2}\sin 2x}$ $= \frac{2}{\tan 2x}$	✓ Simplification *Vereenvoudiging* ✓ $\cos 2x$ ✓ $\frac{1}{2}\sin 2x$ (3)
5.5.1	$\sin 68^0 = \cos 22^0$ $= p$ **OR/*OF***  $\sin 68^0 = p$	✓ $\cos 22^0$ ✓ p **OR/*OF*** ✓ $y = \sqrt{1 - p^2}$ ✓ $\sin 68^0 = p$ (2)
5.5.2	$\cos 16^0 = \cos(38^0 - 22^0)$ $= \cos 38^0 . \cos 22^0 + \sin 38^0 . \sin 22^0$ $= \sqrt{1 - q^2} . p + q . \sqrt{1 - p^2}$ $= p\sqrt{1 - q^2} + q\sqrt{1 - p^2}$	✓ $\cos(38^0 - 22^0)$ ✓ Expansion / *Uitbreiding* ✓ $\cos 38^0$ i.t.o / *i.t.v* q ✓ $\sin 22^0$ i.t.o / *i.t.v* p (4)
		[22]

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QUESTION 6 / *VRAAG 6*

6.1

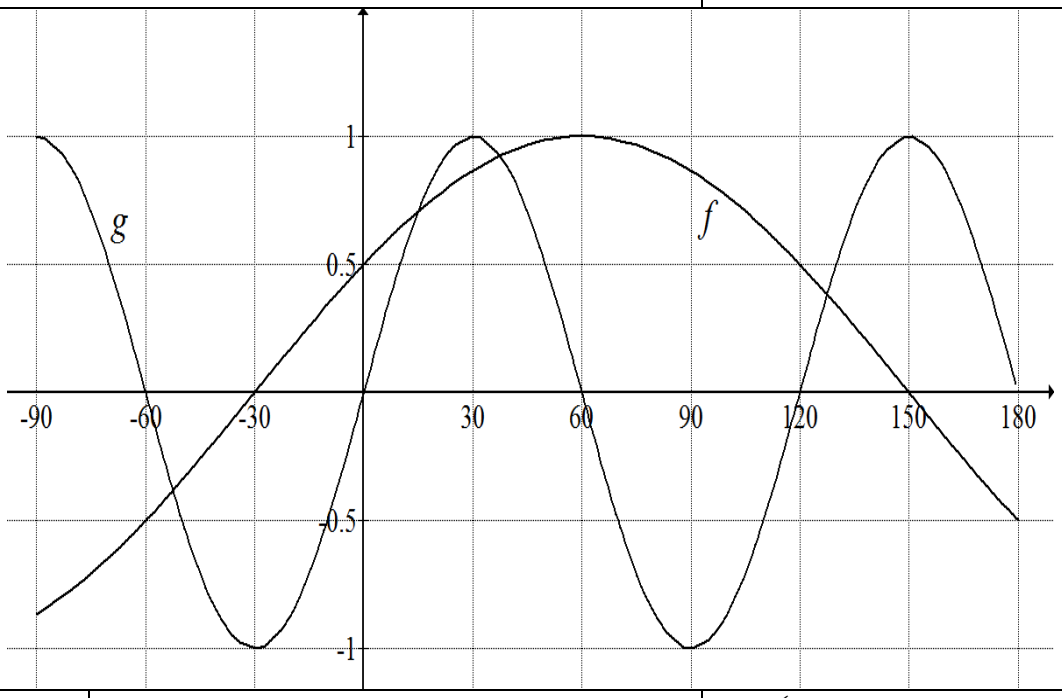
$a = -2$

$p = 30^0$

✓ $a = -2$

✓ $p = 30^0$ (2)

6.2



6.2.1 $x = 60^0$ ✓ (1)

6.2.2

$\cos(x - 60) = \sin 3x$

$\cos(x - 60) = \cos(90^0 - 3x)$

$x - 60 = \pm(90^0 - 3x) + 360^0.k \quad k \in Z$

$\therefore 4x = 150^0 + 360^0.k$

OR

$-2x = -30^0 + 360^0.k \quad k \in Z$

$\therefore x = 37,50^0 + 90^0.k$ or $x = 15^0 - 180^0.k$

$\therefore x = -52,50^0$ and $x = 15^0$

✓ co-function / *ko-funksie*

✓ *ref* ∠

✓ $4x = 150^0 + 360^0.k$

✓ &

$-2x = -30^0 + 360^0.k$

✓ $x = 15^0$

✓ $x = -52,50^0$ (6)

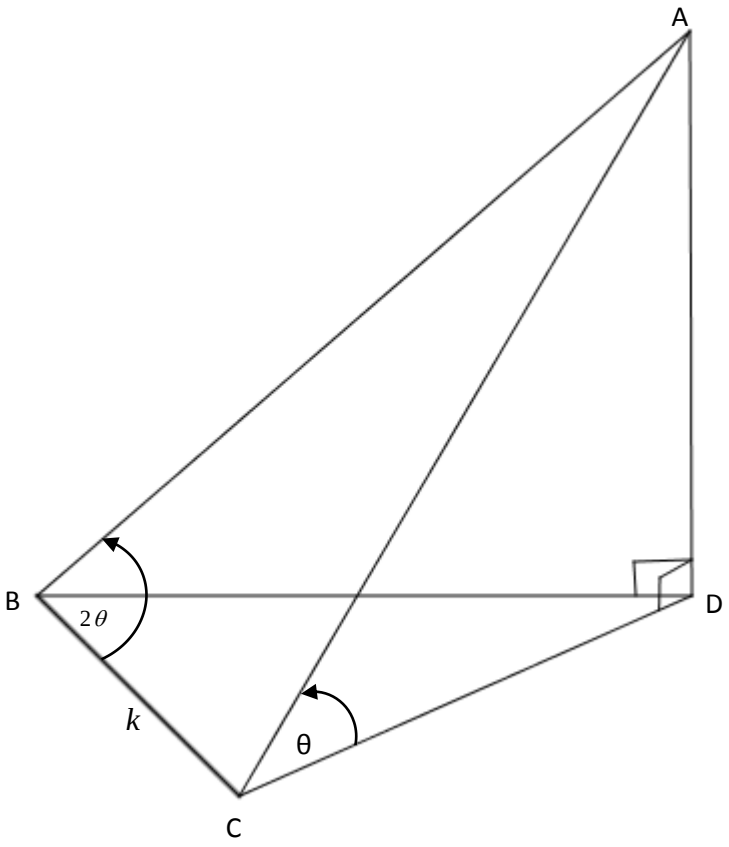
6.2.3 $-52,50^0 < x < 15^0$

✓ both critical values / *beide kritiese waardes*

✓ notation / *notasie* (2)

[11]

QUESTION 7 / *VRAAG 7*



7.1	$AB = k$ $AC = \sqrt{(2k)^2 + k^2 - 2.2k.k.\cos 2\theta}$ $= \sqrt{5k^2 - 4k^2.\cos 2\theta}$ $= \sqrt{k^2\left(5 - 4\left(1 - 2\sin^2\theta\right)\right)}$ $= \sqrt{k^2\left(5 - 4 + 8\sin^2\theta\right)}$ $= k\sqrt{1 + 8\sin^2\theta}$	✓ AB i.t.o / *i.t.v* k ✓ cosine rule formula in Δ ABC *kosinusreël formule in* Δ *ABC* ✓ correct subst. / *korrekte vervanging* ✓ $\cos 2\theta = 1 - 2\sin^2\theta$ ✓ simplification / *vereenvoudiging*	(5)
7.2	$AC = 139,5\sqrt{1 + 8\sin^2(42^0)}$ $\approx 299 \ m$	✓ correct substitution/*korrekte vervanging* ✓ answer/*antwoord*	(2)
			[7]

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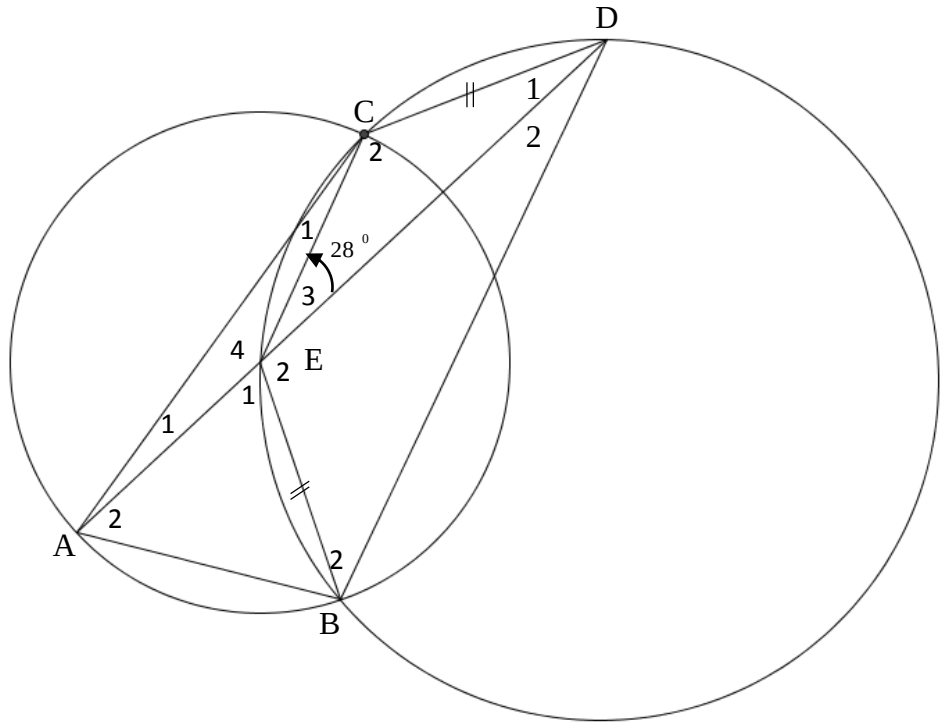
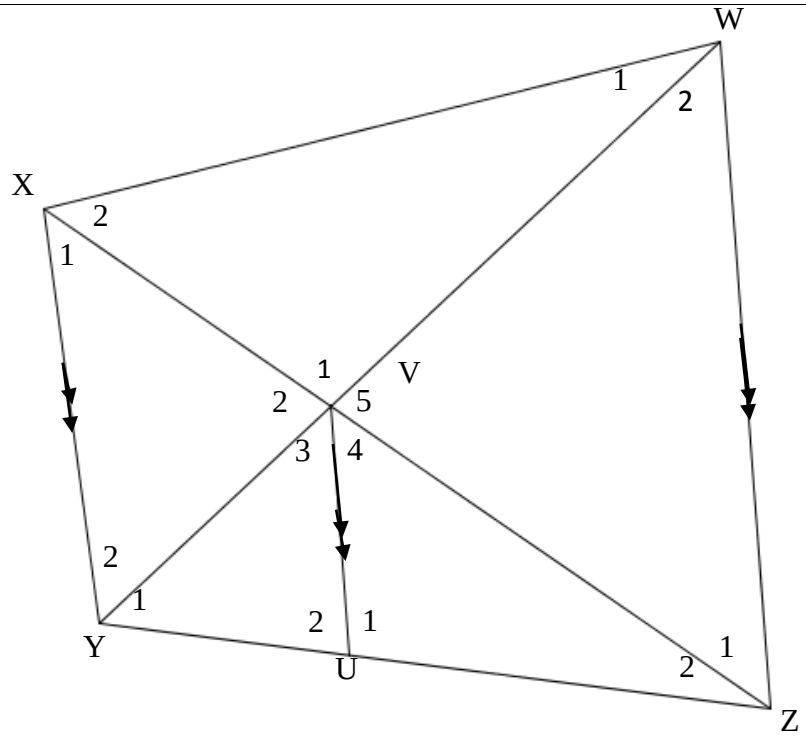
QUESTION 8 / *VRAAG 8*



8.1	$\hat{D}_2 = 28^0$ [∠s subt by = chords]/[∠e onderspan deur = koorde]	✓ S ✓ R (2)
8.2	Alternate ∠s = / *Verwisselende* ∠e	✓ R (1)
8.3	EB = EC [radii / *radiusse*] but/*maar* EB = CD [given / *gegee*] ∴ EC = CD	✓S ✓R (2)
8.3.1	$\hat{D}_1 = 28^0$ [∠s opp = sides]/[∠e teenoor = sye] $\hat{C}_2 = 124^0$ [∠s of a Δ]/[∠e van 'n Δ] ∴ $\hat{B}_2 = 56^0$ [opp.∠s of a cyclic quad]/ [*teenoorst.∠e van koordevierhoek*]	✓ S/R ✓ S ✓S ✓R (4)
8.3.2	$\hat{E}_2 = 96^0$ [∠s of a Δ]/[∠e van 'n Δ] ∴ $B\hat{A}C = \frac{1}{2}(96^0 + 28^0)$ [∠ at centre = 2 × ∠ at circumf] $= 62^0$ [*Middelpunts∠ = 2 × Omtrekshoek*]	✓ S ✓S ✓R (3)
		[12]

QUESTION 9 / *VRAAG 9*



9.1

$\frac{YU}{UZ} = \frac{XV}{VZ}$ [prop theo, $UV \parallel YX$ or line $\parallel$ to one side of a Δ] [*Eweredigh.Stelling, UV ||YX of lyn ||aan een sy van 'n Δ*]

$= \frac{YV}{VW}$ [prop theo, $UV \parallel ZW$ or line $\parallel$ to one side of a Δ] [*Eweredigh.Stelling, UV ||ZW of lyn ||aan een sy van 'n Δ*]

$\therefore \frac{XV}{VZ} = \frac{YV}{VW}$

✓ S ✓R

✓ S

(3)

9.2

$\frac{\text{Area of/van } \Delta XVY}{\text{Area of/van } \Delta WVZ} = \frac{\frac{1}{2} \times 3r \times 3s \times \sin \hat{V}_2}{\frac{1}{2} \times 4r \times 4s \times \sin \hat{V}_5}$

but/*maar* $\hat{V}_2$ [vert.opp.∠s]/[*regoorst. ∠e*]

$= \frac{9}{16}$

✓substitution / *vervanging*

✓substitution / *vervanging*

✓S/R

✓answer / *antwoord*

(4)

9.3

$\hat{X}_1 = \hat{V}_4$ [alt.∠s, $XY \parallel WX$]/[*verw.∠e, XY ||WX*]

$\hat{V}_3 = \hat{V}_4$ [given]/[gegee]

$\hat{V}_3 = \hat{W}_2$ [corresp ∠s, $WZ \parallel UV$]/[*ooreenk.∠e, WZ ||UV*]

$\therefore \hat{X}_1 = \hat{W}_2$

WXYZ is a cyclic quad [converse ∠s same segment or line subt = ∠s]

WXYZ is 'n koordevierhoek

[omgekeerde ∠e in dieselfde segment of lyn onderspan = ∠e]

✓ S/R

✓ S/R

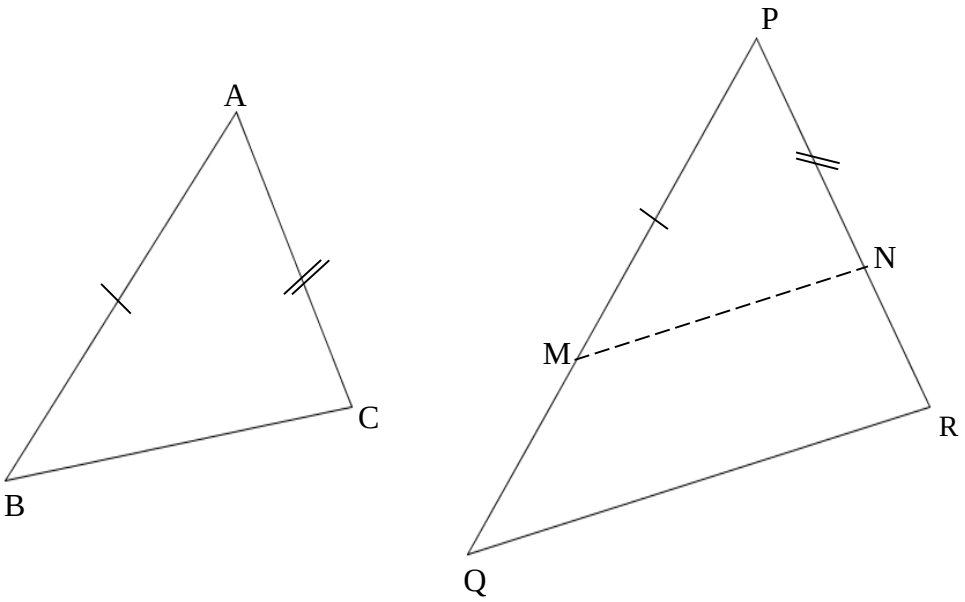
✓ R

(3)

9.4	$\hat{V}_3 = \hat{V}_4 = \hat{X}_1$ $\therefore$ UV is a tangent to circle XYV [converse of tan chord theo.] *UV is 'n raaklyn aan die sirkel XYZ* [*omgekeerde van raaklyn – koord stelling*]	✓ S ✓ R	(2)
			[12]

QUESTION 10 / *VRAAG 10*

10.1



Constr: Mark M on PQ and N on PR such that PM = AB and PN = AC

Konstr: Merk M op PQ en N op PR sodat PM = AB en PN = AC

✓constr
konstr.

Proof/*Bewys*: In ΔABC and/*en* ΔPMN

$AB = PM$ [constr / *konstr.*]

✓ SSS

$\hat{A} = \hat{P}$ [given / *gegee*]

$AC = PN$ [constr / *konstr.*]

$\therefore \Delta ABC \equiv \Delta PMN$ [S∠S]

$\therefore \hat{B} = P\hat{M}N$

✓ R

$= \hat{Q}$ [given / *gegee*]

$\therefore MN \parallel QR$ [corresp ∠s = /*ooreenk.∠e* =]

$\frac{PM}{PQ} = \frac{PN}{PR}$ [prop theo/*eweredigh. stelling*, $MN \parallel QR$]

✓ S/R

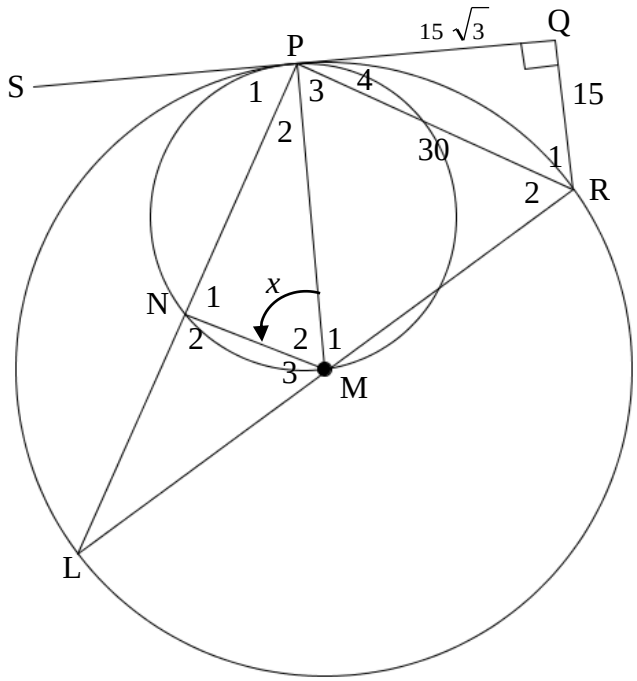
but/*maar* $AB = PM$ and/*en* $AC = PN$ [constr / *konstr.*]

✓ S/R

$\therefore \frac{AB}{PQ} = \frac{AC}{PR}$

✓ S

(6)

10.2			
10.2.1	$\hat{N}_1 = 90^0$ [∠ subt by diameter / ∠ in semi – circle] [∠ *onderspan deur middellyn* / ∠ *in semi-sirkel*] $\therefore$ LN = NP [line from centre ⊥ to chord] / [*lyn vanaf die middelpunt* ⊥ *op koord*]	✓ S ✓ R ✓ R	(3)
10.2.2	$\hat{P}_4 = \hat{L}$ [tangent chord theorem]/[*raaklyn - koord stelling*] $L\hat{P}R = 90^0$ [∠ subt by diameter]/[∠ *onderspan deur middellyn*] $\therefore \hat{R}_2 = 90^0 - \hat{P}_4$ [∠s/*e* of/*van* Δ LPR] $\hat{R}_1 = 90^0 - \hat{P}_4$ [∠s/*e* of/*van* Δ RPQ]	✓ S ✓ R ✓ S/R ✓ S	(4)
10.2.3	$\hat{N}_1 = \hat{Q}$ [both = 90^0 / *beide* = 90^0] $\hat{P}_2 = \hat{L}$ [∠s opp. = sides] / [∠*e teenoor* = *sye*] $= \hat{P}_4$ $\hat{M}_2 = \hat{R}_1$ [3rd/de ∠] $\therefore$ Δ PNM ||| Δ PQR [∠∠∠]	✓ S ✓S ✓ R ✓R	(4)

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10.2.4	In ΔPLR and/*en* ΔQPR $L\hat{P}R = \hat{Q}$ [both/*beide* $= 90^0$] $\hat{R}_2 = \hat{R}_1$ [proved/*alreeds bewys*] $\hat{L} = \hat{P}_4$ [$3^{rd/de}$ $\angle$] $\Delta PLR \|\|\| \Delta QPR$ [$\angle\angle\angle$] $\therefore \frac{LR}{PR} = \frac{PR}{QR}$ $LR = \frac{30^2}{15}$ $= 60$	✓ SSS ✓ R ✓ ratios / *verhoudings* ✓substitution / *vervanging* ✓LR	(5)
10.2.5	$NM \parallel PR$ [co – int $\angle$s supp OR corresp $\angle$s =] [*ko - binne* $\angle$*e suppl.OF ooreenk.*$\angle$*e* =] $\therefore NM = \frac{1}{2}PR$ [midpoint theorem / middelpunt stelling] $\sin x = \frac{30\sqrt{3}}{15}$ $x = 60^0$	✓ R ✓ R ✓ ratio/*verhouding* ✓ value of *x* / *waarde van x*	(4)
			[26]

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