Province of the
EASTERN CAPE
EDUCATION

NATIONAL SENIOR CERTIFICATE/ *NASIONALE SENIOR SERTIFIKAAT*

GRADE/*GRAAD* 12

JUNE/JUNIE 2019

MATHEMATICS P2/*WISKUNDE V2* MARKING GUIDELINE/*NASIENRIGLYN*

MARKS/*PUNTE*: 150

This marking guideline consists of 17 pages./
Hierdie nasienriglyn bestaan uit 17 bladsye.

QUESTION 1/*VRAAG 1*

1.1	Mean/*Gemiddelde* = 48	✓ 48 (1)
1.2	SD/SA = 22,08 Penalty of 1 mark for incorrect rounding *Penaliseer 1 punt vir verkeerde afronding*	✓✓22,08 (2)
1.3	Girls performed better. The girls' mean percentage is bigger than that of boys and the girls standard deviation is smaller than that of boys *Meisies het beter gevaar.* *Die meisies se gemiddelde persentasie is groter as dié van die seuns en die standaardafwyking is kleiner as dié van die seuns.*	✓Girls / *Meisies* ✓Reason / *Rede* (2)
1.4	51 – 48 = 3 ∴ each boy's percentage must be increased by 3. *∴elke seun se persentasie moet met 3 vermeerder word.*	✓3 (1)
1.5	Boys' standard deviation will remain the same *Die seuns se standaardawyking sal dieselfde bly.*	✓remain the same *dieselfde bly* (1)
		[7]

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QUESTION 2/*VRAAG 2*

2.1

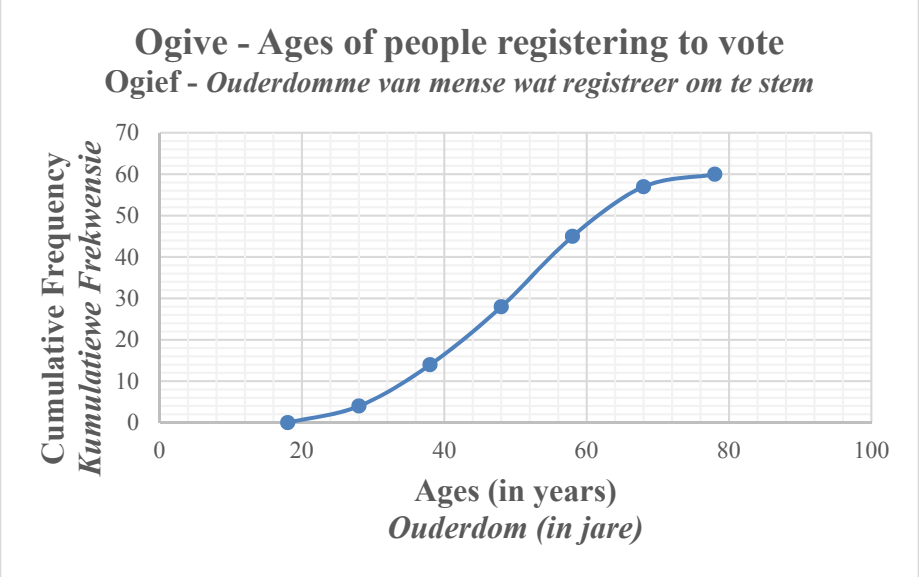
Ages (in years) *Ouderdom (in jare)*	**Frequency** *Frekwensie*	**Cumulative Frequency** *Kumulatiewe Frekwensie*
$18 \le x < 28$	4	4
$28 \le x < 38$	10	14
$38 \le x < 48$	14	28
$48 \le x < 58$	17	45
$58 \le x < 68$	12	57
$68 \le x < 78$	3	60

✓frequency *frekwensie*

✓cumulative frequency *kumulatiewe frekwensie*

(2)

2.2



✓grounding at / *anker by* (0 ; 18)

✓upper limits *boonste limiete*

✓shape / *vorm*

(3)

2.3 $48 \le x < 58$

✓answer / *antwoord* (1)

2.4 60 – 49 = 11 senior citizens / *senior burgers*

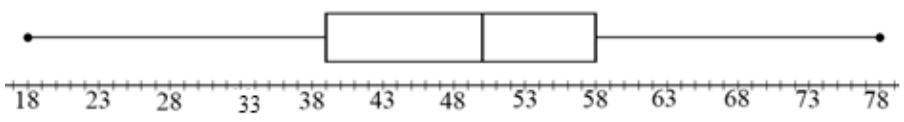
✓49
✓answer / *antwoord* (2)

2.5 $Q_1 = 39$
$Q_2 = 50$
$Q_3 = 58$

✓value of / *waarde van* Q_1
✓value of / *waarde van* Q_2
✓value of / *waarde van* Q_3

(3)

2.6



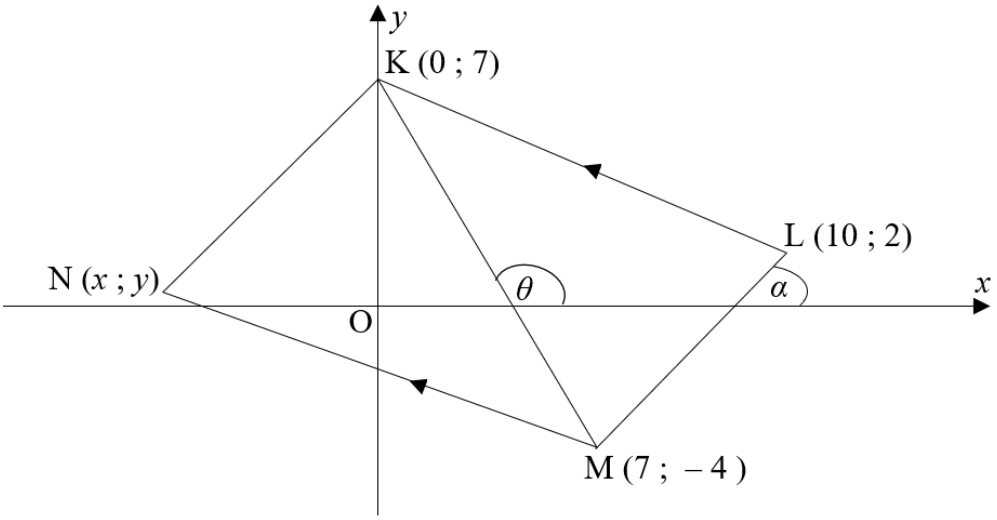
✓minimum and maximum *minimum en maksimum*

✓ box / *boks*

(2)

[13]

QUESTION 3/*VRAAG 3*



3.1.1	$KL = \sqrt{(x_2 - x_1)^2 + (y_2 - y_1)^2}$ $= \sqrt{(10-0)^2 + (2-7)^2}$ $= \sqrt{125} = 5\sqrt{5}$	✓substitution / *vervanging* ✓answer / *antwoord* (2)
3.1.2	$m_{KM} = \frac{y_2 - y_1}{x_2 - x_1}$ $= \frac{-4-7}{7-0}$ $= -\frac{11}{7}$	✓substitution / *vervanging* ✓gradient of KM / *gradiënt van* KM (2)
3.1.3	$m_{LM} = \frac{y_2 - y_1}{x_2 - x_1}$ $= \frac{-4-2}{7-10}$ $= 2$ $\tan\alpha = 2$ $\therefore \alpha = 63,43°$	✓gradient of LM / *gradiënt van* LM ✓ $\tan\alpha = 2$ ✓value of α / *waarde van* α (3)
3.1.4	$\tan\theta = -\frac{11}{7}$ $\text{Ref}\angle = 57,53°$ $\therefore \theta = 122,47°$ $\hat{LMK} = 122,47° - 63,43°$ $= 59,04°$	✓ $\tan\theta = -\frac{11}{7}$ ✓reference angle / *verwysingshoek* ✓value of θ / waarde van θ ✓value of $\hat{LMK}$ / *waarde van* $\hat{LMK}$ (4)

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3.2	$m_{KN} = m_{LM}$ $\frac{y-7}{x-0} = 2$ $y = 2x+7$ $m_{MN} = m_{KL}$ $\frac{y+4}{x-7} = -\frac{1}{2}$ $y = -\frac{x}{2}-\frac{1}{2}$ $2x+7 = -\frac{x}{2}-\frac{1}{2}$ $\therefore y = -\frac{x}{2}+\frac{13}{2}$ $4x+14 = -x-1$ $x = -3$ $y = 1$ OR / *OF* $m_{KL} = \frac{-5}{10}$ $m_{NM} = \frac{-5}{10}$ Hence the coordinates of/ *Vervolgens die koördinate van* N(-3;1) OR/*OF* Midpoint of KM/ *Middelpunt van KM* $= \left(\frac{7}{2};\frac{3}{2}\right)$ Midpoint of LN/ *Middelpunt van LN* $= \left(\frac{7}{2};\frac{3}{2}\right)$ $\therefore \frac{x+10}{2} = \frac{7}{2}$ and/*en* $\frac{y+2}{2} = \frac{3}{2}$ $\therefore$ N$(-3;1)$	✓ $y = 2x+7$ ✓ $y = -\frac{x}{2}-\frac{1}{2}$ ✓value of x / *waarde van x* ✓value of y / *waarde van y* OR/*OF* ✓ $m_{KL} = \frac{-5}{10}$ ✓ $m_{NM} = \frac{-5}{10}$ ✓value of x / *waarde van x* ✓value of y / *waarde van y* OR/*OF* ✓ Midpoint of KM / *Middelpunt van KM* $= \left(\frac{7}{2};\frac{3}{2}\right)$ ✓ Midpoint of LN / *Middelpunt van LN* $= \left(\frac{7}{2};\frac{3}{2}\right)$ ✓value of x / *waarde van x* ✓value of y / *waarde van y* (4)

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3.3	$m_{LM} \times m_{MN} = 2 \times \left(-\frac{1}{2}\right)$ $= -1$ $\therefore \hat{LMN} = 90°$ OR/OF $\hat{LMN}$ is a right angle / *is 'n regtehoek*	✓product of gradients / *produk van gradiënte* ✓conclusion / *gevolgtrekking* (2)
3.4	KL = NM = $5\sqrt{5}$ $KM = \sqrt{7^2 + 11^2}$ $= \sqrt{170}$ $\hat{LMN} = 90°$ and / *en* $\hat{LMK} = 59,04°$ $\therefore \hat{KMN} = 90° - 56,04° = 30,96°$ Area of $\Delta KMN = \frac{1}{2} \times \sqrt{170} \times 5\sqrt{5} \times \sin 30,96°$ $= 37,50$ square units / *vierkante eenhede*	✓ KL = NM = $5\sqrt{5}$ ✓ KM = $\sqrt{170}$ ✓ $\hat{LMN} = 90°$ and/*en* $\hat{LMK} = 59,04°$ ✓ $\hat{KMN} = 30,96°$ ✓ Area of ΔKMN (5)
		[22]

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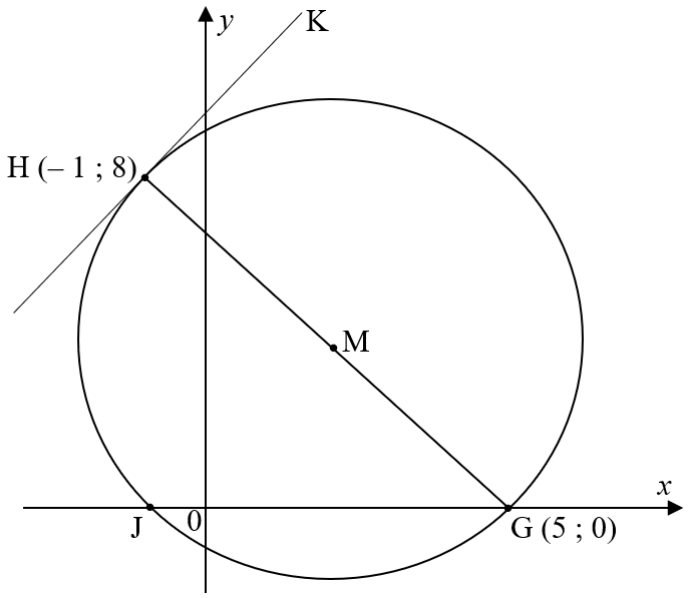
QUESTION 4/*VRAAG 4*



4.1	M (2 ; 4)	✓value of x / *waarde van x* ✓value of y / *waarde van y* (2)
4.2	$r^2 = (5-2)^2 + (0-4)^2$ $= 25$ $\therefore (x-2)^2 + (y-4)^2 = 25$	✓$(x-2)^2$ ✓$(y-4)^2$ ✓25 (3)
4.3	$m_{GH} = \frac{8-0}{-1-5}$ $= -\frac{8}{6} = -\frac{4}{3}$ $m_{\tan} = \frac{3}{4}$ $y - 8 = \frac{3}{4}(x+1)$ $\therefore y = \frac{3}{4}x + \frac{35}{4}$	✓m_{GH} ✓$m_{\tan}$ ✓substitution / *vervanging* ✓equation / *vergelyking* (4)
4.4	At/By J, $y = 0$ $(x-2)^2 + (0-4)^2 = 25$ $(x-2)^2 = 9$ $x-2 = \pm 3$ $x = 5$ or/*of* $x = -1$ $\therefore$ J$(-1;0)$	✓$y = 0$ ✓substitution / *vervanging* ✓$x = -1$ (3)

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4.5	HJG is a right angled triangle (8, 6 and 10). So the rotation of J around M will complete a rectangle Hence, j $((-1+6;0+8)) = J^{|}(5;8)$ *HJG is 'n reghoekige driehoek (8, 6 en 10)* *Dus sal die rotasie van J om M die reghoek voltooi.* *Vervolgens is* $J^{|}((-1+6;0+8)) = J^{|}(5;8)$	✓value of *x / waarde van x* ✓value of *y / waarde van y* (2)
4.6	$x^2 + y^2 - 12x - 2y + 17 = 0$ $x^2 - 12x + y^2 - 2y = -17$ $x^2 - 12x + 36 + y^2 - 2y + 1 = -17 + 36 + 1$ $(x-6)^2 + (y-1)^2 = 20$ Distance between the centres: *Afstand tussen die middelpunte*: $\sqrt{(2-6)^2 + (4-1)^2} = 5$ ∴ the centre lies on the original circle */ die middelpunt lê op die omtrek van die oorspronklike sirkel*	✓completing the square / *voltooiing van die vierkant* ✓factorisation/*faktorisering*: *x* ✓factorisation/*faktorisering*: *y* ✓distance formula / *afstand formule* ✓conclusion / *gevolgtrekking* (5)
		[19]

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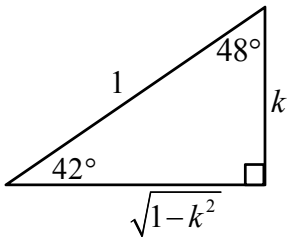
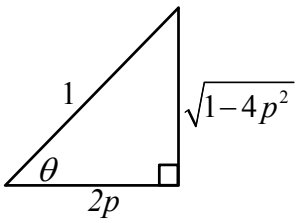
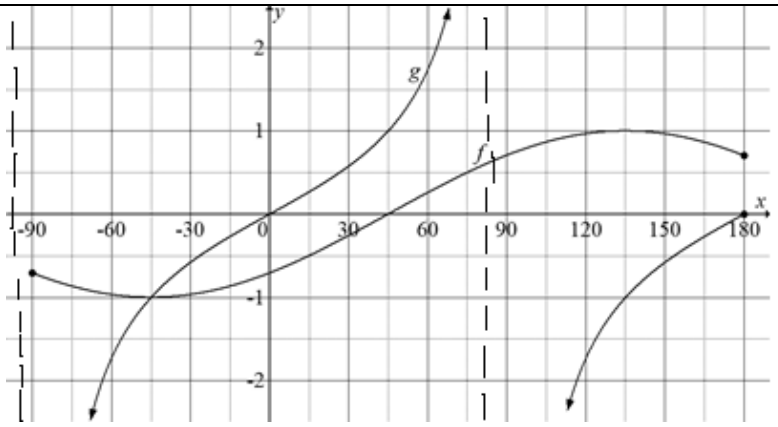
QUESTION 5/*VRAAG 5*

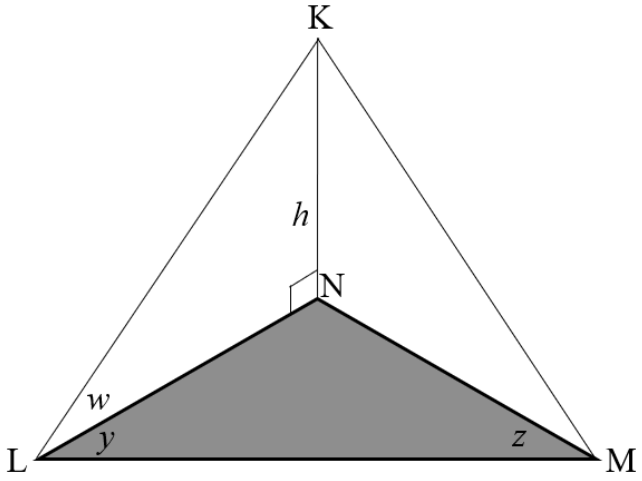
5.1.1	$\sin 42° = \frac{k}{1}$ $\tan 42° = \frac{k}{\sqrt{1-k^2}}$  	✓ $\sqrt{1-k^2}$ ✓ $\tan 42° = \frac{k}{\sqrt{1-k^2}}$ (2)
5.1.2	$\sin 84° = \sin 2 \times 42°$ $= 2\sin 42° \cos 42°$ $= 2.k.\sqrt{1-k^2} \quad = 2k\sqrt{1-k^2}$	✓double angle/*dubbelhoek* ✓expansion/*identiteit/ontwikkeling* ✓substitution/*vervanging* (3)
5.1.3	$\sin 3° = sin(45° - 42°)$ $= sin\,45° cos\,42° - cos\,45° sin\,42°$ $= \frac{\sqrt{2}}{2}.\sqrt{1-k^2} - \frac{\sqrt{2}}{2}.k$	✓$3° = 45° - 42°$ ✓expansion/*identiteit/ontwikkeling* ✓substitution/*vervanging* ✓substitution/*vervanging* (4)
5.2	$\frac{\sin(x-450°).\tan(180°+x).\sin(90°-x)}{\cos(-x)}$ $\frac{-\cos x.\tan x.\cos x}{\cos x}$ $-\cos x.\frac{\sin x}{\cos x}$ $-\sin x$	✓ $-\cos x$ ✓ $\tan x$ ✓ $\cos x$ ✓ $\cos x$ ✓ $\frac{\sin x}{\cos x}$ ✓answer / *antwoord* (6)
5.3.1	$\cos(A+B) = \cos A \cos B - \sin A \sin B$	✓expansion/*identiteit/ontwikkeling* (1)
5.3.2	LHS/LK $= cos\,3\alpha$ $= \cos(2\alpha + \alpha)$ $= \cos 2\alpha \cos\alpha - \sin 2\alpha \sin\alpha$ $= (2\cos^2\alpha - 1).\cos\alpha - 2\sin\alpha\cos\alpha.\sin\alpha$ $= 2\cos^3\alpha - \cos\alpha - 2\sin^2\alpha\cos\alpha$ $= 2\cos^3\alpha - \cos\alpha - 2(1-\cos^2\alpha)\cos\alpha$ $= 2\cos^3\alpha - \cos\alpha - 2\cos\alpha + 2\cos^3\alpha$ $= 4\cos^3\alpha - 3\cos\alpha$ = RHS/*RK*	✓compound angle identity *saamgesteldehoek-identiteit* ✓cos double angle identity cos *dubbelhoek-identiteit* ✓sin double angle identity sin *dubbelhoek-identiteit* ✓$(1-\cos^2\alpha)$ (4)

5.4	$\cos\theta = \frac{2p}{1}$  	✓ $\sqrt{1-4p^2}$
	$\cos 2\theta = \cos^2\theta - \sin^2\theta$	✓ $\cos 2\theta = \cos^2\theta - \sin^2\theta$
	$7p = (2p)^2 - \left(\sqrt{1-4p^2}\right)^2$	✓substitution/*vervanging*
	$7p = 4p^2 - 1 + 4p^2$ $8p^2 - 7p - 1 = 0$ $(8p+1)(p-1) = 0$ $\therefore p = -\frac{1}{8}$ *or / of* $p = 1$	✓standard form *standaardvorm* ✓values of p / *waardes van p*
	OR/*OF*	OR/*OF*
	$\cos 2\theta = 2\cos^2\theta - 1$ $7p = 2.(2p)^2 - 1$ $7p = 8p^2 - 1$ $8p^2 - 7p - 1 = 0$ $(8p+1)(p-1) = 0$ $\therefore p = -\frac{1}{8}$ *or / of* $p = 1$	✓ $2\cos^2\theta - 1$ ✓✓substitution/*vervanging* ✓standard form / *standaardvorm* ✓values of p / *waardes van p*
	OR/*OF*	OR/OF
	$\cos 2\theta = 1 - 2\sin^2\theta$ $7p = 1 - 2.\left(\sqrt{1-4p^2}\right)^2$ $7p = 1 - 2(1-4p^2)$ $7p = 1 - 2 + 8p^2$ $8p^2 - 7p - 1 = 0$ $(8p+1)(p-1) = 0$ $\therefore p = -\frac{1}{8}$ *or / of* $p = 1$	✓ $\sqrt{1-4p^2}$ ✓ $1 - 2\sin^2\theta$ ✓substitution / *vervanging* ✓standard form *standaardvorm* ✓values of p / *waardes van p*
		(5)
		[25]

QUESTION 6/*VRAAG 6*

6.1	$y \in [-1;1]$ OR/*OF* $-1 \le y \le 1$	✓answer / *antwoord* (1)
6.2		*g:* ✓asymptotes at -90° and 90° *asymptote vir* -90° en -90° ✓*x*-intercepts *x-afsnitte* ✓shape / *vorm* (3)
6.3	180°	✓180° (1)
6.4	$x = -45°$	✓$-45°$ (1)
6.5	$x \in [45°; 90°)$ OR/*OF* $45° \le x < 90°$	✓critical values *kritiese waardes* ✓notation / *notasie* (2)
6.6	$h(x) = \sin(x - 45°) + 1$	✓$h(x) = \sin(x - 45°) + 1$ (1)
		[9]

QUESTION 7/*VRAAG 7*



7.1	In ΔKLN : $\tan w = \frac{h}{LN}$ $LN = \frac{h}{\tan w}$	✓ $LN = \frac{h}{\tan w}$ (1)
7.2	In ΔNLM $\frac{LM}{\sin \hat{N}} = \frac{LN}{\sin \hat{M}}$ $\frac{LM}{\sin(180° - (y+z))} = \frac{LN}{\sin z}$ $\therefore LM = \frac{LN.\sin(y+z)}{\sin z}$ But $LN = \frac{h}{\tan w}$ $\therefore LM = \frac{h \sin(y+z)}{\tan w \sin z}$	✓correct sine rule *korrekte sinusreël* ✓substitution *vervanging* ✓isolating LM *isoleer* LM ✓answer / *antwoord* (4)
7.3	$LM = \frac{h \sin(y+z)}{\tan w \sin z}$ and/*en* $h = 38\,\text{m},\ w = 21^\circ,\ y = 52^\circ$ and/*en* $z = 59$ $\therefore\ LM = \frac{38.\sin(52° + 59°)}{\tan 21° \sin 59°}$ $= 107,82\ \text{m}$	✓substitution *vervanging* ✓answer / *antwoord* (2)
		[7]

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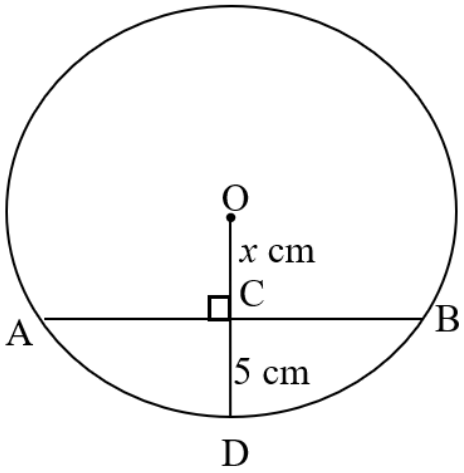
QUESTION 8/*VRAAG 8*



8.1	the centre of a circle / *die middelpunt van die sirkel*	✓answer / *antwoord* (1)
8.2.1	AC = 10 cm (line from centre ⊥ chord) (*lyn vanaf die middelpunt ⊥ op koord*)	✓length of AC *lengte van* AC ✓Reason/*rede* (2)
8.2.2	$(x+5)^2 = 10^2 + x^2$ $x^2 + 10x + 25 = 100 + x^2$ $10x = 75$ $\therefore \quad x = 7,5\,\text{cm}$ $\therefore$ radius $= 7,5\,\text{cm} + 5\,\text{cm}$ $= 12,5\,\text{cm}$	✓radius = $(x + 5)$ ✓applying Pythagoras theorem / *toepassing van Pythagoras se stelling* ✓value of x / *waarde van x* ✓length of radius / *lengte van radius* (4)
		[7]

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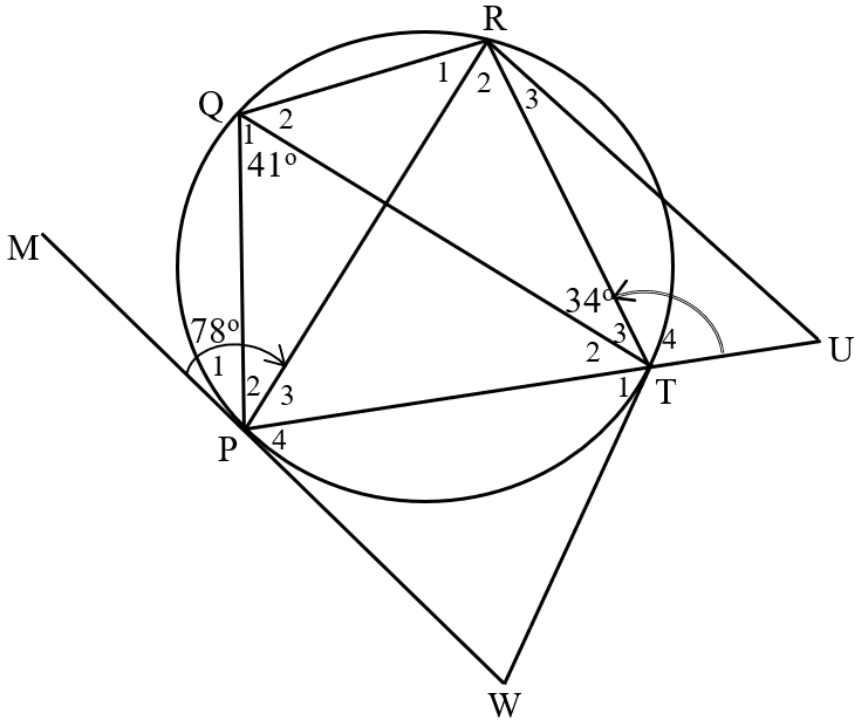
QUESTION 9/*VRAAG 9*



9.1	interior opposite angle / *teenoorstaande binnehoek*	✓answer / *antwoord* (1)
9.2.1	$\hat{R}_2 = \hat{Q}_2 = 41°$ (∠s in the same seg)/(*∠e in dieselfde seg.*) $\hat{P}_4 = \hat{Q}_1 = 41°$ (tan-chord theorem)/(*raaklyn-koord stelling*) $\hat{T}_1 = \hat{P}_4 = 41°$ (∠s opp. = sides)/(*∠e teenoor gelyke sye*) OR/*OF* $\hat{T}_1 = \hat{R}_2 = 41°$ (tan – chord theorem)/(*raaklyn-koord stelling*)	✓Statement / *bewering* ✓Reason / *rede* ✓Statement / *bewering* ✓Reason / *rede* ✓Statement / *bewering* ✓Reason / *rede* (6)
9.2.2(a)	$\hat{T}_2 + 34° = 78°$ (tan – chord theorem)/(*raaklyn - koord stelling*) $\therefore \hat{T}_2 = 44°$	✓Statement / *bewering* ✓Reason / *rede* (2)
9.2.2(b)	$41° + \hat{Q}_2 + 44° + 34° = 180°$ (opp. ∠s of a cyclic quad.) $\therefore \hat{Q}_2 = 61°$ (*teenoorst.∠e van 'n koordevierhoek*)	✓Statement / *bewering* ✓Reason / *rede* (2)
9.2.2(c)	$\hat{T}_4 = 41° + 61°$ (ext. ∠s of a cyclic quad.) $\therefore \hat{T}_4 = 102°$ (*buite ∠ van koordevierhoek*) OR/*OF* $\hat{T}_4 + 44° + 34° = 180°$ (int. ∠s of a Δ) $\therefore \hat{T}_4 = 102°$ (*binne ∠e van 'n Δ*)	✓Statement / *bewering* ✓Reason / *rede* OR/*OF* ✓Statement / *bewering* ✓Reason / *rede* (2)

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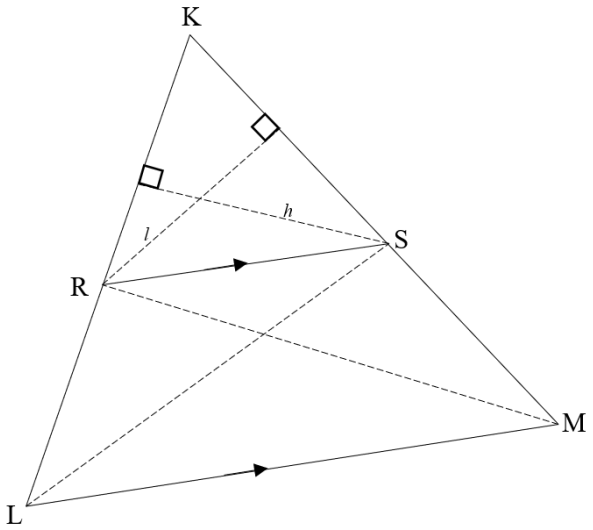
9.2.2(d)	$\hat{W} + 41° + 41° = 180°$ (int . ∠s of a Δ)/*(binne ∠e van Δ*) $\therefore \hat{W} = 98°$	✓Statement /*bewering* ✓ Reason / *rede* (2)
9.2.3(a)	$\hat{Q}_2 = 61°$ and/*en* $\hat{T}_2 = 44°$ $\therefore \hat{Q}_2 \neq \hat{T}_2$ ∴ QR is not parallel to PT (alt. ∠s are not equal) *QR is nie ewewydig aan PT nie* (*verw. ∠e is nie gelyk nie*)	✓$\hat{Q}_2 \neq \hat{T}_2$ ✓ alt. ∠s are not equal *verw. ∠e is nie gelyk nie* (2)
9.2.3(b)	$\hat{R}_2 + \hat{W} = 41° + 98°$ $= 139° \neq 180°$ ∴ PRTW is not a cyclic quad. (Opp. ∠s are not supp.) *PRTW is nie 'n koordevierhoek nie* (*teenoorst. ∠e is nie* supplementêr *nie*)	✓$\hat{R}_2 + \hat{W} \neq 180°$ ✓ PRTW is not a cyclic quad. *PRTW is nie 'n koordevierhoek nie* (2)
9.2.3(c)	$\hat{R}_1 = \hat{T}_2 = 44°$(∠s in same seg)/*(∠e in dieselfde segment*) $\hat{R}_1 + \hat{R}_2 = 44° + 41°$ $= 95° \neq 90°$ ∴ TQ is not a diameter (∠subt. by TQ is not a right angle) *∴TQ is nie 'n middellyn nie (∠onder span deur TQ* is *nie 'n reghoek nie)*	✓$\hat{R}_1 + \hat{R}_2 \neq 90°$ ✓ TQ is not a diameter *TQ is nie 'n middellyn nie* (2)
		[21]

QUESTION 10/*VRAAG 10*

10.1



Construction: Draw heights h and l on KR and KS respectively. Join LS and MR
Konstruksie: Teken hoogtes h en l op KR en KS onderskeidelik. Verbind LS en MR

✓construction / *konstruksie*

Proof/*Bewys*:

$$\frac{\text{Area of } \Delta KRS}{\text{Area of } \Delta LRS} = \frac{\frac{1}{2}.KR.h}{\frac{1}{2}.RL.h} = \frac{KR}{RL}$$

✓ratio of areas / *verhouding van oppervlaktes*

$$\frac{\text{Area of } \Delta KRS}{\text{Area of } \Delta MSR} = \frac{\frac{1}{2}.KS.l}{\frac{1}{2}.SM.l} = \frac{KS}{SM}$$

✓ratio of areas *verhouding van oppervlaktes*

But/*Maar*: area of/*van* ΔLRS = area of/*van* ΔMSR

✓same base and same height *dieselfde basis en hoogte*

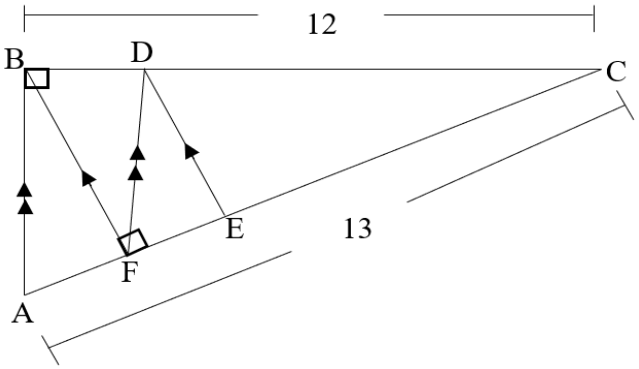
$$\frac{\text{Area of } \Delta KRS}{\text{Area of } \Delta LRS} = \frac{\text{Area of } \Delta KRS}{\text{Area of } \Delta MSR}$$

✓ $\frac{\text{Area of } \Delta KRS}{\text{Area of } \Delta LRS} = \frac{\text{Area of } \Delta KRS}{\text{Area of } \Delta MSR}$

$$\therefore \frac{KR}{RL} = \frac{KS}{SM}$$

(5)

10.2



10.2.1	AB = 5 units	✓5 (1)
10.2.2(a)	$\hat{C}$ is common / *is gemeen* $C\hat{B}A = C\hat{F}B$ (both/*beide* = 90°) $C\hat{A}B = C\hat{B}F$ (sum of ∠s of Δ)/(*som van die ∠e van* Δ) ∴ ΔCBA ||| ΔCFB (∠,∠,∠)	✓Statement/*bewering*/ Reason / *rede* ✓Statement/*bewering*/ Reason / *rede* ✓Reason/*Rede* (3)
10.2.2(b)	$\frac{CB}{CF} = \frac{CA}{CB}$ (|||s) $CB^2 = CF.CA$ $\therefore CF = \frac{CB^2}{CA}$	✓proportion / *verhouding* ✓reason / *rede* ✓ $CB^2 = CF.CA$ (3)
10.2.3	$CF = \frac{CB^2}{CA}$ $CF = \frac{(12)^2}{13}$ ≈ 11 units/*eenhede*	✓substitution / *vervanging* ✓length of CF / *lengte van* CF (2)
10.2.4	AF = 13 – 11 = 2 units/*eenhede*	✓length of AF / *lengte van* AF (1)
10.2.5	$\frac{CB}{BD} = \frac{CA}{AF}$ (prop. theorem/*verh. stelling*; DF || BA) $\frac{12}{BD} = \frac{13}{2}$ $\therefore BD = \frac{24}{13}$ $\frac{CF}{FE} = \frac{CB}{BD}$ (prop. theorem/*verh. stelling*; DF || BA) $\frac{11}{FE} = \frac{12}{\frac{24}{13}}$ $\therefore FE = \frac{22}{13}$ units/*eenhede*	✓proportion / *verhouding* ✓reason / *rede* ✓length of BD / *lengte van* BD ✓proportion / *verhouding* ✓length of FE / *lengte van* FE (5)
		[20]
		TOTAL/*TOTAAL*: 150

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