



Province of the
EASTERN CAPE
EDUCATION

NATIONAL SENIOR
CERTIFICATE/
*NASIONALE SENIOR
SERTIFIKAAT*

GRADE/GRAAD 11

NOVEMBER 2019

**TECHNICAL MATHEMATICS P2/TEGNIESE WISKUNDE V2
MARKING GUIDELINE/NASIENRIGLYN**

MARKS/PUNTE: 150

This marking guideline consists of 17 pages./*Hierdie nasienriglyn bestaan uit 17 bladsye.*

NOTE:

- If a learner makes more than one attempt at answering a question and does not cancel any of them out, only the first attempt will be marked irrespective of which of the attempt(s) may be the correct answer.
- Consistent accurate marking regarding calculations will be followed in the following cases:
 - Subquestion to subquestion: When a certain variable is incorrectly calculated in one subquestion and needs to be substituted into another subquestion full marks can be awarded for the subsequent subquestions provided the methods used are correct and the calculations are correct.
 - Assuming values/answers to solve a problem is unacceptable.

LET WEL:

- *Indien 'n leerder meer as eenkeer 'n poging aanwend om 'n vraag te beantwoord sonder om enige van die pogings te kanselleer, dan word slegs die eerste poging nasien, ongeag watter poging korrek mag wees.*
- *Deurlopende akkuraatheid nasien ten opsigte van berekening word toegepas op die volgende voorvalle:*
 - *Sub-vraag tot sub-vraag: Wanneer 'n sekere veranderlike verkeerd bereken was in een sub-vraag en benodig word in die volgende sub-vraag, dan kan volpunte toegeken word in die daaropvolgende sub-vraag opvoorwaarde dat die metode en die berekeninge korrek is.*
 - *Aanvaarding van waardes/antwoorde om 'n probleem op te los is onaanvaarbaar.*

Symbol/simbool	Explanation/Verduideliking
M	Method / metode
MA	Method with accuracy / metode met akkuraatheid
A	Accuracy / akkuraatheid
AO	Answer only / Slegs Antwoord
CA	Consistent accuracy / Deurlopende akkuraatheid
NPU	No penalty for units / Geen penalisering vir eenhede weggelaat
NPR	No penalty for rounding / Geen penalisering vir afronding
SI	Simplification / Vereenvoudiging
ST	Statement / Bewering
R	Rounding / afronding
RE	Reason / rede
SR	Statement AND correct reason / Bewering EN korrekte rede
SF	Substitution into correct formula / Vervanging in korrekte formule
ID	Identity / Identiteit

QUESTION/VRAAG 1		
1.1	$\begin{aligned} D &= \left(\frac{x_1 + x_2}{2}, \frac{y_1 + y_2}{2} \right) \\ &= \left(\frac{6+8}{2}; \frac{9-1}{2} \right) \\ &= (7;4) \end{aligned}$	✓ x-coordinate / koördinaat ✓ y-coordinate / koördinaat AO A (2)
1.2	$\begin{aligned} m_{FD} &= m_{CD} = \frac{y_2 - y_1}{x_2 - x_1} \\ &= \frac{4-3}{7-4} \\ &= \frac{1}{3} \end{aligned}$ <p>Equation of FD: / Vergelyking van FD:</p> $y - y_1 = m(x - x_1)$ $y - 3 = \frac{1}{3}(x - 4)$ $y = \frac{1}{3}x - \frac{4}{3} + 3$ $= \frac{1}{3}x + \frac{5}{3}$ <p>OR / OF</p> $\begin{aligned} m_{FD} &= m_{CD} = \frac{y_2 - y_1}{x_2 - x_1} \\ &= \frac{4-3}{7-4} \\ &= \frac{1}{3} \end{aligned}$	✓ m_{FD} ✓ value / waarde ✓ substitute grad / vervang grad & (4; 3) or/of (7; 4) ✓ simplification / vereenvoudiging ✓ equation / vergelyking SI CA OR/OF SF CA (5)

	$y = mx + c$ $3 = \frac{1}{3}(4) + c$ $c = \frac{5}{3}$ $y = \frac{1}{3}x + \frac{5}{3}$	✓ substitute grad / vervang grad en (4; 3) or/of (7; 4) ✓ simplification / vereenvoudiging ✓ equation / vergelyking	SF A SI CA CA	
1.3	$m_{BE} = \frac{y_2 - y_1}{x_2 - x_1}$ $= \frac{3+1}{4-8}$ $= -1$	✓ m_{BE} ✓ value / waarde	SF A CA	(2)
1.4	$m_{AF} = 1$ (product of gradients = -1) Equation of AF: $y - y_1 = m(x - x_1)$ $y - 9 = 1(x - 6)$ $y = x - 6 + 9$ $= x + 3$	✓ m_{AF} ✓ substitute grad vervang grad en (6; 9) ✓ equation / vergelyking	CA SF CA CA	(3)
1.5	From 1.3 & 1.4 $x + 3 = \frac{1}{3}x + \frac{5}{3}$ $3x + 9 = x + 5$ $2x = -4$ $x = -2$ $y = 1$ $\therefore F(-2; 1)$	✓ equating equations / gelykstel van vergelykings ✓ Simplification / vereenvoudiging ✓ x-value / waarde ✓ y-value / waarde	M CA CA CA	(4)
1.6	inclination of BE = $\tan^{-1}(-1) = 135^\circ$ $m_{AB} = \frac{10}{-2} = -5$ inclination of AB = $\tan^{-1}(-5)$ $= 180 - 78,69^\circ$ $= 101,3^\circ$ $A\hat{B}E = 135^\circ - 101,3^\circ = 33,7^\circ$	✓ $m = \tan \theta$ ✓ 135° ✓ $m_{AB} = -5$ ✓ $78,69^\circ$ ✓ $101,3^\circ$ ✓ $33,7^\circ$	M CA A CA CA CA	(6)
				[22]

QUESTION/VRAAG 2			
2.1.1	$3 \cot \theta = -2$ $\cot \theta = \frac{-2}{3}$ $r = \sqrt{2^2 + (-3)^2}$ $= \sqrt{13}$ $\cos(180^\circ + \theta) = -\cos \theta$ $= -\left(\frac{2}{\sqrt{13}}\right)$ $= -\frac{2}{\sqrt{13}}$	✓ $\cot \theta = \frac{-2}{3}$ ✓ $r = \sqrt{13}$ ✓ Sketch correct quadrant / skets korrekte kwadrant ✓ $-\cos \theta$ ✓ $-\frac{2}{\sqrt{13}}$	A CA A A CA (5)
2.1.2	$9 \operatorname{cosec}^2 \theta + 4 \sec^2 \theta$ $= 9 \left(\frac{\sqrt{13}}{-3} \right)^2 + 4 \left(\frac{\sqrt{13}}{2} \right)^2$ $= 9 \left(\frac{13}{9} \right) + 4 \left(\frac{13}{4} \right)$ $= 26$	✓ $\frac{\sqrt{13}}{-3}$ ✓ $\frac{\sqrt{13}}{2}$ ✓ Simplification / Vereenvoudiging ✓ Value / waarde	A A SI CA CA (4)
2.2	$\frac{\cos(180^\circ + \theta) \cdot \tan(360^\circ - \theta) \cdot \cos^2(360^\circ + \theta)}{\sin(180^\circ - \theta)} + \sin^2 \theta$ $= \frac{(-\cos \theta)(-\tan \theta)(\cos \theta)^2}{\sin \theta} + \sin^2 \theta$ $= \frac{-\cos \theta}{\sin \theta} \times \frac{-\sin \theta}{\cos \theta} \times \cos^2 \theta + \sin^2 \theta$ $= \cos^2 \theta + \sin^2 \theta$ $= 1$	✓ $-\cos \theta$ ✓ $-\tan \theta$ ✓ $\cos \theta$ ✓ $\sin \theta$ ✓ $\frac{-\sin \theta}{\cos \theta}$ ✓ 1	ID A ID A ID A ID A ID A ID A (6)
2.3.1	$\cos 50^\circ = \sqrt{1 - \sin^2 50^\circ}$ $= \sqrt{1 - t^2}$	✓ $\sqrt{1 - \sin^2 50^\circ}$ ✓ $\sqrt{1 - t^2}$	ID A CA (2)

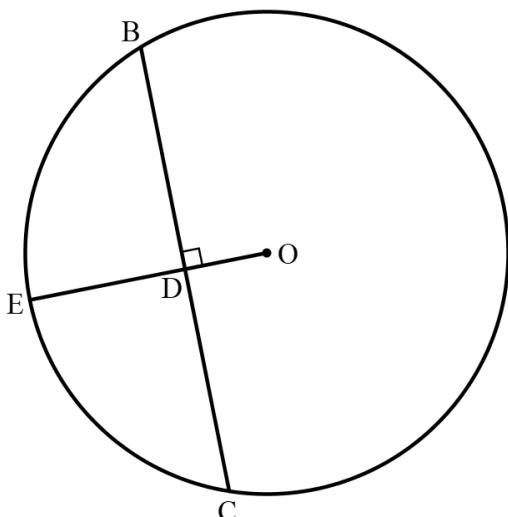
2.3.2	$\begin{aligned} \tan 230^\circ &= \tan 50^\circ \\ &= \frac{\sin 50^\circ}{\cos 50^\circ} \\ &= \frac{t}{\sqrt{1-t^2}} \end{aligned}$ <p style="text-align: center;">OR / OF</p> $\begin{aligned} \tan 230^\circ &= \tan 50^\circ \\ &= \sqrt{\sec^2 50^\circ - 1} \\ &= -\sqrt{\frac{1}{\cos^2 50^\circ} - 1} \\ &= \sqrt{\frac{1}{1-t^2} - 1} \\ &= \sqrt{\frac{1-1+t^2}{1-t^2}} \\ &= \frac{t}{\sqrt{1-t^2}} \end{aligned}$	$\begin{aligned} &\checkmark \tan 50^\circ \\ &\checkmark \frac{\sin 50^\circ}{\cos 50^\circ} \\ &\checkmark \frac{t}{\sqrt{1-t^2}} \end{aligned}$ <p style="text-align: center;">OR / OF</p> $\begin{aligned} &\checkmark \tan 50^\circ \\ &\checkmark \sqrt{\sec^2 50^\circ - 1} \\ &\checkmark \frac{t}{\sqrt{1-t^2}} \end{aligned}$	ID A ID A SI OR / OF ID A ID A SI	
2.4	$\begin{aligned} 2 \tan(x+10^\circ) &= -3,46 \\ \tan(x+10^\circ) &= -1,73 \\ \text{Ref } \angle &= \tan^{-1} 1,73 = 59,97^\circ \\ x+10^\circ &= 180^\circ - 59,97^\circ \text{ or } 360^\circ - 59,97^\circ \\ x &= 110,03^\circ \text{ or } 290,03^\circ \end{aligned}$	$\begin{aligned} &\checkmark \text{RHS} = -1,73 \\ &\checkmark \text{Ref } \angle = 59,97^\circ \\ &\checkmark 180^\circ - \\ &\checkmark 360^\circ - \\ &\checkmark \text{both answers / beide antwoorde} \end{aligned}$	SI A CA CA CA CA	(5)

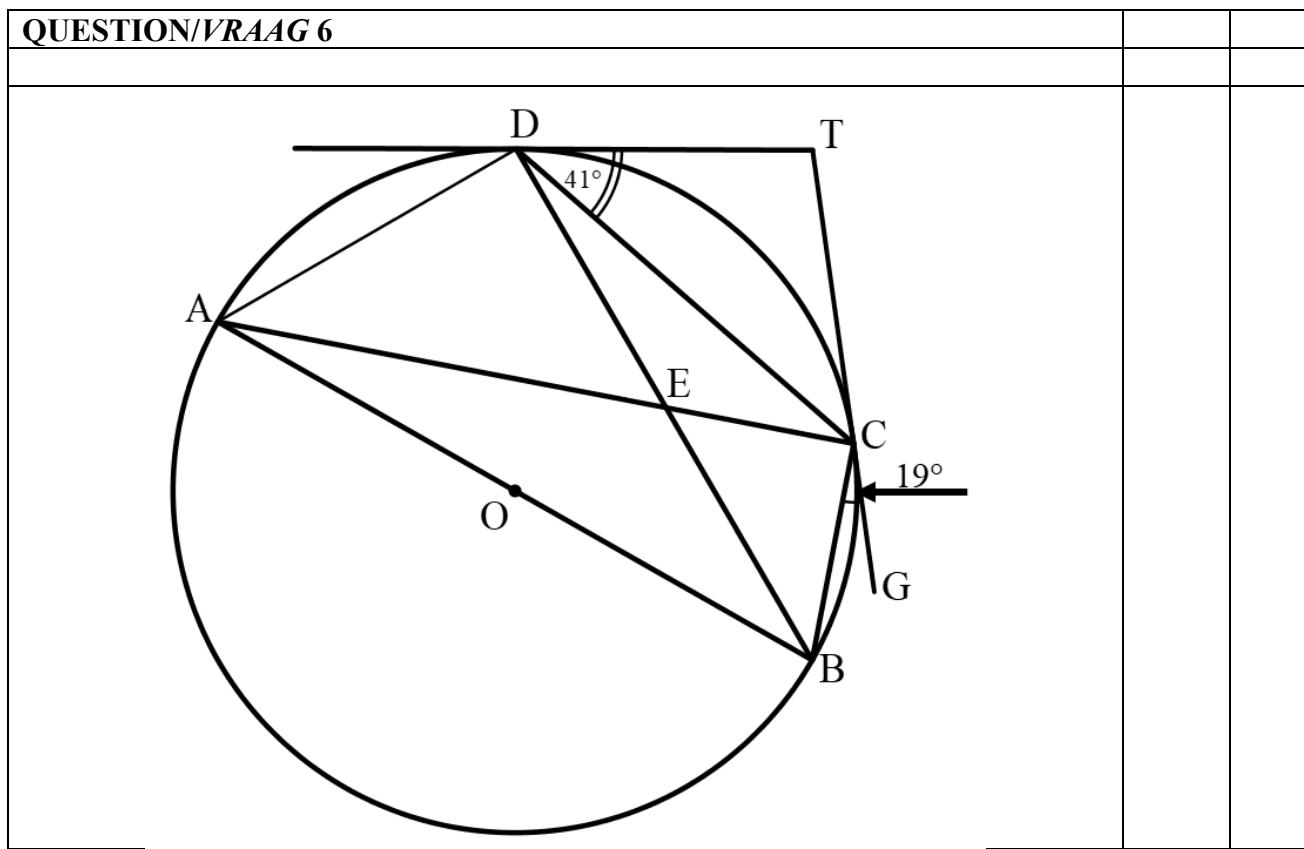
QUESTION/VRAAG 3

3.1	<p>$f(x) = 2 \cos(x^\circ)$</p> <p>$g(x) = \sin(x^\circ + 30^\circ)$</p>	<p>f:</p> <ul style="list-style-type: none"> ✓ x-intercepts / x-afsnitte ✓ y-intercept / y-afsnit ✓ TP / DP <p>g:</p> <ul style="list-style-type: none"> ✓ x-intercepts / x-afsnitte ✓ y-intercept / y-afsnit ✓ TP / DP 	A A A	(6)
3.2	$x = 60^\circ$ or 240°	✓ 60° ✓ 240°	CA CA	(2)
3.3	$0^\circ \leq x \leq 90^\circ$ OR $150^\circ \leq x \leq 270^\circ$	Interval 1: ✓ End points ✓ Notation Interval 2: ✓ End points ✓ Notation	CA CA CA CA	(4)
3.4	$0^\circ < x < 60^\circ$ OR $240^\circ < x < 360^\circ$	Interval 1: ✓ End points ✓ Notation Interval 2: ✓ End points ✓ Notation	CA CA CA CA	(4)
				[16]

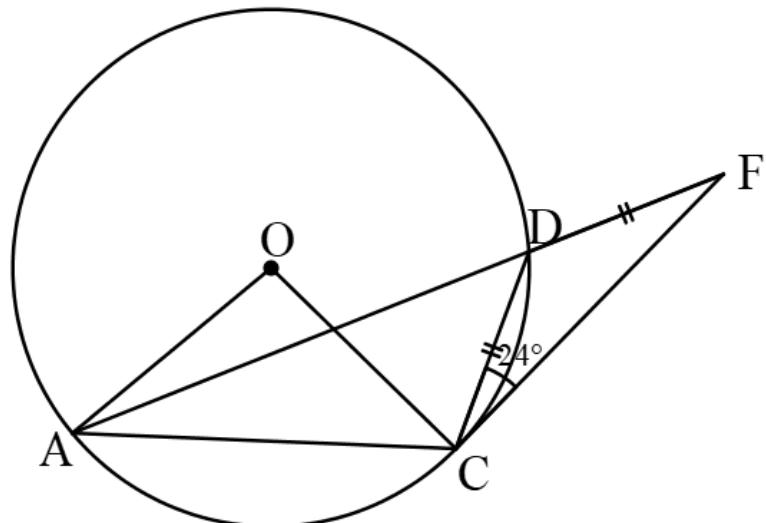
QUESTION/VRAAG 4			
4.1.1	$\frac{BC}{\sin B\hat{A}C} = \frac{AC}{\sin B\hat{B}}$ $BC = \frac{7,07 \sin 75,96^\circ}{\sin 59,04^\circ}$ $= 8 \text{ cm}$	✓ Sin-rule/reël ✓ Substitution / vervanging ✓ Value of BC / waarde van BC NPU	M SF CA (3)
4.1.2	$\text{Area } \Delta ABC = \frac{1}{2} AB \times AC \sin B\hat{A}C$ $= \frac{1}{2}(5,83)(7,07) \sin 75,96^\circ$ $= 19,993 \dots \text{ cm}^2$ $\text{Area } ABCD = 2 \times \text{Area } \Delta ABC$ $= 2 \times 19,993 \dots$ $= 39,99 \text{ cm}^2$	✓ Area rule/reël ✓ Substitusie / vervanging ✓ Value / waarde ✓ Method ✓ Value / waarde NPU	M SF CA M CA (5)
4.2	$PT^2 = PC^2 + TC^2 - 2PC \cdot TC \cos \theta$ $\cos \theta = \frac{PC^2 + TC^2 - PT^2}{2PC \cdot TC}$ $= \frac{1540^2 + 1829^2 - 738^2}{2 \times 1540 \times 1829}$ $= 0,918 \dots$ $\theta \approx 23,3^\circ$	✓ cos-rule/reël ✓ Substitution / vervanging ✓ 0,918... ✓ Value / waarde CA	M A CA CA (4)
			[12]

QUESTION/VRAAG 5		
5.1.1	$A\hat{E}G = 58^\circ$ (alt \angle s/verw \angle e; AB \square)	✓SR
5.1.2	$E\hat{A}D = 58^\circ$ (AE bisect/halveer $B\hat{A}D$) $D\hat{G}H = 116^\circ$ (ext \angle of cyclic quad BADG) <i>(buite \angle van kdvk BADG)</i>	✓ST ✓ST ✓RE
5.1.3	$D\hat{G}I = 58^\circ$ (ext \angle of cyclic quad AEGD) <i>(buite \angle van kdvk AEGD)</i> $B\hat{G}E = B\hat{A}E = 58^\circ$ (\angle s in same segm) <i>(\anglee in dies segm)</i> $I\hat{G}H = B\hat{G}E = 58^\circ$ (vert opp \angle s) <i>(regoorst \anglee)</i> $D\hat{G}I = I\hat{G}H$ OR / OF $I\hat{G}H = B\hat{G}E$ (vert opp \angle s) <i>(regoorst \anglee)</i> $B\hat{G}E = B\hat{A}E = 58^\circ$ (\angle s in same segm) <i>(\anglee in dies segm)</i> $\therefore I\hat{G}H = 58^\circ$ but $D\hat{G}H = 116^\circ$ (prove in/bewys in 5.1.2) $\therefore D\hat{G}I = 116^\circ - I\hat{G}H = 58^\circ$ $\therefore D\hat{G}I = I\hat{G}H$	✓SR ✓ST ✓RE ✓ST OR / OF ✓SR ✓ST ✓RE ✓ST

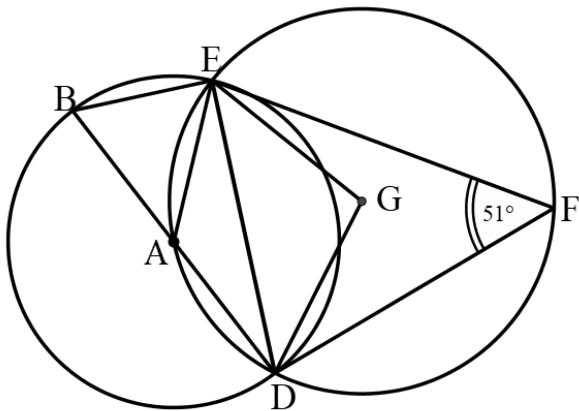
5.1.4	From 5.1.2 and 5.1.3 $A\hat{E}G = 58^\circ = D\hat{G}I$ $\therefore AE \parallel DG$ (equal corresponding \angle s) <i>(gelyke ooreenkomsige \anglee)</i>	✓ST ✓RE		(2)
5.1.5	$A\hat{D}G = 122^\circ$ (opp \angle s of cyclic quad) <i>(teenoorst \anglee van kdvk)</i>	✓ST ✓RE		(2)
5.2	 <p>Let $OD = x$ cm $\therefore ED = 2x$ cm $BO = EO = 3x$ = radius $BD = 6$ cm (line from centre \perp chord) <i>(loodlyn uit midpt \perp na koord)</i> $OB^2 = BD^2 + OD^2$ (Pyth) $(3x)^2 = 6^2 + x^2$ $9x^2 = 36 + x^2$ $8x^2 = 36$ $x^2 = 4,5$ $x = 2,12$ $BO = 6,36$ cm</p>	✓BD = 6 ✓RE ✓Pyth SR ✓set up equation /opstel van vergelyking ✓ SI ✓ value / waarde CA NPU	(6)	[18]



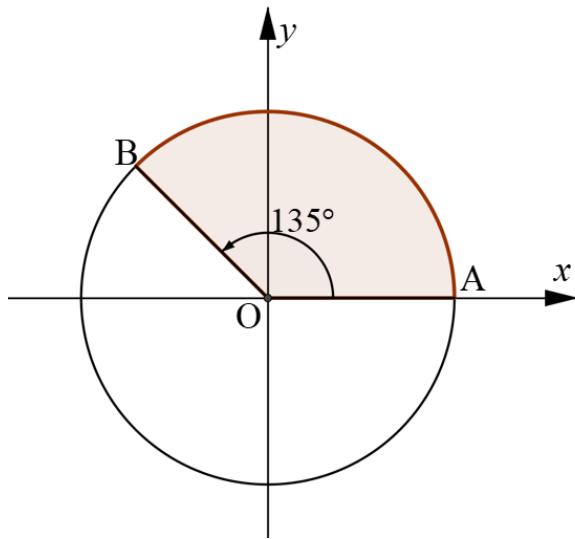
6.1.1	$\hat{D}AC = 41^\circ$ (tan-chord th) <i>(raaklyn-koord st)</i> $\hat{D}BC = 41^\circ$ (tan-chord th) or (\angle s in same seg) <i>(raaklyn-koord st) of (\angle in dieselfde seg)</i> $\hat{T}CD = 41^\circ$ (tan-chord th) <i>(raaklyn-koord st)</i>	✓ST ✓RE ✓ST ✓RE ✓SR	(5)
6.1.2	$\hat{A}DB = 90^\circ$ (\angle in semi-circle) <i>(\angle in semi-sirkel)</i> $\hat{C}AB = 19^\circ$ (tan-chord th) <i>(raaklyn-koord st)</i> $\hat{A}BE = 30^\circ$ (Int \angle s of Δ) <i>(Binne \anglee van Δ)</i>	✓ST ✓RE ✓SR ✓SR	(4)

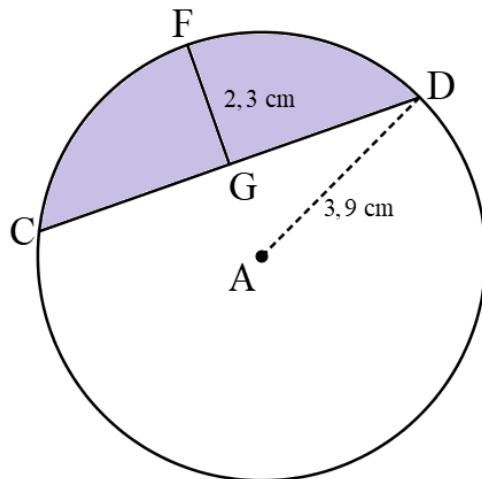


6.2.1	$\hat{F} = 24^\circ$ (\angle s opp = sides) $(\angle e \text{ teenoor} = \text{sye})$ $\hat{ADC} = 48^\circ$ (ext \angle of Δ) $(\text{buite } \angle \text{ van } \Delta)$ $\hat{AOC} = 96^\circ$ (\angle at centre = $2 \times \angle$ at circum) $(\text{middelpuntshoek} = 2 \times \text{omtrekshoek})$	✓SR ✓SR ✓ST ✓RE (4)
6.2.2	$\hat{DAC} = 24^\circ$ (tan-chord) (raaklyn - koord) $AC = CF$ (sides opp = \angle s) $(\text{sye teenoor} = \text{hoeke})$	✓SR ✓RE (2)



6.3.1	$\hat{D}AE = 129^\circ$ (opp \angle s of cyclic quad) $(teenoorste \angle e van kdvh)$ $\hat{B} = \hat{B}EA$ ($AB = AE$ radii; \angle s opp = sides) $(AB = AE$ strale; $\angle e$ teenoor = sye) $\hat{B} = \frac{129^\circ}{2}$ (ext \angle of Δ) $(buite \angle van \Delta)$ $\hat{B} = 64,5^\circ$ OR / OF $\hat{D}AE = 129^\circ$ (opp \angle s of cyclic quad) $(teenoorste \angle e van kdvh)$ In ΔAED : $\hat{A}DE = \hat{A}ED$ ($AD = AE$ radii; \angle s opp = sides) $(AD = AE$ strale; $\angle e$ teenoor = sye) $\hat{A}DE = 25,5^\circ$ (Int \angle s of Δ) $(Binne \angle e van \Delta)$ $\hat{B}ED = 90^\circ$ (\angle in semi-circle) $(\angle in halwe-sirkel)$ $\therefore \hat{B} = 64,5^\circ$ (Int \angle s of Δ) $(Binne \angle e van \Delta)$	✓ST ✓RE ✓SR ✓SR ✓ST OR / OF ✓ST ✓RE	
6.3.2	$\hat{G} = 102^\circ$ (\angle at centre = $2 \times \angle$ at circum) $(middelpunts\angle = 2 \times omtreks\angle)$ $\hat{B} + \hat{G} = 64,5^\circ + 102^\circ = 166,5^\circ$ $\therefore EBDG$ not cyclic (opp \angle s NOT suppl) $\therefore EBDG$ nie-siklies (teen $\angle e$ NIE sup pl)	✓ST ✓RE ✓RE	(5) (3) [23]

QUESTION/VRAAG 7			
			
7.1.1	$135^\circ = 135^\circ \times \frac{\pi}{180^\circ}$ $= \frac{3\pi}{4}$	✓ $\times \frac{\pi}{180^\circ}$ ✓ $\frac{3\pi}{4}$	A A (2)
7.1.2	Area of sector $= \frac{r^2\theta}{2}$ $r^2 \left(\frac{3\pi}{4} \right)$ $18,85 = \frac{r^2}{2}$ $r^2 = 16,0003\dots$ $r = 4 \text{ cm}$	✓ area formula / formule ✓ substitution/ vervanging ✓ simplification / vereenvoudiging ✓ value / waarde ✓ R	M SF SI CA CA (5)
7.1.3	$x^2 + y^2 = 16$	✓ equation / vergelyking	A (1)
7.1.4	$s = r\theta$ $= 4 \times \frac{3\pi}{4}$ $= 3\pi \text{ cm}$	✓ formula / formule ✓ substitution / vervanging ✓ value / waarde	A SF CA CA (3)



7.2	$4h^2 - 4dh + x^2 = 0$ $4(2,3)^2 - 4(7,8)(2,3) + x^2 = 0$ $x^2 - 50,6 = 0$ $x \approx 7,1 \text{ cm}$	✓ formula / <i>formule</i> ✓ substitution / <i>vervanging</i> ✓ simplification / <i>vereenvoudiging</i> ✓ value / <i>waarde</i>	A SF A SI CA NPR	(4)
				[15]

QUESTION/VRAAG 8																		
8.1.1	<table border="1" style="display: inline-table; vertical-align: middle;"> <tr> <td>x</td><td>3,5</td><td>4,5</td><td>5,5</td><td>6,5</td><td>7,5</td></tr> <tr> <td>$f(x)$</td><td>12,25</td><td>20,25</td><td>30,25</td><td>42,25</td><td>56,25</td></tr> </table> $\checkmark \checkmark$ Substitution / vervanging	x	3,5	4,5	5,5	6,5	7,5	$f(x)$	12,25	20,25	30,25	42,25	56,25		A A	(2)		
x	3,5	4,5	5,5	6,5	7,5													
$f(x)$	12,25	20,25	30,25	42,25	56,25													
8.1.2	$A_T = a \left(\frac{o_1 + o_n}{2} + o_2 + o_3 + o_4 + \dots + o_{n-1} \right)$ $\text{Area} = 1 \left(\frac{12,25 + 56,25}{2} + 20,25 + 30,25 + 42,25 \right)$ $= 127 \text{ sq units}$ <p>OR / OF</p> $A_T = a(m_1 + m_2 + m_3 + \dots + m_n)$ $\text{Area} = 1 \left(\frac{12,25 + 20,25}{2} + \frac{20,25 + 30,25}{2} + \frac{30,25 + 42,25}{2} + \frac{42,25 + 56,25}{2} \right)$ $= 127 \text{ sq units}$	\checkmark formula / formule \checkmark $a = 1$ \checkmark Substitution / vervanging \checkmark value / waarde OR / OF \checkmark formula / formule \checkmark $a = 1$ \checkmark substitution / vervanging \checkmark value / waarde	A SF CA OR / OF A SF CA NPU															
8.1.3	Error difference/ fout verskil = $161,7 - 127 = 34,7$ sq units	\checkmark	A NPU	(1)														

8.2.1	<p>Angular velocity = $\omega = 2\pi n = 360^0 n$</p> $n = 42 \text{ rpm} = \frac{42r}{1m} \times \frac{1m}{60s} = 0,7 \text{ rps}$ $\begin{aligned}\omega &= 2\pi n \\ &= 2\pi(0,7) \\ &= 1,4\pi \text{ rad/s}\end{aligned}$	<ul style="list-style-type: none"> ✓ $n = 0,7 \text{ rps}$ ✓ formula/ <i>formule</i> ✓ substitution / <i>vervanging</i> ✓ value / <i>waarde</i> 	A A SF CA NPU	(4)
8.2.2	$38,5 \text{ cm} = \frac{38,5 \text{ cm}}{1} \times \frac{1 \text{ m}}{100 \text{ cm}} = 0,385 \text{ m}$ <p>Circumferential velocity = $v = \pi Dn$</p> $\begin{aligned}&= \pi(0,385)(0,7) \\ &= 0,2695\pi \text{ m/s}\end{aligned}$	<ul style="list-style-type: none"> ✓ conversion / <i>herlei</i> ✓ formula/ <i>formule</i> ✓ substitution / <i>vervanging</i> ✓ value / <i>waarde</i> 	A A SF A NPU	(4)
8.3	<p>Volume of prism = lwh</p> $\begin{aligned}&= 43 \times 22 \times 8 \\ &= 7568 \text{ cm}^3\end{aligned}$ <p>Volume of one sphere = $\frac{4}{3}\pi r^3$</p> $\begin{aligned}&= \frac{4}{3}\pi \left(\frac{3}{2}\right)^3 \\ &= \frac{9}{2}\pi \text{ cm}^3\end{aligned}$ <p>Tot spheres casted = $\frac{7568}{\frac{9}{2}\pi}$</p> ≈ 535	<ul style="list-style-type: none"> ✓ vol prism ✓ vol ONE sphere / <i>vol EEN sfeer</i> ✓ $\frac{9}{2}\pi$ ✓ value/ <i>waarde</i> 	A A CA NPU	(4)
				[19]
			TOTAL/TOTAAL:	150