Province of the
EASTERN CAPE
EDUCATION

NATIONAL SENIOR CERTIFICATE

GRADE 12

SEPTEMBER 2019

TECHNICAL MATHEMATICS P2

MARKS: **150**

TIME: **3 hours**

TMATE2

This question paper consists of 16 pages, including 1 information sheet and a special answer book.

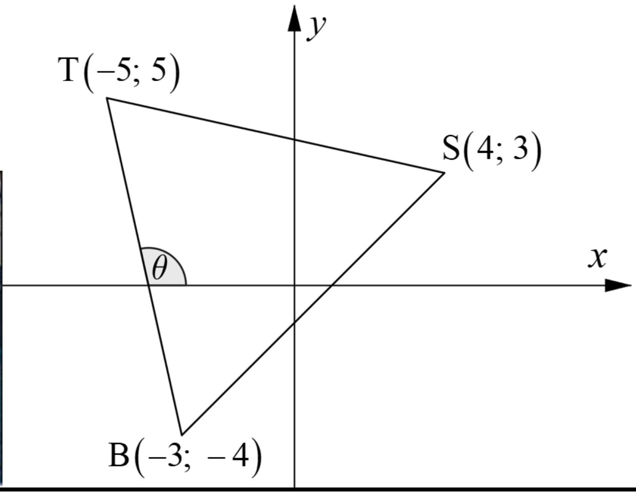
INSTRUCTIONS AND INFORMATION

Read the following instructions carefully before answering the questions.

1. This question paper consists of 11 questions.
2. Answer ALL the questions in the SPECIAL ANSWER BOOK provided.
3. Clearly show ALL calculations, diagrams, graphs, et cetera which you have used in determining the answers.
4. Answers only will NOT necessarily be awarded full marks.
5. You may use an approved scientific calculator (non-programmable and non-graphical) unless stated otherwise.
6. If necessary, round off your answers to TWO decimal places, unless stated otherwise.
7. Diagrams are NOT necessarily drawn to scale.
8. An information sheet with formulae is included at the end of the question paper.
9. Write neatly and legibly.

QUESTION 1

The picture below is of two tugboats pulling a ship into the harbour.
The diagram next to it is a representation of the situation, where S represents the ship and T and B, the tugboats.



1.1 Calculate the length of ST, rounded off to TWO decimal places. (3)

1.2 Calculate the gradient of BT. (2)

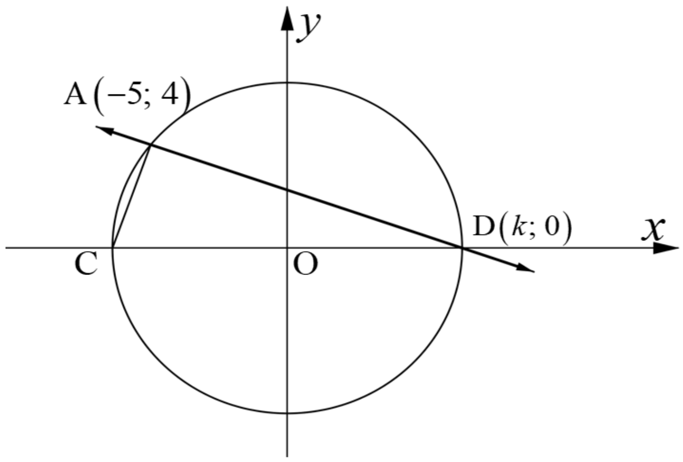
1.3 Hence, calculate the size of θ with the positive x-axis, rounded off to ONE decimal place. (4)

1.4 Determine the equation of the straight line through S parallel to BT. (3)

[12]

QUESTION 2

In the diagram below, circle ACD with centre O at the origin, cuts straight line AD at $A(-5;\ 4)$ and $D(k;\ 0)$. D is on the *x*-axis.



Determine:

2.1 The equation of the circle (2)

2.2 The numerical value of *k* (3)

2.3 The gradient of OA (1)

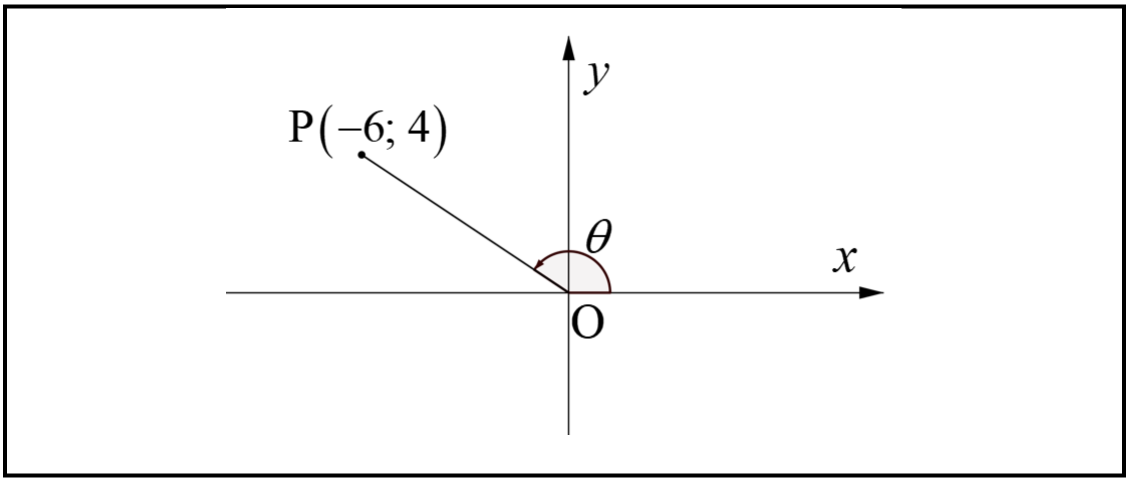
2.4 The equation of the tangent to the circle at A (3)

2.5 Analytically that $D\hat{A}C = 90°$ (4)

[13]

QUESTION 3

3.1 In the diagram below, P is the point $(-6;\ 4)$. θ is the angle between the x-axis and the straight line, OP.



Calculate the numerical value of the following (leave your answer in surd form):

3.1.1 OP (2)

3.1.2 $\cos^2\theta - \sin^2\theta$ (4)

3.1.3 $\cot\theta - 2$ (2)

3.2 If θ = 64,5° and β = 73,2°, calculate the following, correct to TWO decimal places (Show all your workings):

$\cot^2 2\beta - \text{cosec}^2\theta$ (6)

3.3 Simplify:

$\sin(180° - x).\cos(180° + x).\sec(360° + x).\sin^2 \dfrac{\pi}{3}$ (7)

3.4 Solve for θ, give your answer correct to ONE decimal place:

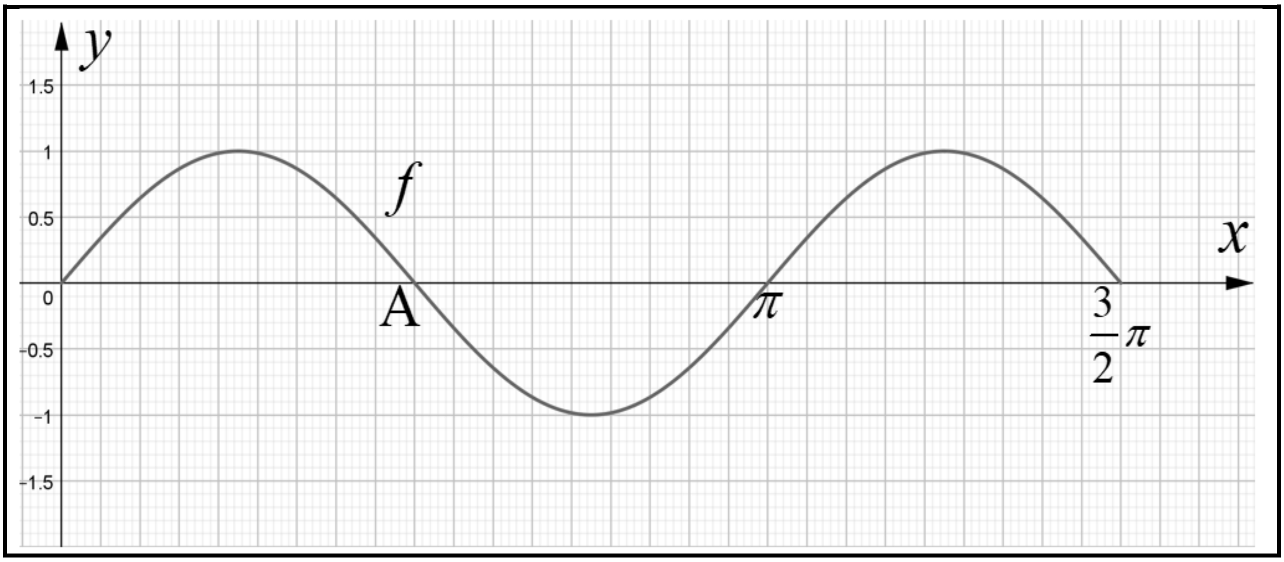
3.4.1 $4\cos(2\theta + 20°) = 2{,}178 \qquad (2\theta + 20°) \in [0°;\ 180°]$ (3)

3.4.2 $\text{cosec}\ (\theta - 30°) = 1{,}57 \qquad \theta \in [0°;\ 360°]$ (6)

[30]

QUESTION 4

Given $f(x) = \sin px$ and $g(x) = \tan x$ for $x \in \left[0;\ \frac{3}{2}\pi\right]$



4.1 Write down:

4.1.1 The value of A (1)

4.1.2 The value of *p* (1)

4.1.3 The minimum value of *f* (1)

4.2 Draw the graph of *g* on the same system of axis as provided in your ANSWER BOOK. Clearly indicate ALL critical points. (3)

4.3 Use the graphs to determine for which value(s) of *x* is:

4.3.1 $f(x) - g(x) = 0$, where $x \in \left[\pi;\ \frac{3}{2}\pi\right]$ (2)

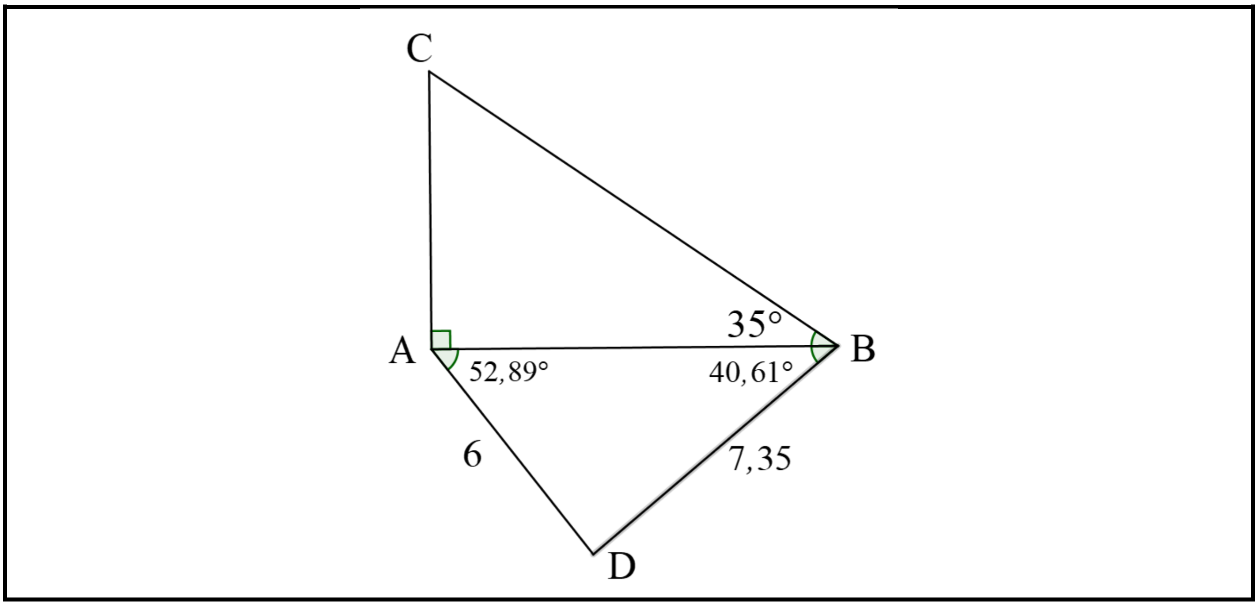
4.3.2 $f(x) \geq g(x)$, where $x \in [0;\ \pi]$ (4)

[12]

QUESTION 5

In the diagram below, A, B and D are three points in the horizontal plane. AC is a vertical tower and the angle of elevation from B to C is 35°.

- $A\hat{B}D = 40{,}61°$
- $B\hat{A}D = 52{,}89°$
- AD = 6 m and BD = 7,35 m

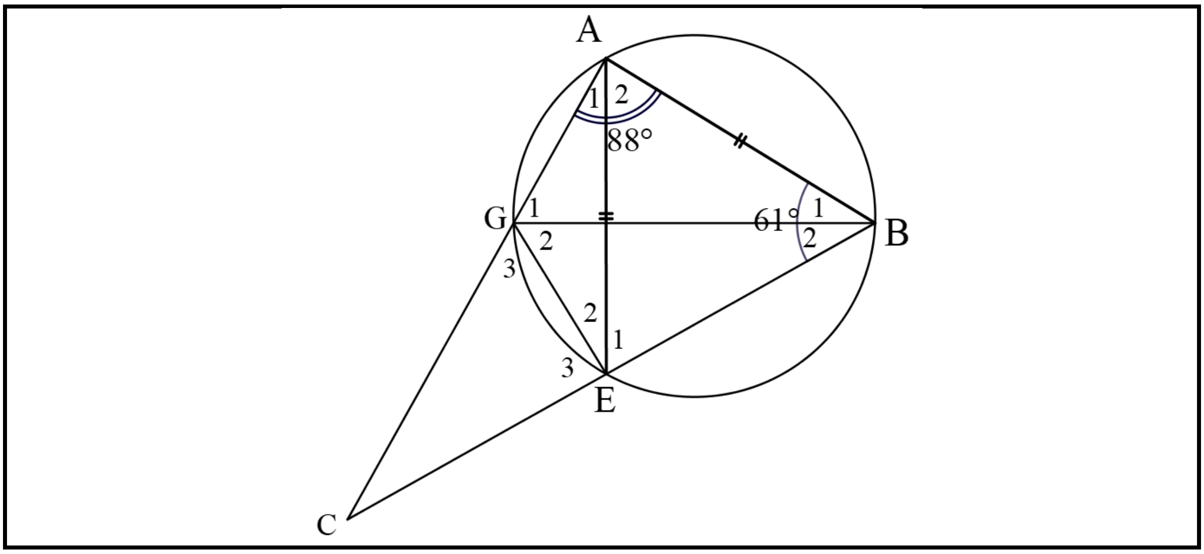


Determine the height of the tower CA, correct to ONE decimal place. (6)

[6]

QUESTION 6

In the diagram below, ABEG is a cyclic quadrilateral with AB = AE and $A\hat{B}E = 61°$.
AG and BE are produced to meet at C.



6.1 Name, with reasons, three other angles equal to 61°. (5)

6.2 If $B\hat{A}G = 88°$, determine, stating reasons, the size of:

6.2.1 $\hat{E}_2$ (2)

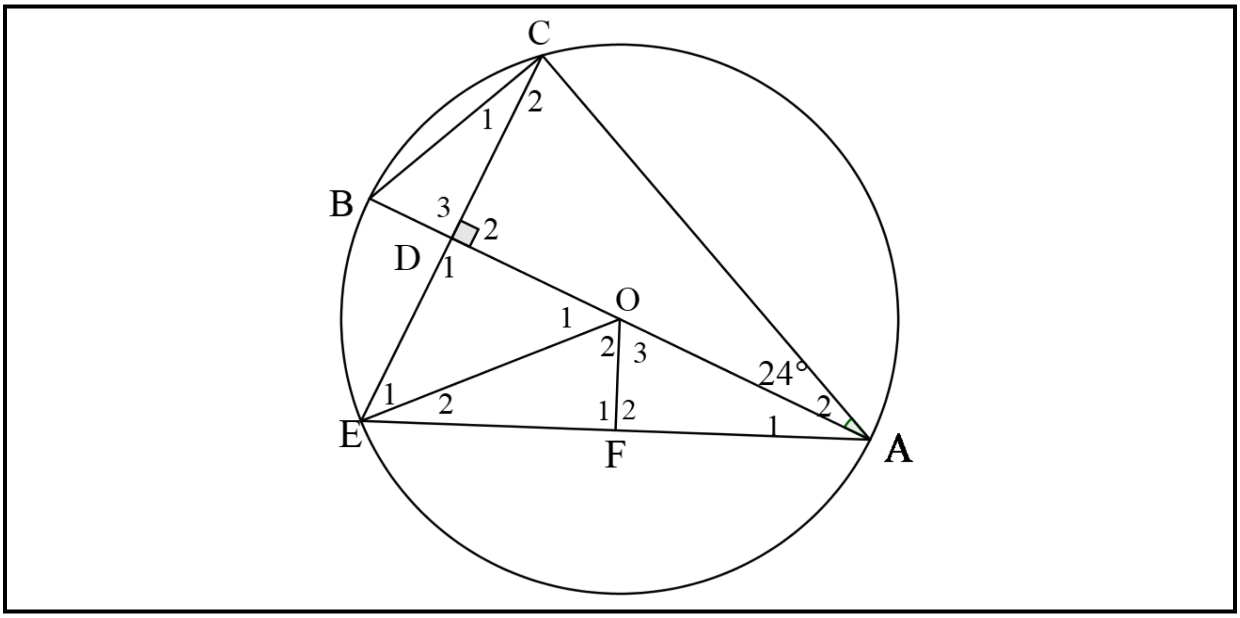
6.2.2 $\hat{B}_2$ (3)

[10]

QUESTION 7

In the diagram, the vertices of ΔACE lie on the circle with centre O.

- Diameter AB and chord CE intersect at D.
- $DO \perp CE$
- $\hat{A}_2 = 24°$



7.1 Determine, stating reasons, the size of $\hat{B}$. (3)

7.2 7.2.1 Show that ΔADC ≡ ΔADE. (4)

7.2.2 Hence, show that DA bisect $\hat{A}$. (1)

7.2.3 Determine, stating reasons, the size of $\hat{O}_1$. (2)

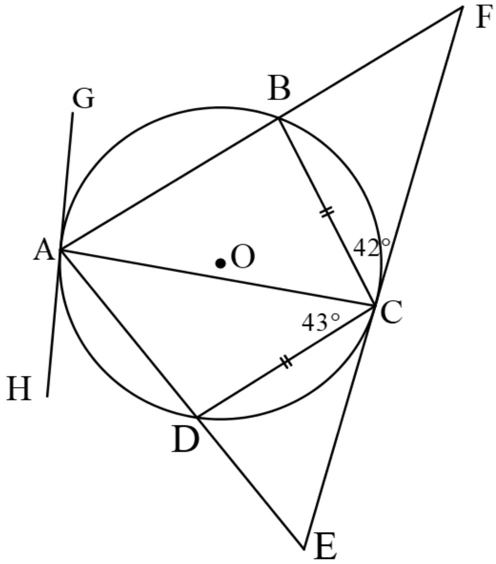
7.3 If it is further given that EF = FA, prove that DOFE is a cyclic quadrilateral. (3)

[13]

QUESTION 8

In the diagram, ABCD is a cyclic quadrilateral with BC = CD.

- The tangent at C intersect with AB produced in F and AD produced in E.
- GAH is a tangent to the circle at A.
- $B\hat{C}F = 42°$
- $A\hat{C}D = 43°$



8.1 Determine, stating reasons, the size of:

8.1.1 $D\hat{A}C$ (4)

8.1.2 $\hat{F}$ (4)

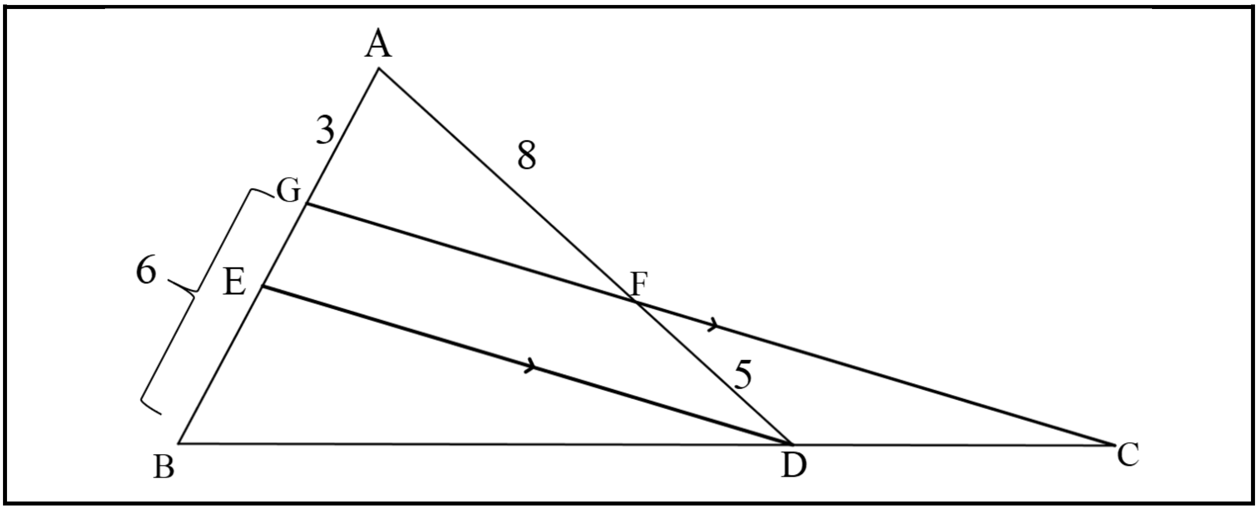
8.2 Prove that GAH is a tangent to circle AFE. (3)

[11]

QUESTION 9

In the diagram below, side BD of ΔABD is produced to C.

- F is a point on AD so that AF = 8 and FD = 5.
- CF produced cuts AB at G so that AG = 3 and GB = 6.
- E is a point on GB such that ED || GC.



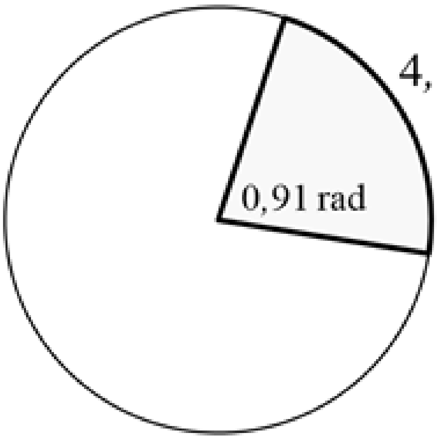
9.1 In ΔAED, determine the value of GE, with reasons, to the nearest integer. (5)

9.2 Determine, with reasons, the numerical value of $\frac{BC}{BD}$. (3)

[8]

QUESTION 10

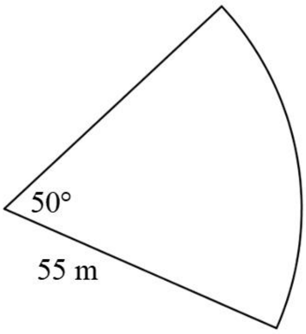
10.1 In the diagram below, an arc of length 4,75 cm subtends an angle of 0,91 radians.



10.1.1 Determine the length of the radius of the circle. (4)

10.1.2 Hence, determine the circumference of the circle. (3)

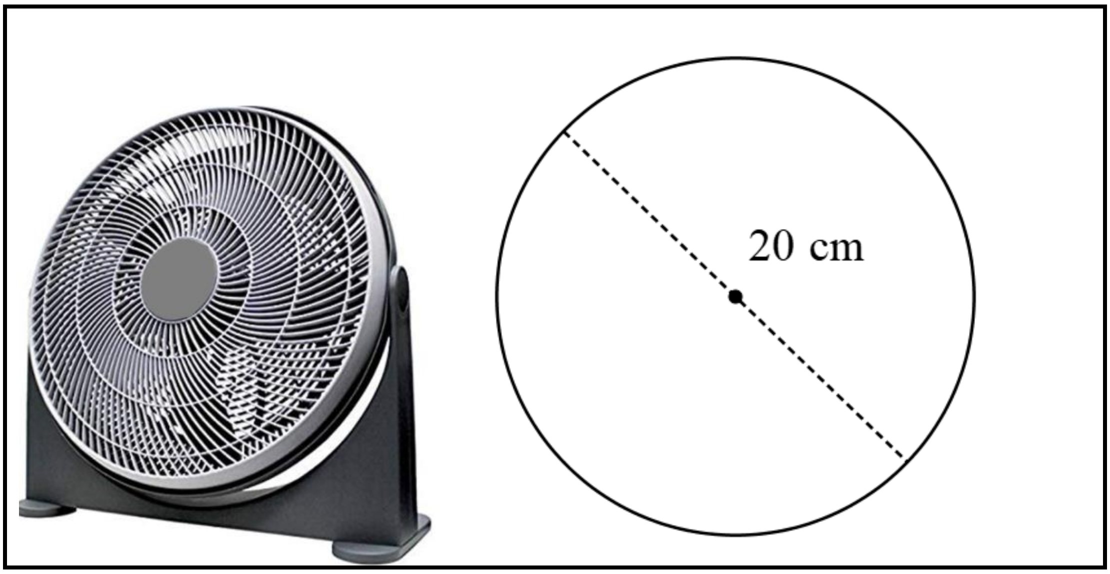
10.2 A football stadium floodlight can spread its illumination over an angle of 50° to a distance of 55 m. Represented by the pictures and diagram below.



10.2.1 Convert 50° to radians, rounded off to THREE decimals. (3)

10.2.2 Determine the maximum area that is floodlit. (3)

10.3 A desk fan with diameter 20 cm is rotating 215 times every minute.



10.3.1 Calculate the length of a fin. (1)

10.3.2 Calculate the angular velocity. (3)

10.3.3 Calculate the linear speed of the fan. (3)

10.3.4 Determine the linear speed to the nearest km/h. (4)

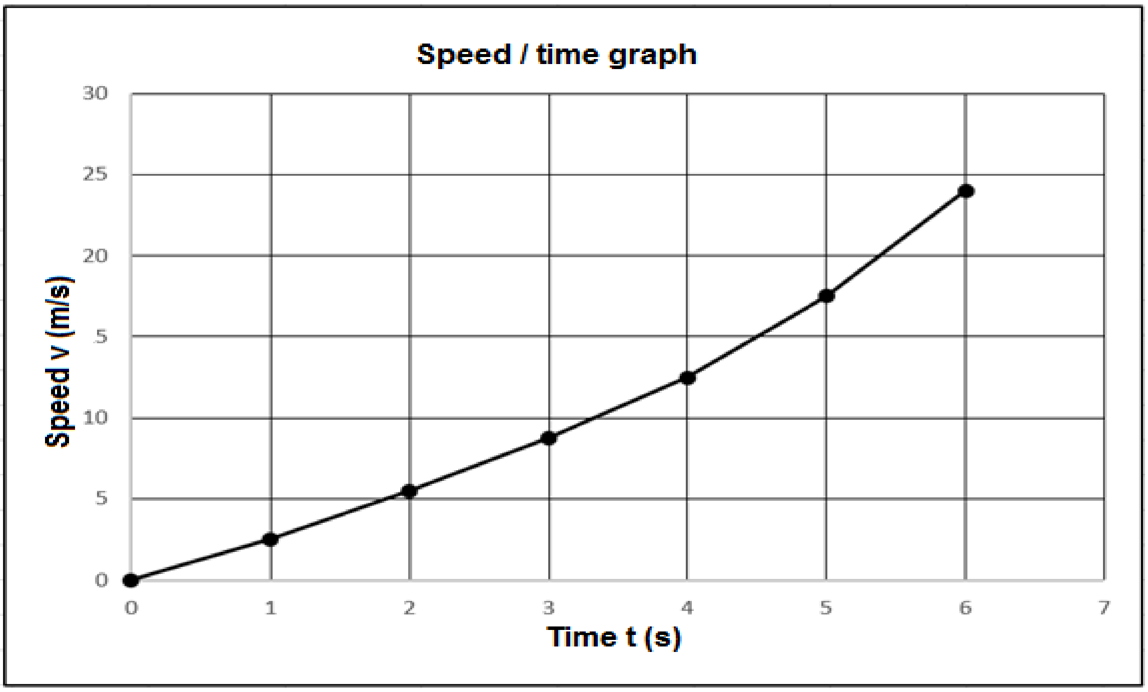
[24]

QUESTION 11

11.1 A car starts from rest and its speed is measured every second for 6 s. This data is represented in the table below.

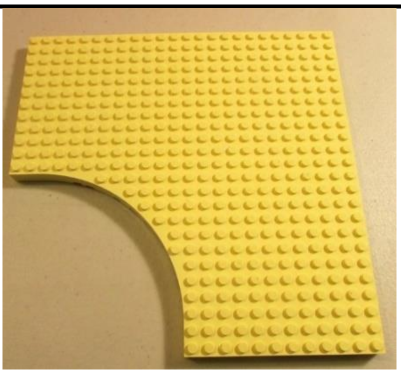
Time *t* (s)	0	1	2	3	4	5	6
Speed v (m/s)	0	2,5	5,5	8,75	12,5	17,5	24

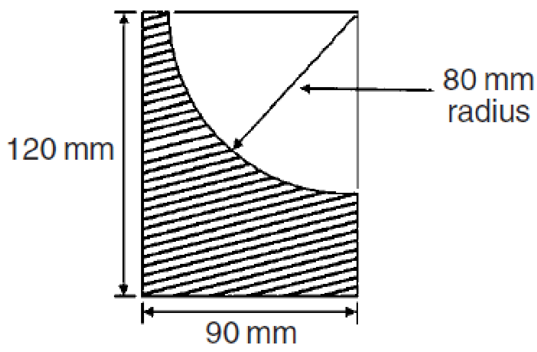
This data is further represented by a speed-time graph below.



Determine the distance travelled in 6 seconds (i.e. the area under the speed-time graph) by the mid-ordinate rule. (4)

11.2 The picture below is of a Lego brick. The diagram next to it represents the Lego brick with measurements.





Determine the area of the Lego brick as shown in the diagram. (7)

[11]

TOTAL: 150

INFORMATION SHEET

$x=\dfrac{-b\pm\sqrt{b^2-4ac}}{2a}$ $\quad x=-\dfrac{b}{2a}$ $\quad y=\dfrac{4ac-b^2}{4a}$

$a^x=b \Leftrightarrow x=\log_a b$ $a>0$, $a\neq 1$ and $b>0$

$A=P(1+ni)$ $\quad A=P(1-ni)$

$A=P(1+i)^n$ $\quad A=P(1-i)^n$ $\quad i_{eff}=\left(1+\dfrac{i^m}{m}\right)^m-1$

$d=\sqrt{\left(x_2-x_1\right)^2+\left(y_2-y_1\right)^2}$ $\quad \mathrm{M}\left(\dfrac{x_1+x_2}{2};\dfrac{y_1+y_2}{2}\right)$

$y=mx+c$ $\quad y-y_1=m\left(x-x_1\right)$ $\quad m=\dfrac{y_2-y_1}{x_2-x_1}$

$m=\tan\theta$ $\quad \dfrac{x^2}{a^2}+\dfrac{y^2}{b^2}=1$

$f'(x)=\lim\limits_{h\to 0}\dfrac{f(x+h)-f(x)}{h}$ $\quad \int x^n dx=\dfrac{x^{n+1}}{n+1}+C$, $n\neq -1$

$\int\dfrac{1}{x}dx=\ln(x)+C,\quad x>0$ $\quad \int a^x dx=\dfrac{a^x}{\ln a}+C$, $a>0$

$\pi rad=180°$

Angular velocity $=\omega=2\pi n=360^0 n$ where n = rotation frequency

Circumferential velocity $=v=\pi Dn$ where D = diameter and n = rotation frequency

$s=r\theta$ where r = radius and θ = central angle in radians

$4h^2-4dh+x^2=0$ where h = height of segment, d = diameter of circle and x = length of chord

Area of a sector $=\dfrac{rs}{2}=\dfrac{r^2\theta}{2}$ where r = radius, s = arc length and θ = central angle in radians

In ΔABC:

$\dfrac{a}{\sin \mathrm{A}}=\dfrac{b}{\sin \mathrm{B}}=\dfrac{c}{\sin \mathrm{C}}$ $\quad a^2=b^2+c^2-2bc.\cos \mathrm{A}$ $\quad \text{Area}=\dfrac{1}{2}ab.\sin \mathrm{C}$

$\sin^2\theta+\cos^2\theta=1$ $\quad 1+\tan^2\theta=\sec^2\theta$ $\quad \cot^2\theta+1=\mathrm{cosec}^2\theta$

$\mathrm{A}_T=a\left(\dfrac{o_1+o_n}{2}+o_2+o_3+o_4+\ldots+o_{n-1}\right)$ where a = equal parts, o_i = i^{th} ordinate and n = number of ordinates

OR

$\mathrm{A_T}=a\left(m_1+m_2+m_3+\ldots+m_n\right)$ where a = equal parts, $m_1=\dfrac{o_1+o_2}{2}$ and n = number of ordinates