Province of the
EASTERN CAPE
EDUCATION

EXAMINATIONS AND ASSESSMENT CHIEF DIRECTORATE
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REPUBLIC OF SOUTH AFRICA, Website: www.ecdoe.gov.za

2022 NSC CHIEF MARKER'S REPORT

SUBJECT:		**TECHNICAL MATHEMATICS**
PAPER:		**2**
DURATION OF PAPER:		**3 hours**
DATES OF MARKING:		**09 - 21 December 2022**

SECTION 1: (General overview of Learner Performance in the question paper as a whole)

The number of Eastern Cape NSC, SC and MEO candidates that wrote the final NSC Technical Mathematics 2022 paper was 2762, which is a 197 more than in 2021.

A sample of 100 scripts was collected during the marking process. The selected sample comprises of scripts that were moderated by the Internal Moderator and/or Chief Marker, and/or the Senior Marker and some non-moderated scripts.

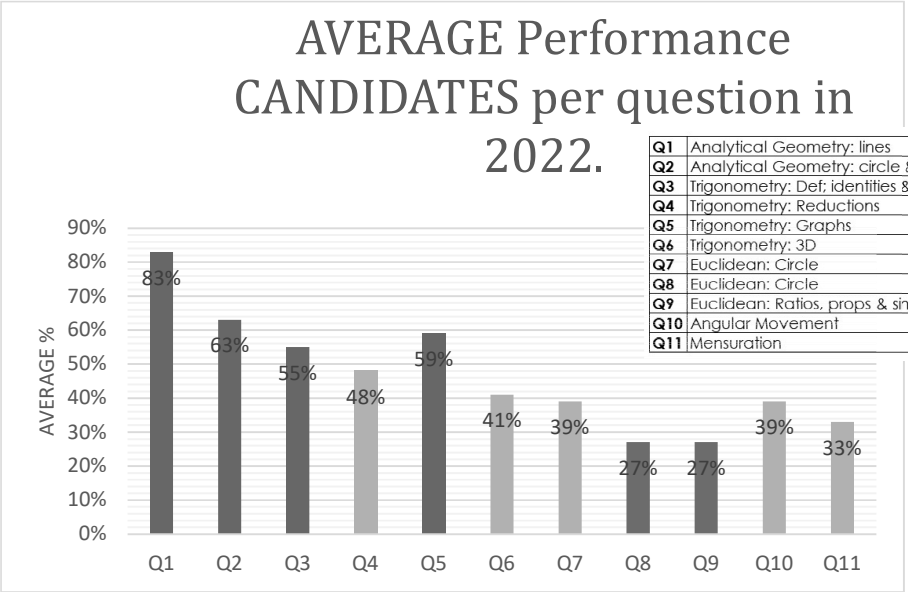
The graphical representation in the report will be based on the 100 sampled candidates' responses which were selected as depicted in the next table:

	[0; 44]	[45; 59]	[60; 74]	[75; 89]	[90; 104]	[105; 119]	[120; 150]	TOTAL
Required	15	15	20	20	20	5	5	100
Actual	23	12	20	16	16	8	5	100
Percentage	23%	12%	20%	16%	16%	8%	5%	

The 2022 cohort performed better than the cohort of 2021, when looking at the pass %. Looking at the 7-point scale, candidates are also performing better than the 2021 cohort and previous years.

Technical Mathematics Paper 2 is, unfortunately, still failing in its aim as quoted in the CAPS document ("*(d) The National Curriculum Statement Grades R – 12 aims to produce candidates that are able to: • identify and solve problems and make decisions using critical and creative thinking;*"), as the bulk of the candidates still performs at level 1.

The



average performance of the sampled 100 candidates of the questions, is depicted in the graph below:

KEY:

Blue – > 50%

Green - < 50%, but > 30%

Red - < 30%

Questions 1, 2, 3 and 5 was the best performing questions, with Question 8 and 9 being the worst performing questions of the sampled candidates. The performance per question of Questions 4 and 6 were relatively close to each other in percentage, as well as question 7, 10 and 11. Question 1 was the overall best performing question with an average above 80%, which is ± 20% more than the average in 2021. Questions 2, 3 and 5 performed with an average above 50%. Questions 4, 6, 7, 10 and 11 performed with averages less than 50%, which is also an improvement from 2021. Questions 8 and 9 were the two worst performing questions, below 30%.

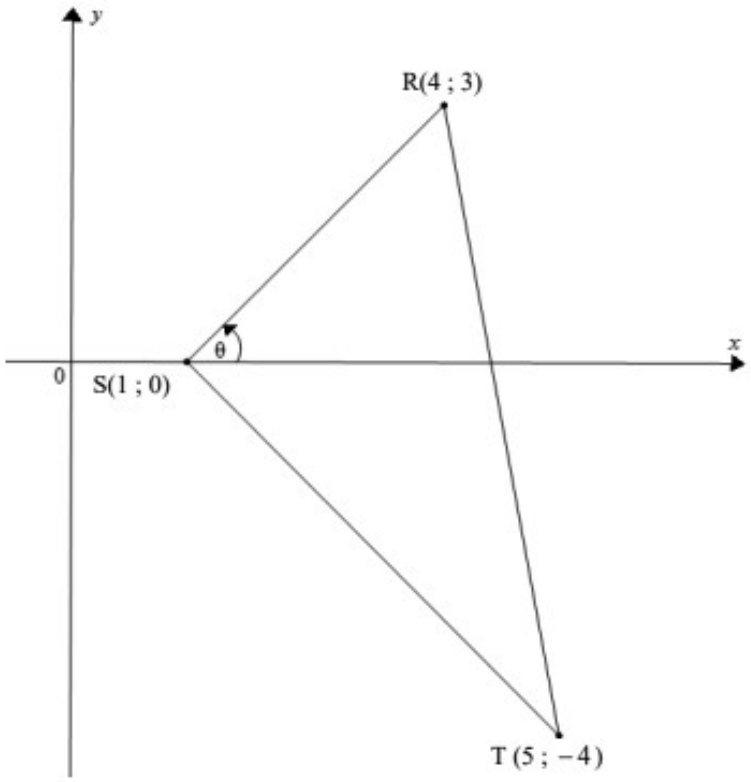
The best answered topic for 2022 was Analytical Geometry (Questions 1 and 2). Trigonometry (Question 3 – 6), Euclidean Geometry (Question 7), Circles, Angles and Angular Movement (Question 10) and Measurement (Question 11) were well answered questions, in the sense that candidates wrote a lot, but made unnecessary mistakes, which cost them marks and lead to the questions being poorly answered. Euclidean Geometry (Questions 8 and 9) were poorly answered, and candidates scored little to no marks. This makes these two questions the worst performing questions for 2022, which is in line with the performance of 2021.

SECTION 2: Comment on candidates' performance in individual questions (It is expected that a comment will be provided for <u>each question</u>).

QUESTION 1 [Total marks 13]

QUESTION 1

In the diagram below, ΔRST with vertices R(4 ; 3), S(1 ; 0) and T (5 ; −4) is given. The angle of inclination of RS with the positive x-axis is θ.



1.1 Determine the gradient of RS. (2)

1.2 1.2.1 Write down the formula required to calculate the angle of inclination of a line. (1)

1.2.2 Hence, determine the value of θ. (2)

1.3 Calculate the length of RT in simplified surd form. (2)

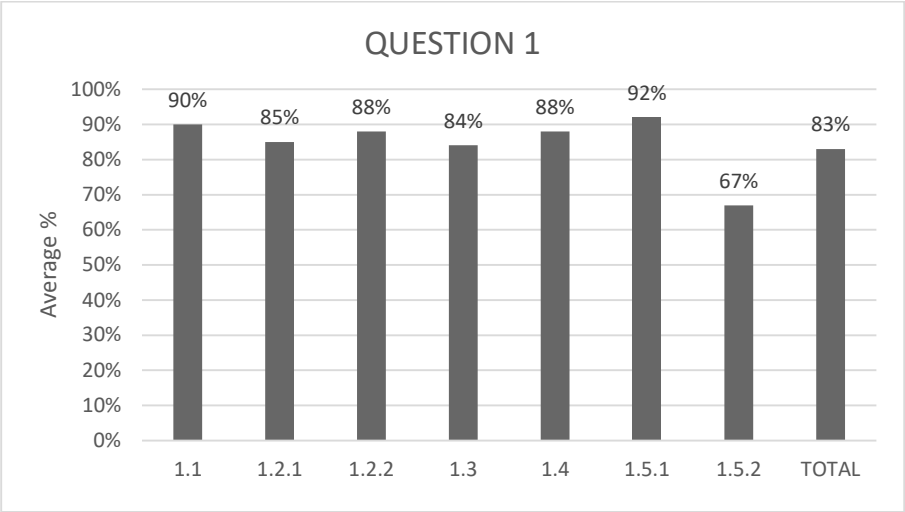
1.4 Determine the coordinates of the midpoint of ST. (2)

1.5 A line is drawn parallel to RS passing through the midpoint of ST.

1.5.1 Complete the statement:

If two lines are parallel, then their gradients are … (1)

1.5.2 Hence, determine the equation of the line parallel to RS passing through the midpoint of ST in the form $y = \ldots$ (3)
[13]



- This was the overall best answered question by the sampled candidates.
- Average percentage for this question is 83%.
- It was a good, easy question to start the paper off with.
- Candidates performed well in all the sub-questions relating to Grades 10 and 11 work.
- Most candidates answered this question.

Common errors and misconceptions

i.) Candidates could not identify the inclination angle formula in 1.2.1, yet they could calculate the inclination angle in 1.2.2. Candidates copy formula incorrectly or change the signs of the formula.

ii.) Some candidates did not give the answer in 1.3 in surd form and hence lost a mark as they gave the answer in decimal form.

iii.) Candidates struggled to determine the equation of the straight line in 1.5.2 as they did not correctly understand that they specifically had to use the coordinate of the mid-point and then substituted the incorrect coordinate to calculate the c-value. This caused unnecessary loss of marks.

iv.) Candidates are also substituting coordinates incorrectly, by swopping the values for x and y.

Suggestions for improvement

Educators should focus on the following during their contact time with candidates:

i.) Inclination angles:
   a. The general formula for the inclination angle should be emphasised as $tan\theta = m$.
   b. Candidates should be reminded that inclination angles can be acute or obtuse, depending on the gradient being used.
   c. Emphasis should be placed on what it means when there is a negative gradient and what the negative sign is used for.

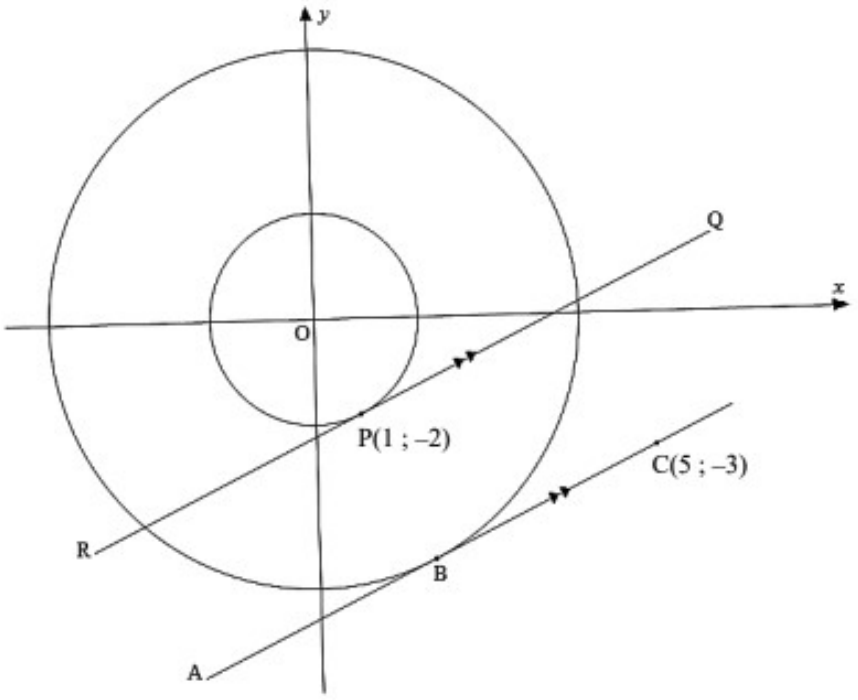
ii.) Candidates must be reminded to always answer the question being asked. If they are required to give the answer in simplified surd form, they must remember to do so.

iii.) Coordinates:
   a. Clearly distinguish between which coordinates to use when the equation of a specific line is being asked.
   b. Emphasise which value in a coordinate is x and which is y, as many candidates are still swopping the values around when substituting into various formulae.

QUESTION 2 [Total marks 12]

QUESTION 2

2.1 In the diagram below, O is the centre of both the smaller and the larger circles.
RQ is a tangent to the smaller circle at point $P(1\ ;\ -2)$.
AC is a tangent to the larger circle at point B with $C(5\ ;\ -3)$.
RQ ∥ AC



2.1.1 Determine the equation of the smaller circle. (2)

2.1.2 Write down the gradient of OP. (1)

2.1.3 Give a reason why OP is perpendicular to RQ. (1)

2.1.4 Hence, determine the gradient of AC. (2)

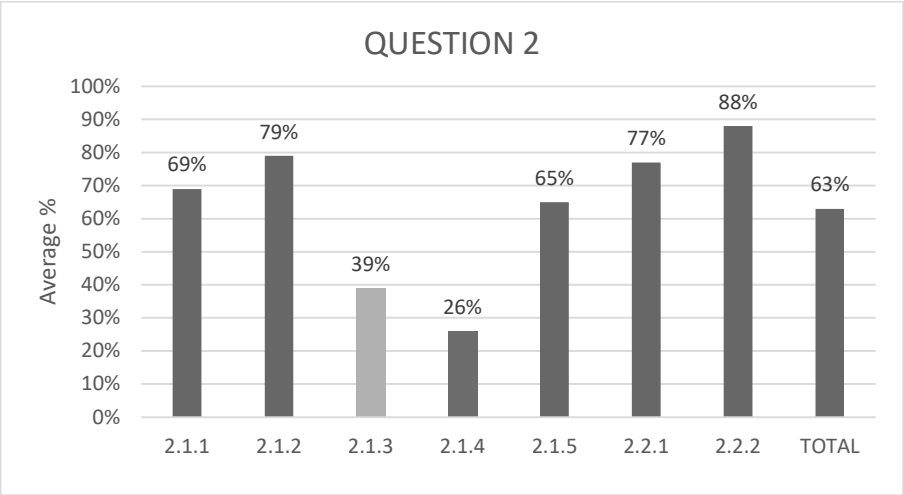
2.1.5 Hence, determine the equation of AC in the form $y = \ldots$ (3)

2.2 Given: $\frac{x^2}{36} + \frac{y^2}{16} = 1$

2.2.1 Express the equation in the form $\frac{x^2}{a^2} + \frac{y^2}{b^2} = 1$ (1)

2.2.2 Hence, sketch the graph defined by $\frac{x^2}{36} + \frac{y^2}{16} = 1$ (2)

[12]



- Along with question 1, this was one of the better answered questions in the question paper.
- This question was easy, as most work tested was Grade 10 and 11 work.
- Overall, the question performance was well above 50%.
- Questions 2.1.3 and 2.1.4 was the two sub-questions that candidates struggled with most.

Common errors and misconceptions

i.) In Q2.1.1 many candidates calculated the radius, instead of the equation of the circle.

ii.) Candidates do not know their basic rules and therefore cannot give reasons as to why certain concepts can be applied. For example, in Q2.1.3 candidates were expected to give a reason why two lines are perpendicular, yet they were not able to do so.

iii.) Because candidates could not provide a reason for Q2.1.3, they struggled to answer Q2.1.4 as they needed to apply the concept in this question.

iv.) In Q2.1.5 candidates substituted any point, instead of a point on the tangent line. Candidates who did substitute the correct coordinate did not simplify correctly when multiplying out the brackets when $y - y_1 = m(x - x_1)$ is used.

Suggestions for improvement

Educators should focus on the following during their contact time with candidates:

i.) Emphasise to candidates that they must always read the question carefully and make sure of what is being asked of them. Not reading the question completely results in unnecessary marks that are being lost due to them not giving the final answer that is expected in the question.

ii.) Emphasise basic rules of Analytical Geometry that was taught in Grade 10, but still plays a vital role in answering certain questions in Grade 12. For example, clearly distinguish between when are lines parallel and when they are perpendicular and how do you show that lines are parallel or perpendicular.
   a. Lines are perpendicular when the product of the two gradients are equal to -1.
   b. Lines are parallel when the gradients of one or more lines are equal.

iii.) Coordinates:
   a. Clearly distinguish between which coordinates to use when the equation of a specific line is being asked.
   b. Emphasise which value in a coordinate is x and which is y, as many candidates are still swopping the values around when substituting into various formulae.

QUESTION 3 [Total marks 13]

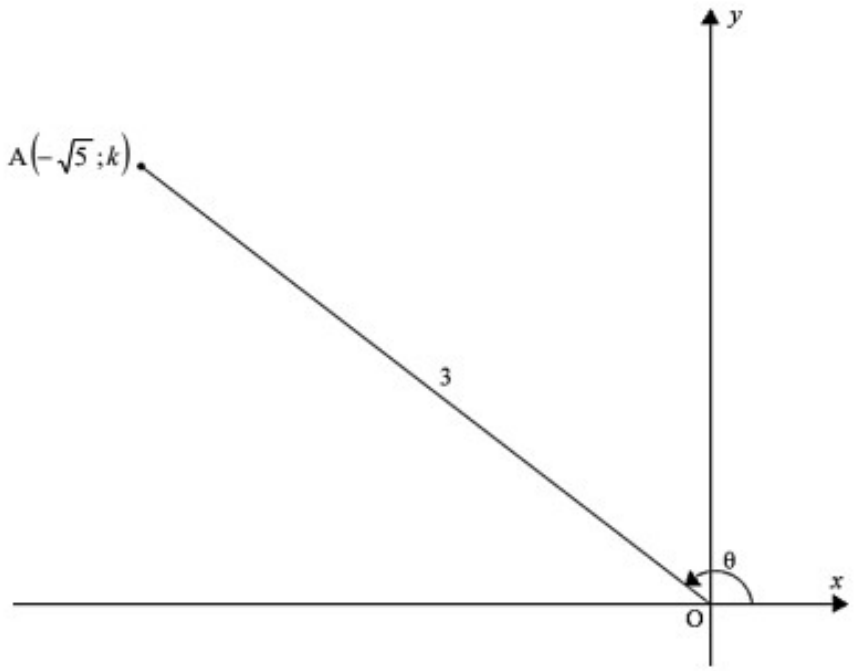
QUESTION 3

3.1 Given: $\hat{P} = \frac{2}{7}\pi$ and $\hat{Q} = 37°$

3.1.1 Convert $\frac{2}{7}\pi$ to degrees. (1)

3.1.2 Determine the value of $\text{cosec P} - \cos Q$. (2)

3.2 In the diagram below, OA = 3 units and $A\left(-\sqrt{5}\,;k\right)$ is a point on the Cartesian plane. The angle of inclination of OA with respect to the positive x-axis is θ.

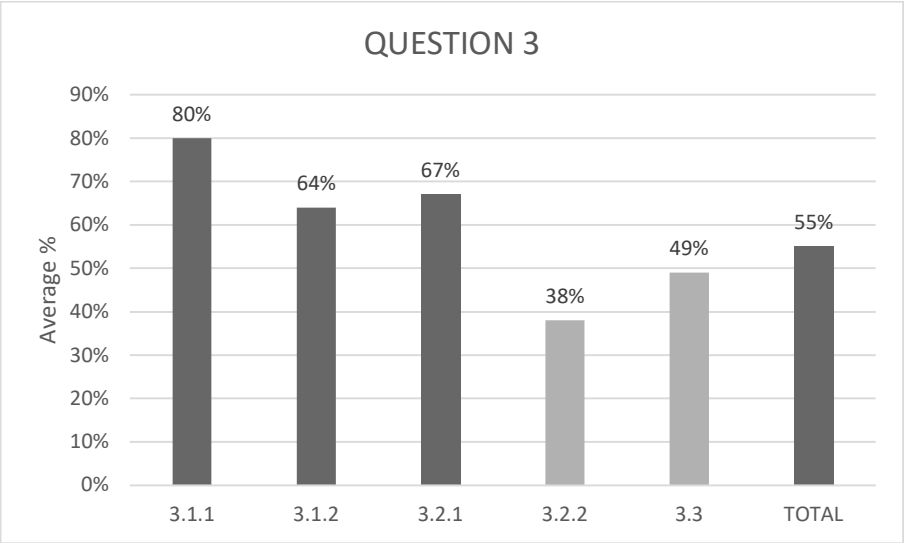


Determine, **without the use of a calculator**, the numerical value of:

3.2.1 k (3)

3.2.2 $\sqrt{5}\cot\theta + 1$ (3)

3.3 Determine the value(s) of x if $3\tan x = -0{,}531$ and $x \in [0°;360°]$. (4)
[13]



- This question was well answered, compared to 2021.
- More candidates were able to find the correct values to Q3.1.1 and Q3.1.2.
- Candidates did however struggle with Q3.2.2 and Q3.3.

Common errors and misconceptions

i.) In Q3.1 the most common error that occurred was candidates not converting the radian measure to degrees before calculating the values required. Candidates are also mixing the different trigonometric ratios and the reciprocals.

ii.) In Q3.2, candidates were given a diagram which they needed to use to find the value of a reciprocals. To do this, candidates needed to apply Pythagoras, which is a concept from Grade 8 and 9. This however proved problematic as candidates struggled to apply Pythagoras correctly as they struggle to identify which side is x, y or r (opposite, adjacent or hypotenuse). Many candidates also made use of the calculator.

iii.) For Q3.3 many candidates could calculate the correct reference angle, but did not know how to proceed form there, thus losing out on marks. Many candidates also gave an answer for x in two quadrants, instead of choosing the correct quadrant. Some candidates also made use of the general solution which is a concept applied in Mathematics.

Suggestions for improvement

Educators should focus on the following during their contact time with candidates:

i.) Emphasise that any value that is in radian measure must always be converted to degrees, when doing Trigonometry.

ii.) Basic Grade 10 trig ratios must be revised and consolidated so that candidates know the ratios as well as their reciprocal ratios.

iii.) When doing questions on completing diagrams, emphasis must be placed on what is required to be done before the question can be answered.

a. If the diagram is given:
    1. The triangle must be completed towards the x-axis.
    2. All values of the triangle must be calculated. Ratio values cannot be given if all values have not been calculated.

b. If the diagram must be sketched by the candidate:
    1. Trig equation given must be simplified so that the trig ratio is clear. i.e.: $3tan\theta - 1 = 0 \Rightarrow tan\theta = \frac{1}{3}$
    2. Diagram should be drawn and completed in the correct quadrant using the sign (+ or –) of the ratio value.

c. Pythagoras is used to calculate the unknown values in the triangle diagram. This basic Grade 8 concept must be emphasised and consolidated even in Grade 12.

iv.) Emphasis must be placed on answering the question. If the reference angle is calculated that does not mean that the question was answered. Classroom practice using and working with their CAST-diagram must be made a priority.

QUESTION 4 [Total marks 14]

QUESTION 4

4.1 Simplify the following:

4.1.1 $\sin(360° - \alpha)$ (1)

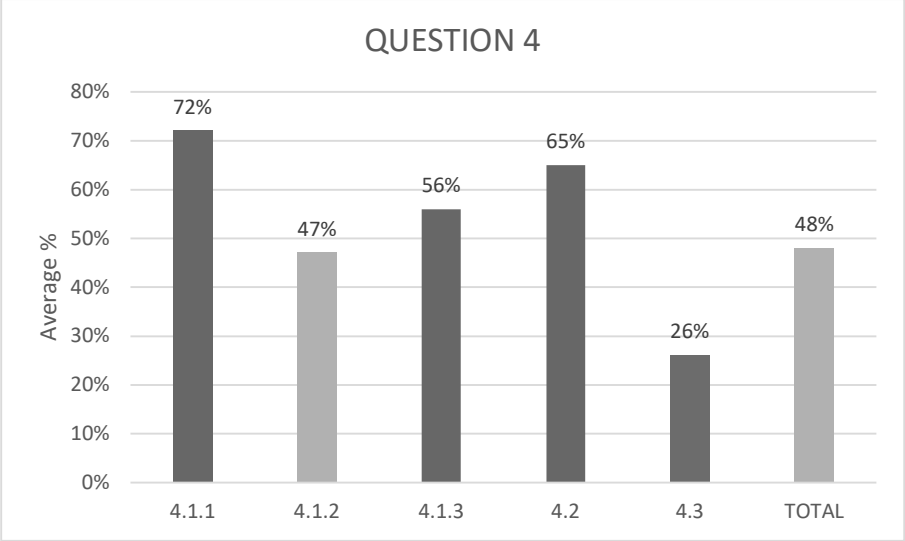
4.1.2 $\tan^2(\pi - \alpha)$ (2)

4.1.3 $\dfrac{\sin(360° - \alpha)\cdot\tan(180° - \alpha)\cdot\text{cosec}(2\pi - \alpha)}{\cos(360° + \alpha)\cdot\text{cosec}(180° - \alpha)\cdot\tan^2(\pi - \alpha)}$ (6)

4.2 Complete the identity: $1 - \sin^2 x = \ldots$ (1)

4.3 Prove the identity that: $\text{cosec}\, x - \sin x = \cot x \cdot \cos x$ (4)

[14]



- Candidates performed better in this question than in 2021.
- Candidates struggled the most with Q4.3.

Common errors and misconceptions

i.) Candidates do not know their identities and reduction formulae and how to apply them in a question that is being asked. When working with the reduction formulae, candidates do not know in which quadrants to work to be able to simplify.

ii.) Candidates are struggling to prove identities as they do not do the left and right part separately and they do not apply the identities they are taught correctly to be able to simplify.

iii.) When it comes to the simplification to prove the identities, candidates are struggling with the basic algebra behind the question.

a. i.e.: $\frac{1}{sin\theta} \times \frac{sin\theta}{cos\theta}$ will be simplified to $cos\theta$ instead of $\frac{1}{cos\theta}$.

b. i.e.: $\frac{1}{sin\theta} - sin\theta$ will be simplified to $\frac{1-sin\theta}{sin\theta}$ or candidates cancel the $sin\theta$ and end up with and answer of 0.

Suggestions for improvement

Educators should focus on the following during their contact time with candidates:

i.) Candidates must know all the square identities. They must know how to change the subject of these identities to recognise, use and apply them in their calculations.

ii.) Candidates must practice enough problems containing reductions on a regular basis, as these are easy marks to score if they are applied correctly. The CAST-diagram can assist in applying the reductions easily.

iii.) A clear connection must be made between Algebra and Trigonometry. Educators must emphasise that even though the question is on simplifying a Trigonometry Ratio or Proving Identities, in many instances there is Algebra involved in order to simplify to the correct answer.

QUESTION 5 [Total marks 11]

QUESTION 5

Given the functions defined by $f(x) = \sin(x + 30°)$ and $g(x) = \cos x$ for $x \in [0°; 360°]$

5.1 Write down:

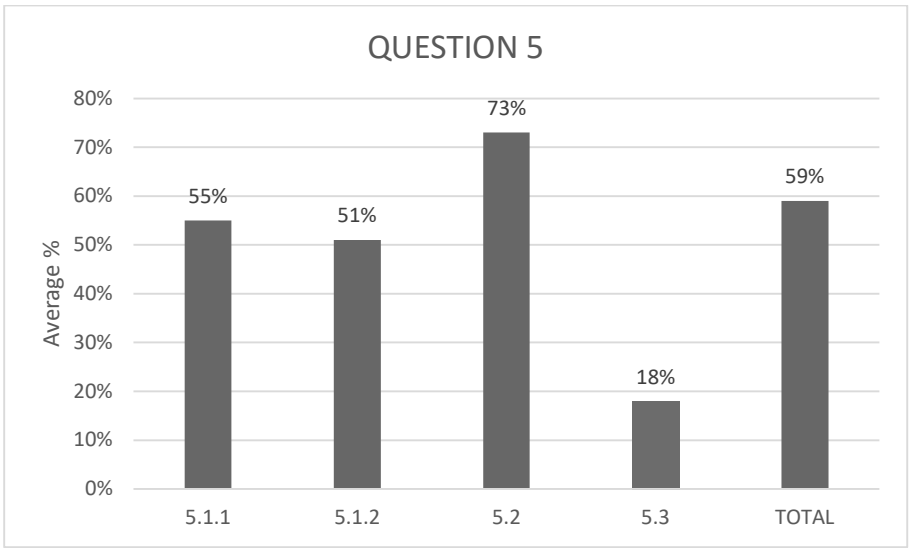
5.1.1 The period of g (1)

5.1.2 The amplitude of f (1)

5.2 Draw a sketch graph of f and g on the same set of axes on the grid provided in the ANSWER BOOK. Clearly indicate ALL turning points, end points and intercepts with the axes. (7)

5.3 Use the graph in QUESTION 5.2 to write down the value(s) of x for which the graph of g is increasing. (2)

[11]



- Q5.2 was the best performing sub-question. Many candidates were able to draw some sort of graph.
- It seems Q5.3 weas very difficult for the candidates. Reading information from the graph poses a huge problem.

Common errors and misconceptions

i.) In Q5.1.1 many candidates wrote the period of the cosine-graph as an interval, instead of a value of $360°$, hence giving the domain of the graph.

ii.) In Q5.1.2 many candidates were unable to give the amplitude of the sine-graph.

iii.) In Q5.2 many candidates were able to correctly draw the cosine-graph but struggled with the sine-graph that was shifted $30°$ to the left. Candidates are also not using the values given on diagram sheet but writing their own.

iv.) Many candidates were unable to correctly sketch the start and endpoints on the graph. They indicated arrows at the end of their graph, instead of only sketching for the given interval given.

v.) Q5.3 was very poorly answered, as candidates do not understand how to look for the values of x, when a specific restriction is given.

vi.) Candidates struggled with the notation in Q5.3, by either using the incorrect brackets or inequality signs.

Suggestions for improvement

Educators should focus on the following during their contact time with candidates:

i.) Candidates must be reminded that the period of a graph is a single value in degrees and that the amplitude of a graph is a single positive value. Clearly distinguish the difference between the domain and period of a function and again range and domain as well.

ii.) Candidates should be taught how to use the table-mode of the calculator to draw accurate graphs.
   a. Horizontal shifting of graphs must be revised
   b. Please note the inconsistency in the CAPS document – page 18 the Overview and the Grade 11 Curriculum Statement
   c. Grade 11 Curriculum statement only refers to $y = \tan x$, $y = \sin(x + p)$ and $y = \cos(x + p)$ (pg. 37)
   d. However, the overview goes even further that $y = \tan kx$ and $y = \tan(x + p)$ must also be studied
   e. The Examination guidelines of 2021 refers only to draw $y = \mathrm{a}\tan x$ and candidates must know the effects of *p* in $y = \tan(x + p)$
   f. So, for teaching purposes consult the Overview and Curriculum statements of the CAPS document, but for examination purposes the Examination Guidelines must also be referred to

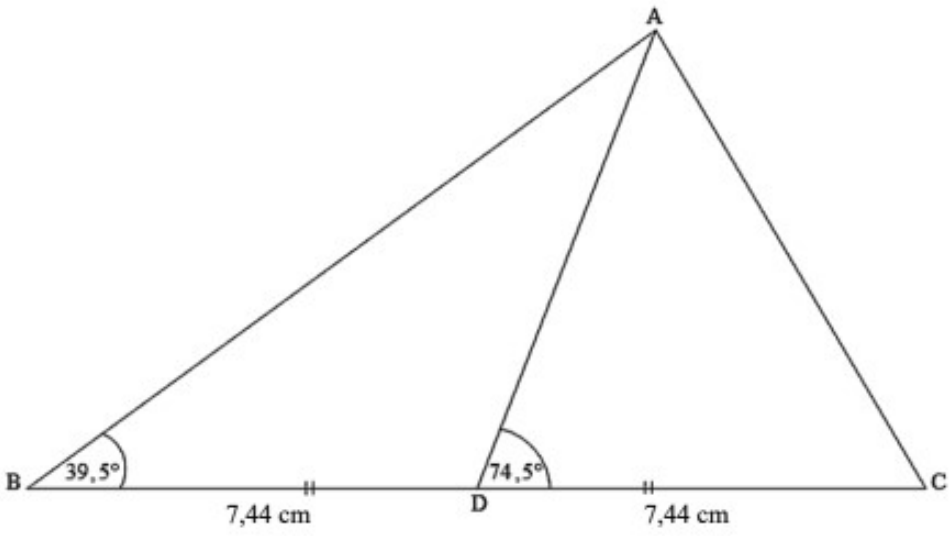
iii.) Interpretation of graphs must be constantly incorporated in graph revision worksheets.

QUESTION 6 [Total marks 11]

QUESTION 6

In the diagram below, BDC is a straight line with BD = DC = 7,44 cm

$\hat{B} = 39{,}5°$ and $A\hat{D}C = 74{,}5°$



6.1 Determine the size of:

6.1.1 $B\hat{A}D$ (1)

6.1.2 $A\hat{D}B$ (1)

6.2 Consider ΔABD.

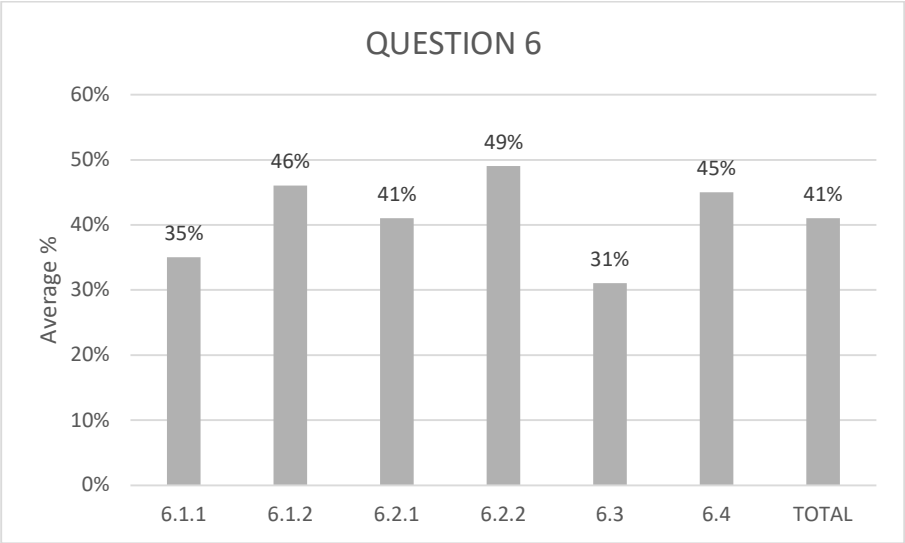
6.2.1 Complete the following with respect to ΔABD: $\frac{\ldots}{\sin B\hat{D}A} = \frac{BD}{\ldots}$ (1)

6.2.2 Hence, determine the length of AB. (2)

6.3 Hence, calculate the length of AC. (3)

6.4 Determine the area of ΔABC. (3)

[11]



- Overall Question 6 was answered well. There was an 11% improvement in the overall performance of the question from last year.
- Candidates lack understanding of 3D Trigonometry.

Common errors and misconceptions

i.) Calculating basic angles in Q6.1 using Grade 8 and 9 Geometry rules was a problem.

ii.) Candidates struggled to choose the correct formula. They mixed up the formulae and used sine-rule when they were supposed to use cosine-rule and vice versa.

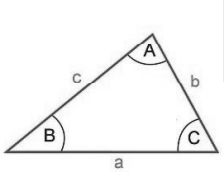
iii.) Candidates also struggled to choose the correct angles and lengths to substitute into the formula.

Suggestions for improvement

Educators should focus on the following during their contact time with candidates:

i.) Candidates must be taught how to decide which formula (cosine or sine rule) to select.
   a. Conditions for the use of the Sine Rule:
      1. Two sides and a non-inclusive angle
      2. Two angles and one side
   b. Conditions for the use of the Cosine Rule:
      1. Two sides and an inclusive angle
      2. Three sides

ii.) Candidates must be taught which angle goes with which side in a specific triangle.



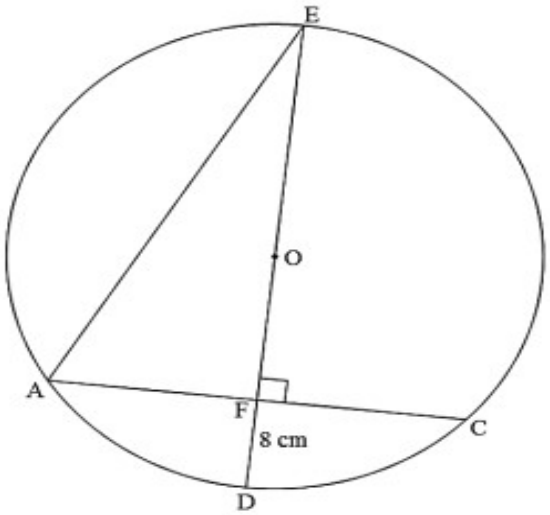
QUESTION 7 [Total marks 6]

QUESTION 7

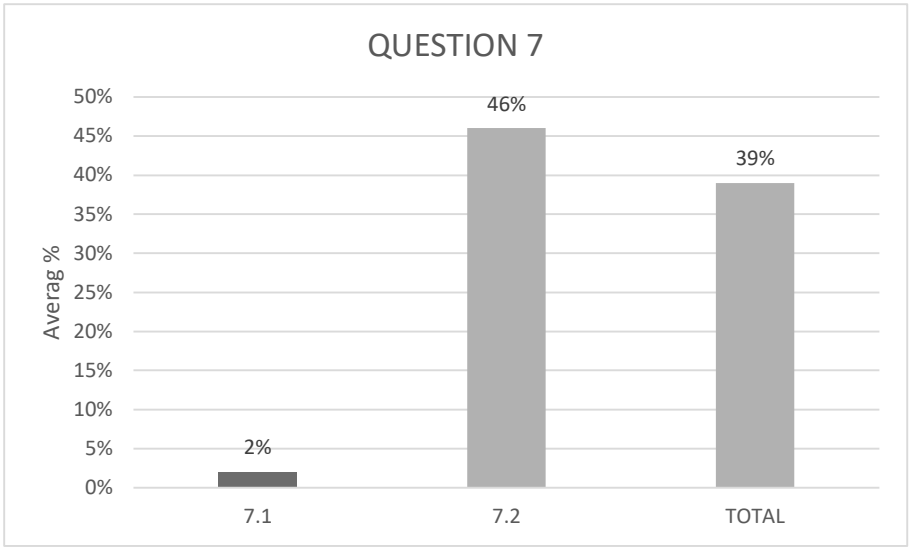
7.1 Fill in the missing word(s) in the following theorem statement:

The … of a chord passes through the centre of the circle. **(1)**

7.2 In the diagram below, AC and AE are chords of the circle with centre O.
Diameter ED is perpendicular to AC at F.
ED = 34 cm, FD = 8 cm and AC = 30 cm



Determine, stating reasons, the length of AE. **(5)**
[6]



- Overall, this was a poorly answered question.
- Completing statements or giving reasons to questions was a problem for candidates.
- Even though the diagram was provided in the answer book to assist candidates in finding answers, they did not make use of the diagram provided.

Common errors and misconceptions

i.) For Q7.1 candidates could not complete the statement given.
ii.) Many candidates attempted Q7.2 and many received min 3 or 4 marks. Candidates lost marks for not giving reasons for certain lengths.

Suggestions for improvement

Educators should focus on the following during their contact time with candidates:

i.) Emphasis must be placed on completing a theorem statement – these questions are classified as level 1 questions – use the proper reasons as provided by the Examination Guidelines. Knowing your theorems are the first path to success to Euclidean Geometry and identifying the applicable theorem in a diagram.

ii.) When the question requires to determine the size of an angle, it means there must be value attach to the angle.

iii.) Assumptions cannot be made if they cannot be substantiated or proved.

iv.) Teach candidates to "break-up" the diagram – in other words look for certain identifying diagrams that relate to the individual theorems – this means ample exercises for the eyes to get used to.

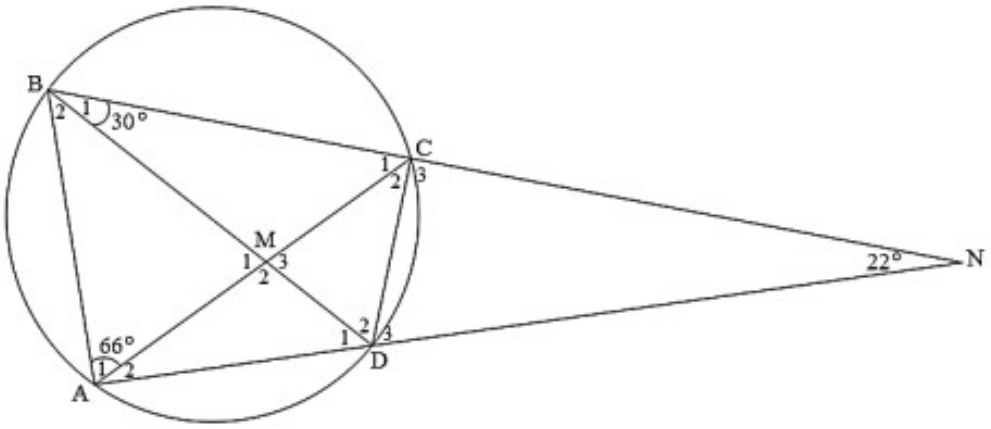
QUESTION 8 [Total marks 18]

QUESTION 8

8.1 Complete the following theorem statement:

The exterior angle of a cyclic quadrilateral is equal to the … (1)

8.2 In the diagram below, ABCD is a cyclic quadrilateral. Diagonals BD and AC intersect at M. Chords BC and AD produced meet at N such that $\hat{N} = 22°$, $\hat{B}_1 = 30°$ and $\hat{A}_1 = 66°$.



8.2.1 Determine, stating reasons, the size of the following angles:

(a) $\hat{A}_2$ (2)

(b) $\hat{C}_1$ (2)

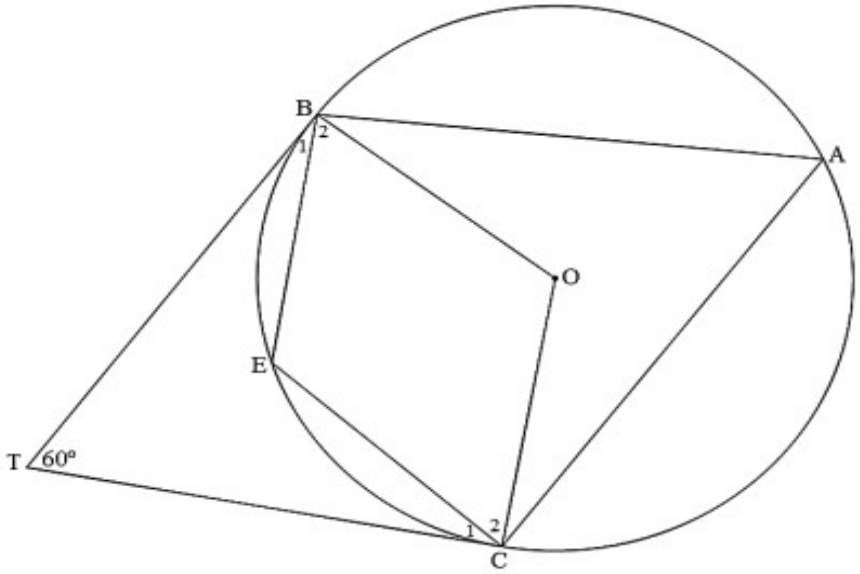
(c) $\hat{C}_3$ (2)

8.2.2 Show, stating reasons, that quadrilateral MCND is NOT a cyclic quadrilateral. (2)

8.3 In the diagram below, O is the centre of circle ABEC.

Tangents TB and TC touch the circle at B and C respectively, such that $\hat{T} = 60°$.

Radii OB and OC are drawn.



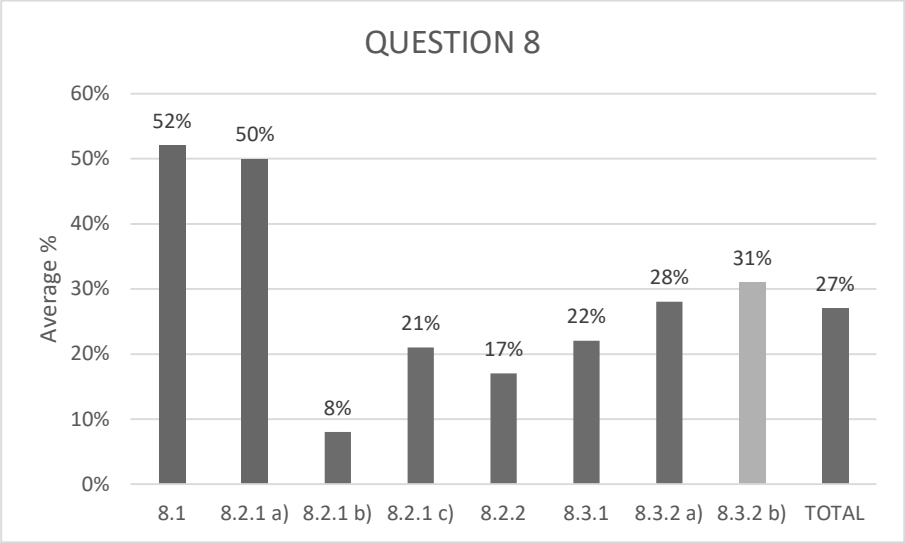
8.3.1 Write down, stating reasons, TWO angles each equal to $90°$. (3)

8.3.2 Determine, stating reasons, the size of the following angles:

(a) $\hat{A}$ (4)

(b) $\hat{E}$ (2)

[18]



- Question 8 was a very poorly answered question.
- A lot of candidates left the question blank and did not even attempt to answer the question.

Common errors and misconceptions

i.) In Q8.1 most candidates know the reason, but they do not write it correctly and hence loose the marks.
   a. i.e.: Candidates write opposite angle instead of opposite interior angle.

ii.) Candidates are struggling to find the sizes of angles within the diagram. To answer this question, they had to use the knowledge from Grade 11 Euclidean Geometry. Q8.2.1b and Q8.2.1c was poorly answered. Candidates managed to score marks due to CA marks that was awarded. Some candidates took a longer approach to Q8.2.1b, but then unfortunately did not make use of the diagram or show all calculations as to how they came about the answer.

iii.) Q8.2.2 was poorly answered and very few candidates attempted the question. Those who did attempt to answer the question did not show the correct calculations, nor did they give the correct reason.

iv.) Q8.3 was poorly answered as well. Candidates are not using correct notation when naming angles.
   a. i.e.: Candidates referred to $O\hat{B}T$ and $O\hat{C}T$ as $\hat{B}_{1+2}$ and $\hat{C}_{1+2}$ respectively instead of $\hat{B}_1 + \hat{B}_2$ and $\hat{C}_1 + \hat{C}_2$.

v.) In Q8.3.2a and Q8.3.2b candidates wrote answers and calculations that was not necessarily referring to the question being asked.

Suggestions for improvement

Educators should focus on the following during their contact time with candidates:

i.) Emphasis must be placed on completing a theorem statement – these questions are classified as level 1 questions – use the proper reasons as provided by the Examination Guidelines. Knowing your theorems are the first path to success to Euclidean Geometry and identifying the applicable theorem in a diagram.

ii.) When the question requires to determine the size of an angle, it means there must be value attach to the angle.

iii.) Assumptions cannot be made if they cannot be substantiated or proved.

iv.) Teach candidates to "break-up" the diagram – in other words look for certain identifying diagrams that relate to the individual theorems – this means ample exercises for the eyes to get used to.

v.) Euclidean Geometry can only be mastered if it is practiced continuously. Candidates must be taught how to transfer given information onto the diagram to assist them in answering given questions.

vi.) Diagrams should be analysed to assist in finding answers, in other words first try to look which theorems can possibly be used in order to find the answers to the questions being asked.

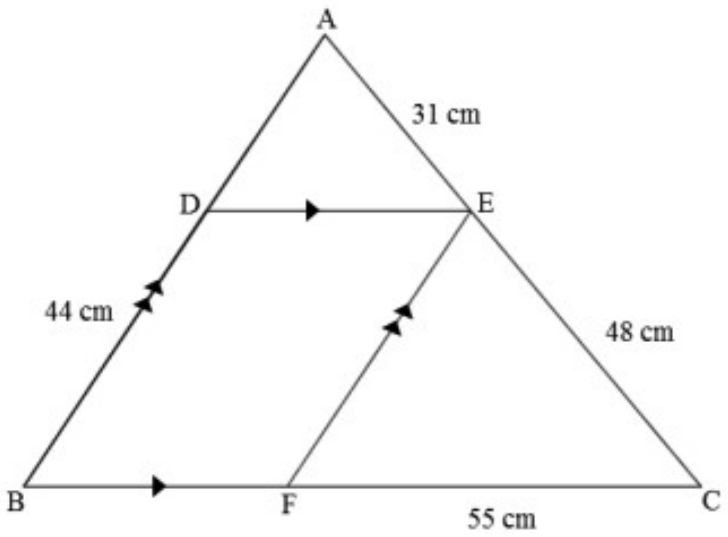
QUESTION 9 [Total marks 17]

QUESTION 9

9.1 Complete the following theorem statement:

A line drawn parallel to one side of a triangle divides the other two sides … (1)

9.2 The diagram below shows ΔABC with points D, E and F on sides AB, AC and BC respectively. DEFB is a parallelogram.
AE = 31 cm, EC = 48 cm, BD = 44 cm and FC = 55 cm.

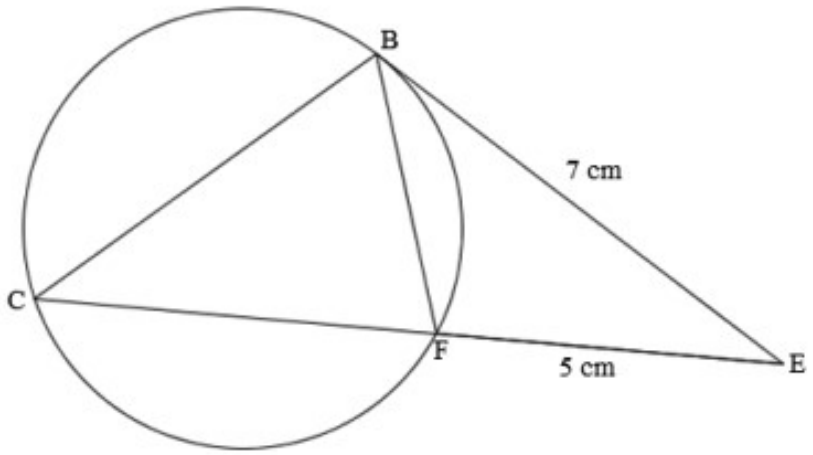


9.2.1 If DE ∥ BF and BD ∥ FE, state any TWO OTHER properties of the parallelogram. (2)

9.2.2 Determine, stating reasons, the length of AD. (3)

9.2.3 Determine the length of DE. (3)

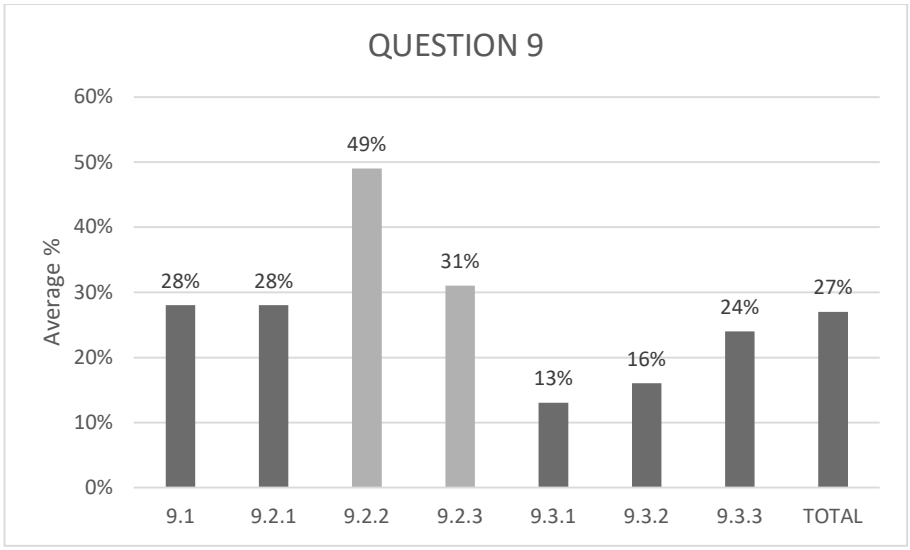
9.3 In the diagram below, BE is a tangent to circle BCF at point B.
Chord CF produced meets BE at E such that BE = 7 cm and FE = 5 cm.



9.3.1 Prove that $\Delta EBF \;|||\; \Delta ECB$. (4)

9.3.2 Hence, deduce that $EB^2 = EC \times EF$ (1)

9.3.3 Determine the length of CF. (3)
[17]



- Together with Question 8, Question 9 was also one of the poorest performing questions in the question paper.
- Candidates are struggling immensely with Euclidean Geometry as a whole. This includes the Grade 11 Euclidean Geometry.
- Candidates are struggling to get even the most basic of marks in the question.

Common errors and misconceptions

i.) In Q9.1 candidates mostly answered equal or equally, as if they got confused with the mid-point theorem of Grade 10.

ii.) Q9.2.1 which was one of the easiest questions in question 9 was very poorly answered as candidates either gave the properties of the parallelogram that was already indicated on the diagram, or they did not give any at all.

iii.) In Q9.2.2 candidates struggled to give the correct answer as they answered Q9.1 incorrectly and did not quite understand the trend of the question. The candidates who managed to give the correct proportions and get the correct answer lost a mark as they did not give the complete reason.

   a. i.e.: Candidates either stated only "prop theorem" or gave the set of parallel lines instead of giving the two parts together for a complete reason (prop thm; $DE \| BC$).

iv.) Because candidates did not know the properties of the parallelogram or how to correctly apply the proportionality theorem, they struggled to give the length of DE in Q9.2.3.

v.) Q9.3.1 seemed to be the most difficult question for candidates to answer, even though it is based on Grades 8 and 9 work on similarity. Candidates are not going back to work done in previous grades when they revise for their Grade 12 examinations.

vi.) Q9.3.2 was also poorly answered as candidates struggled to prove the similarity between the two triangles and then could not apply the similarity properties to show the length of EB^2.

vii.) Q9.3.3 was the easiest sub-question to answer, yet candidates struggled to do so as they could not do Q9.3.1 and Q9.3.2. Candidates however should have been able to answer this sub-question even without having done the above two questions.

Suggestions for improvement

Educators should focus on the following during their contact time with candidates:

i.) Emphasis must be placed on completing a theorem statement – these questions are classified as level 1 questions – use the proper reasons as provided by the Examination Guidelines. Knowing your theorems are the first path to success to Euclidean Geometry and identifying the applicable theorem in a diagram.

ii.) Similarity Theorem:

   a. Even though similarity is a Grade 9 concept, it must be revised again in Grade 12 when doing the similarity theorem. Emphasise that only similarity is used in Grade 12.

   b. When candidates have proven similarity or the following information $\Delta ABC \ \|\|\| \ \Delta ACE$ was provided they should be able to immediately deduce that $\frac{AB}{AC} = \frac{AC}{AE} = \frac{BC}{CE}$, by using any acceptable method.

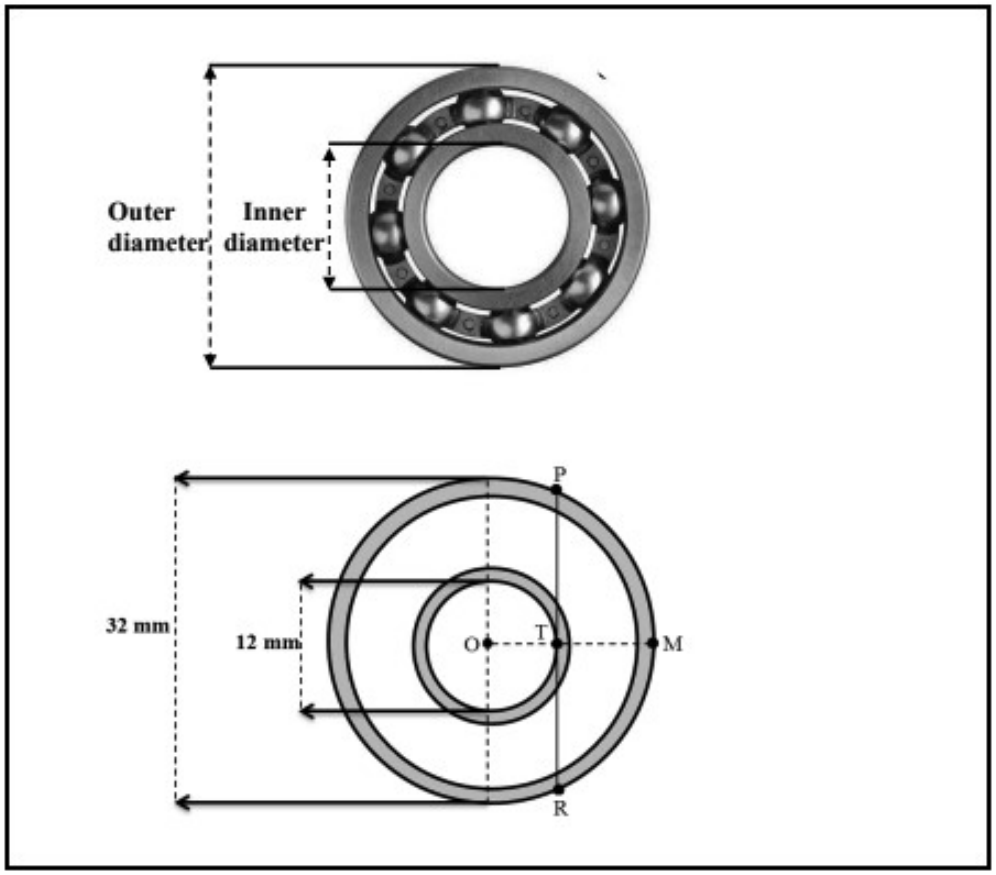
      1. i.e.: $\Delta \overline{ABC} \ \|\|\| \ \Delta \overline{ACE}$

   c. Further, expose candidates to more of these types of questions.

QUESTION 10 [Total marks 20]

QUESTION 10

10.1 Below is a picture of a bearing. The diagram below the picture models the bearing with an outer diameter of 32 mm and an inner diameter of 12 mm.
O is the centre of the circles.
P, M and R are points on the circumference of the bigger circle.
PR is a chord of the outer circle and is a tangent to the inner circle at point T.
TM is the height of segment PMR.



Determine:

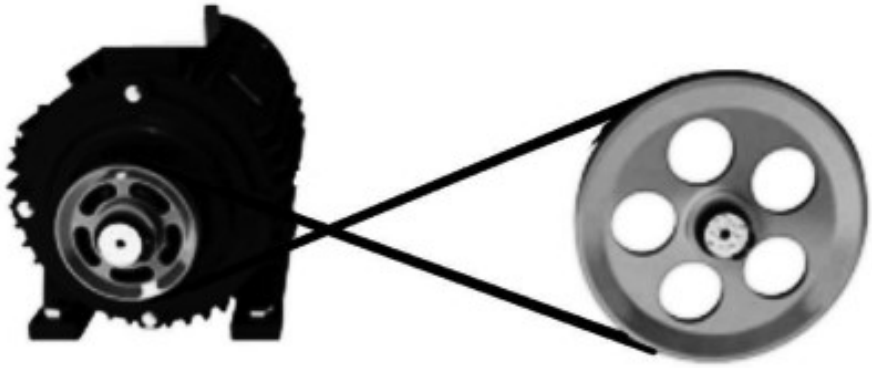
10.1.1 The circumferential velocity (in metres per second) of a particle at point M on the bearing when it rotates at 5 000 revolutions per minute (4)

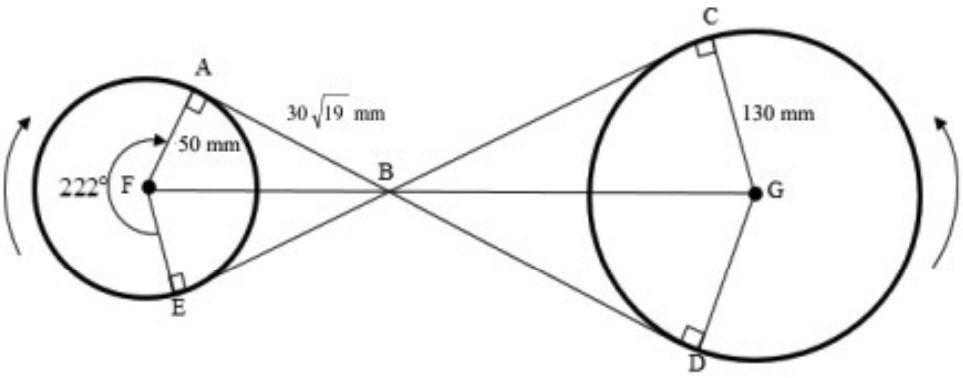
10.1.2 The length, in millimetres, of chord PR (5)

10.2

- The picture and the diagram below show a belt crossing at point B around two pulleys with centres F and G which are 500 mm apart.
- The smaller pulley is connected to a motor.
- As the smaller pulley, with centre F and a radius of 50 mm rotates, it causes the larger pulley, with centre G and a radius of 130 mm, to rotate in the opposite direction.
- Reflex $A\hat{F}E = 222°$
- $AB = 30\sqrt{19}$ mm

The diagram shows the two parallel pulleys.



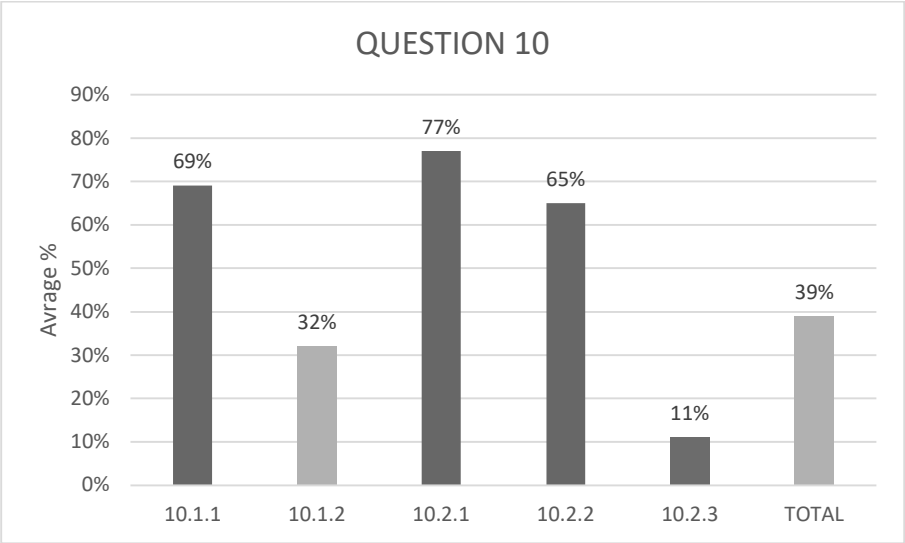


10.2.1 Convert 222° to radians. (1)

10.2.2 Calculate the area of major sector AE. (3)

10.2.3 Determine the total length of the crossed belt if the length of the major arc CD is 503 mm. (7)

[20]



- In comparison with the sample candidates from 2021, this question took a tremendous dive in performance, with only 39% of the candidates being able to answer the question in relation to 52% of candidates answering the question last year.
- 10.2.3 was the poorest performing sub-question.

Common errors and misconceptions

i.) Most candidates did not do the conversions in the Q10.1.1.

ii.) Candidates also substituting π as 180° in formula for circumferential velocity.

iii.) Candidates also mix up the formulae for circumferential velocity and angular velocity. Even though the angular velocity formula could still have been used to answer Q10.1.1, candidates are not substituting the correct information into the formula to get to the answer for circumferential velocity.

iv.) Candidates who choose correct formulae loose marks as they do not substitute the correct information into the formula and then end up calculate something completely different as to what is being assessed in the question.

v.) Q10.2.3 was the worst performing sub-question. Many candidates attempted the question but did not fully understand what was being asked of them and then they did not calculate all the necessary information to answer the question correctly.

Suggestions for improvement

Educators should focus on the following during their contact time with candidates:

i.) Basic conversions between different units (mm, cm, m, etc.); from Degrees to Radians or from Radians to Degrees, must be practiced in class to ensure candidates know these level 1 questions. Candidates cannot afford to lose marks because basic conversions cannot be done.

ii.) When using the formulae $s = r\theta$ and $Area = \frac{rs}{2} = \frac{r^2\theta}{2}$,the formula sheet clearly states that the angle (θ) must be in RADIANS. Educators should emphasise this to candidates as to many loose a mark as the final answer is then not in the correct unit. All measurements must be in the same units – candidates must select a formula and not to use both.

iii.) Emphasis must be placed on formulae for circumferential and angular velocity. Candidates are using the incorrect formula in questions. Each unknown in the formula must be explained, so that candidates substitute the correct information into the formula to find the correct answer.

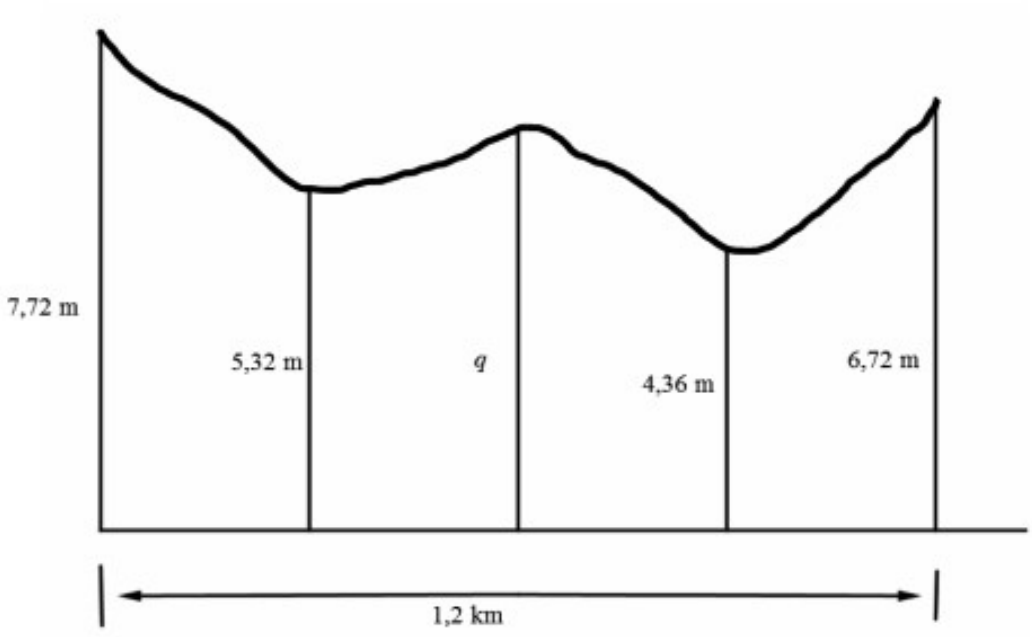
iv.) Problem solving questions in the context of Circles, Angles and Angular Movement should be done in class as part of the class work and not only focussed on during test or exam times. Incorporating these questions into classwork will help candidates become more comfortable with the questions and more of them will then attempt the questions in exams.

QUESTION 11 [Total marks 15]

QUESTION 11

11.1 A farmer has an irregular piece of land on his farm that he wants to use. He determines that one straight side of the land is 1,2 km in length.

He divides this straight side of the land into four equal segments, resulting in five different ordinates of lengths, 7,72 m, 5,32 m, q, 4,36 m and 6,72 m, as shown in the diagram below.



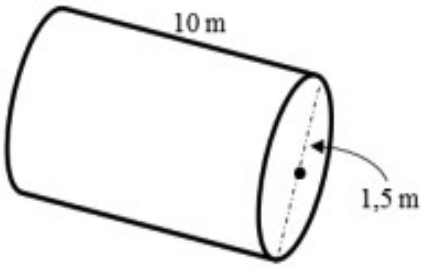
11.1.1 Convert 1,2 km to metres. (1)

11.1.2 If the area of the irregular piece of land is $6\,948\ m^2$, determine the numerical value of q. (5)

11.2 A fuel station uses a horizontal right cylindrical tank to store fuel underground.
The storage tank is filled to contain at most $68\ m^3$ of fuel (FIGURE A).
The fuel tank of a car is filled to contain at most 52 litres of fuel (FIGURE B).

- The height of the cylindrical storage tank is 10 m and the radius is 1,5 m.
- The capacity of the right cylindrical storage tank is $70{,}69\ m^3$
- The capacity of the car fuel tank is 55 litres.

FIGURE A



$V = 70{,}69\ m^3$

FIGURE B

$V = 55$ litres

Total surface area of a closed right cylinder $= 2\pi r^2 + 2\pi r h$

Volume of a right cylinder $= \pi r^2 h$

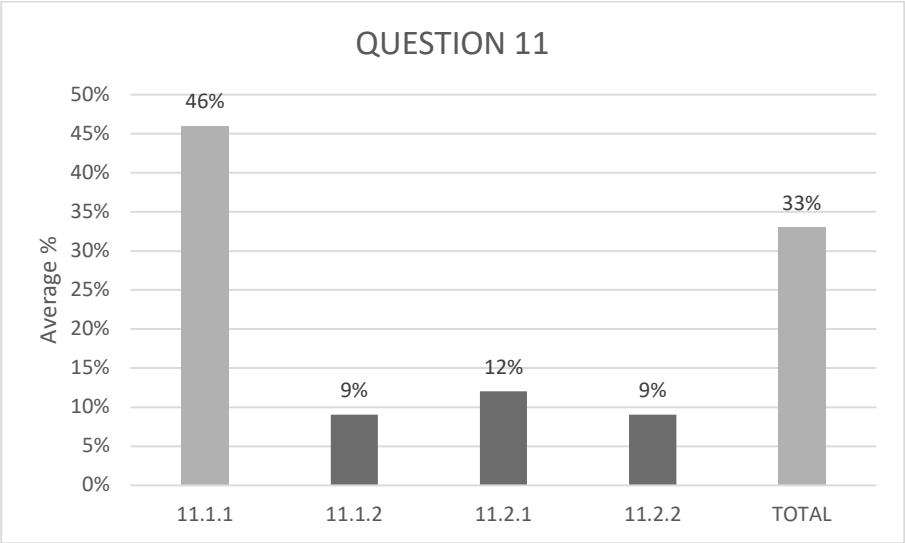
11.2.1 The right cylindrical storage tank is covered with special material to prevent leakage. The material used costs R8,93 per square metre.

Show that the cost of material used to cover the right cylindrical tank will not exceed R1 000. (4)

11.2.2 The right cylindrical storage tank and the car fuel tank have air space when filled to the given capacity.

Determine which ONE of the tanks will have a bigger percentage air space. (5)

[15]



- In this question the biggest problem for candidates was the application questions, Q11.2.1 and Q11.2.2. many candidates just left the questions out completely and those that did attempt them, messed up so badly that no marks could be awarded.
- Q11.1.2 was a huge disappointment in the way the candidates answered it, as usually this is the one question in which they score marks easily.

Common errors and misconceptions

i.) Many candidates struggled with the basic conversion from km to metres. Many divided instead of multiplied by a 1000.

ii.) Candidates are copying formulae incorrectly from the information sheet that is provided.

iii.) When substituting the width of the equal parts learners use the number of equal parts instead of the width or they make use of the total length.
   a. i.e.: 4 and 1200 is substituted instead of 300

iv.) Candidates are incorrectly simplifying. They are not applying basic algebra correctly to solve for the unknown value of q. Addition signs also become multiplication signs during simplification.
   a. i.e.: $\frac{5,32+q}{2} = 5,32q$ instead of $5,32 + q$

v.) in Q11.2.1 many candidates are using the volume formula instead of the total surface area formula. Some candidates that manage to choose the correct formula copy it correctly form the question paper but then substitute incorrectly.
   a. Candidates are getting their units confused and then substitute the price per square metre into the place of the values for height or radius.

vi.) In some instances, candidates are correctly calculating the total surface area required in Q11.2.1 yet they do not fully answer the question. They stop after calculating the TSA instead of calculating the total price to see if it will end up being less than the $R\ 1000$.

vii.) Q11.2.2 proved most difficult for candidates to answer. Many candidates guessed at the answer instead of doing calculations to prove which would have the bigger air space percentage. Many candidates simply left the question blank as well.

Suggestions for improvement

Educators should focus on the following during their contact time with candidates:

i.) Basic conversions between different units (mm, cm, m, etc.) must be practiced in class to ensure candidates know these level 1 questions. Candidates cannot afford to lose marks because basic conversions cannot be done.

ii.) The application of the mid-ordinate rule must be done in all forms so that candidates get used to not only calculating the area itself, but they must be able to calculate any value that is given as an unknown – these are easy marks to get.

i.) Expose candidates to more practical modelling problems, as in Q11.2.1 and Q11.2.2.

ii.) Educators must emphasise to candidates to copy formulas that are given CORRECTLY from the information sheet.

OVERALL COMMENT

- The overall performance of the 2022 cohort was better than in 2021. More candidates received level 6 and 7 in paper 2 than before. There were however 35 candidates who received ZERO for the question paper.
- It is disheartening to see that there are still centres receiving ZERO percent pass rate and candidates achieving single digit totals in the question paper.
- When using formulae, candidates must make sure the units are the same.
- In most cases for Technical Mathematics the angles are in radians, especially in the topic Circles, Angles and Angular Movement as provided and mentioned on the formula sheet.
- Understanding all the formulae on the formula sheet will be a big advantage to the candidates, so that they are able to identify the correct formulae to use in the questions. 9 Marks in this question paper was dedicated to simply choosing the correct formula from the information sheet, yet candidates could not do this and scored 0, where the minimum mark any candidate should have gotten was 9.
- Educators must focus on practicing level 1 and 2 questions with their candidates as many could not even score marks in these questions.
- Grade 11 work must also be thoroughly revised with candidates to ensure all marks asked on Grade 11 work can be scored. The Grade 12 curriculum for Technical Mathematics is structured in such a way that revision of previous grades work is definitely possible.
- All questions must always be attempted by candidates as consistent accuracy marking ensures that marks can be given to candidate answers even if previous answers were completely wrong or incorrect.
- No adjustment is necessary for this year's question paper and the raw marks are accepted.

basic education

Department:
Basic Education
REPUBLIC OF SOUTH AFRICA

NATIONAL SENIOR CERTIFICATE

GRADE 12

TECHNICAL MATHEMATICS P2

NOVEMBER 2022

MARKS: 150

TIME: 3 hours

This question paper consists of 18 pages and a 2-page information sheet.

TMATE2

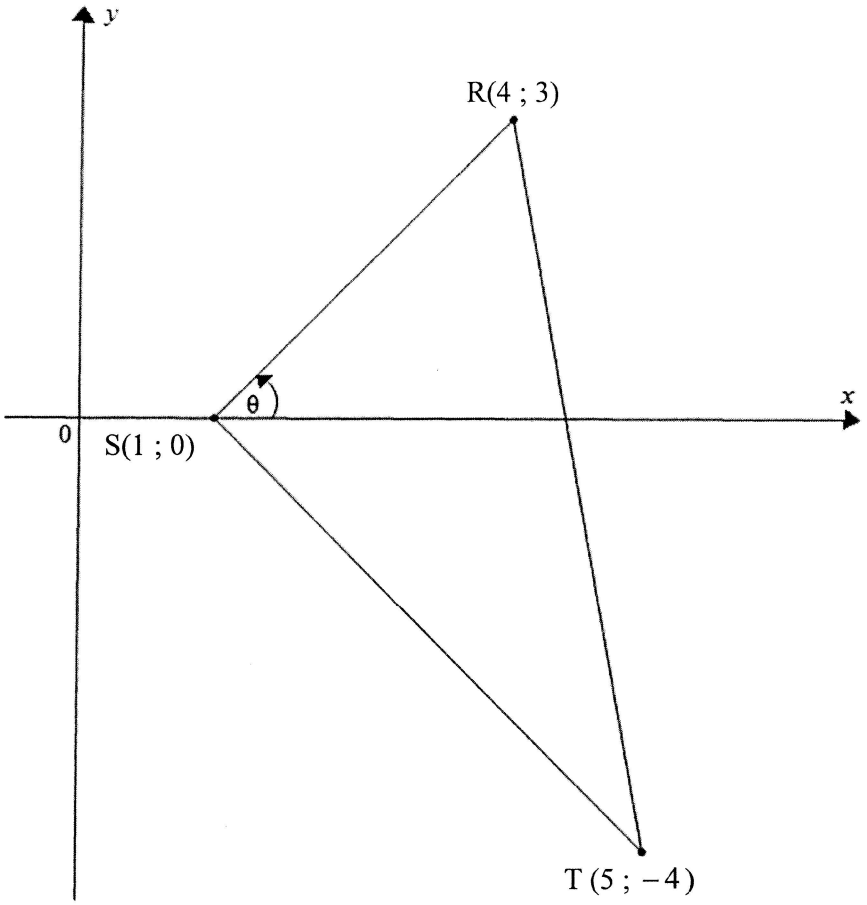
INSTRUCTIONS AND INFORMATION

Read the following instructions carefully before answering the questions.

1. This question paper consists of 11 questions.
2. Answer ALL the questions in the SPECIAL ANSWER BOOK provided.
3. Clearly show ALL calculations, diagrams, graphs, etc. that you used in determining your answers.
4. Answers only will NOT necessarily be awarded full marks.
5. If necessary, round off answers to TWO decimal places, unless stated otherwise.
6. Diagrams are NOT necessarily drawn to scale.
7. You may use an approved scientific calculator (non-programmable and non-graphical), unless stated otherwise.
8. An information sheet with formulae is included at the end of the question paper.
9. Write neatly and legibly.

QUESTION 1

In the diagram below, ΔRST with vertices R(4 ; 3), S(1 ; 0) and T(5 ; −4) is given.
The angle of inclination of RS with the positive x-axis is θ.



1.1 Determine the gradient of RS. (2)

1.2 1.2.1 Write down the formula required to calculate the angle of inclination of a line. (1)

1.2.2 Hence, determine the value of θ. (2)

1.3 Calculate the length of RT in simplified surd form. (2)

1.4 Determine the coordinates of the midpoint of ST. (2)

1.5 A line is drawn parallel to RS passing through the midpoint of ST.

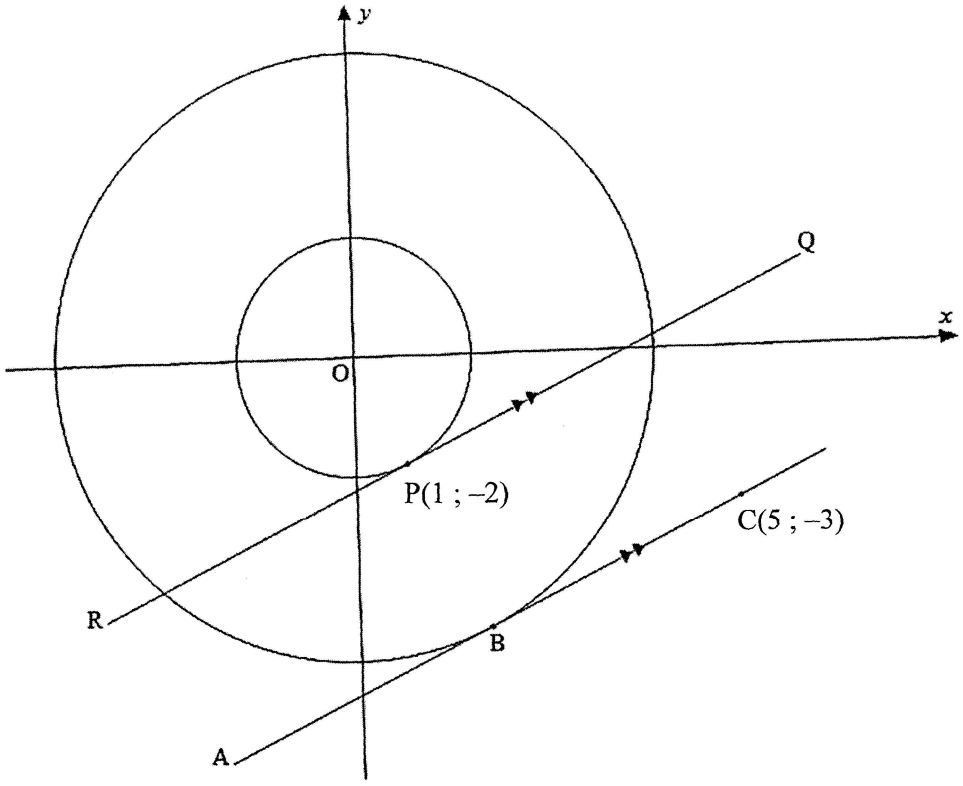
1.5.1 Complete the statement:

If two lines are parallel, then their gradients are … (1)

1.5.2 Hence, determine the equation of the line parallel to RS passing through the midpoint of ST in the form $y = \ldots$ (3)
[13]

QUESTION 2

2.1 In the diagram below, O is the centre of both the smaller and the larger circles.
RQ is a tangent to the smaller circle at point $P(1\ ;\ -2)$.
AC is a tangent to the larger circle at point B with $C(5\ ;\ -3)$.
RQ || AC



2.1.1 Determine the equation of the smaller circle. (2)

2.1.2 Write down the gradient of OP. (1)

2.1.3 Give a reason why OP is perpendicular to RQ. (1)

2.1.4 Hence, determine the gradient of AC. (2)

2.1.5 Hence, determine the equation of AC in the form $y = \ldots$ (3)

2.2 Given: $\frac{x^2}{36}+\frac{y^2}{16}=1$

2.2.1 Express the equation in the form $\frac{x^2}{a^2}+\frac{y^2}{b^2}=1$ (1)

2.2.2 Hence, sketch the graph defined by $\frac{x^2}{36}+\frac{y^2}{16}=1$ (2)
[12]

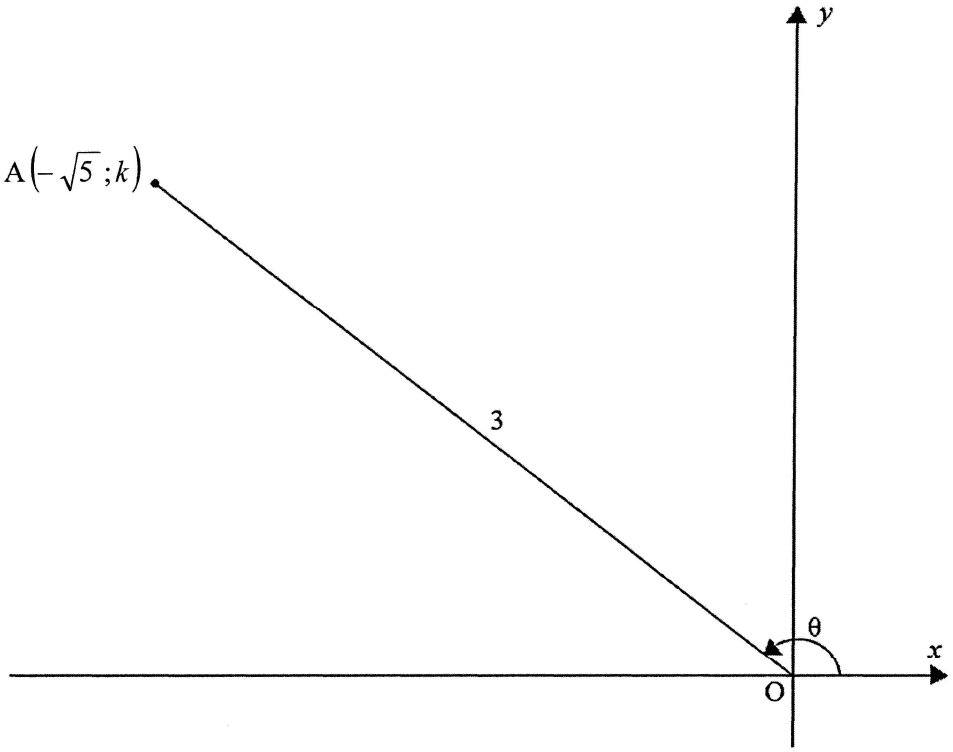
QUESTION 3

3.1 Given: $\hat{P} = \frac{2}{7}\pi$ and $\hat{Q} = 37°$

3.1.1 Convert $\frac{2}{7}\pi$ to degrees. (1)

3.1.2 Determine the value of cosec P – cos Q. (2)

3.2 In the diagram below, OA = 3 units and $A\left(-\sqrt{5}\ ;\ k\right)$ is a point on the Cartesian plane. The angle of inclination of OA with respect to the positive x-axis is θ.



Determine, **without the use of a calculator**, the numerical value of:

3.2.1 k (3)

3.2.2 $\sqrt{5}\cot\theta + 1$ (3)

3.3 Determine the value(s) of x if $3\tan x = -0,531$ and $x \in [0°;360°]$. (4)
[13]

QUESTION 4

4.1 Simplify the following:

4.1.1 $\sin(360° - \alpha)$ (1)

4.1.2 $\tan^2(\pi - \alpha)$ (2)

4.1.3 $$\frac{\sin(360° - \alpha)\cdot\tan(180° - \alpha)\cdot\text{cosec}\,(2\pi - \alpha)}{\cos(360° + \alpha)\cdot\text{cosec}\,(180° - \alpha)\cdot\tan^2(\pi - \alpha)}$$ (6)

4.2 Complete the identity: $1 - \sin^2 x = \ldots$ (1)

4.3 Prove the identity that: $\text{cosec}\,x - \sin x = \cot x \cdot \cos x$ (4)
[14]

QUESTION 5

Given the functions defined by $f(x) = \sin(x + 30°)$ and $g(x) = \cos x$ for $x \in [0°; 360°]$

5.1 Write down:

5.1.1 The period of g (1)

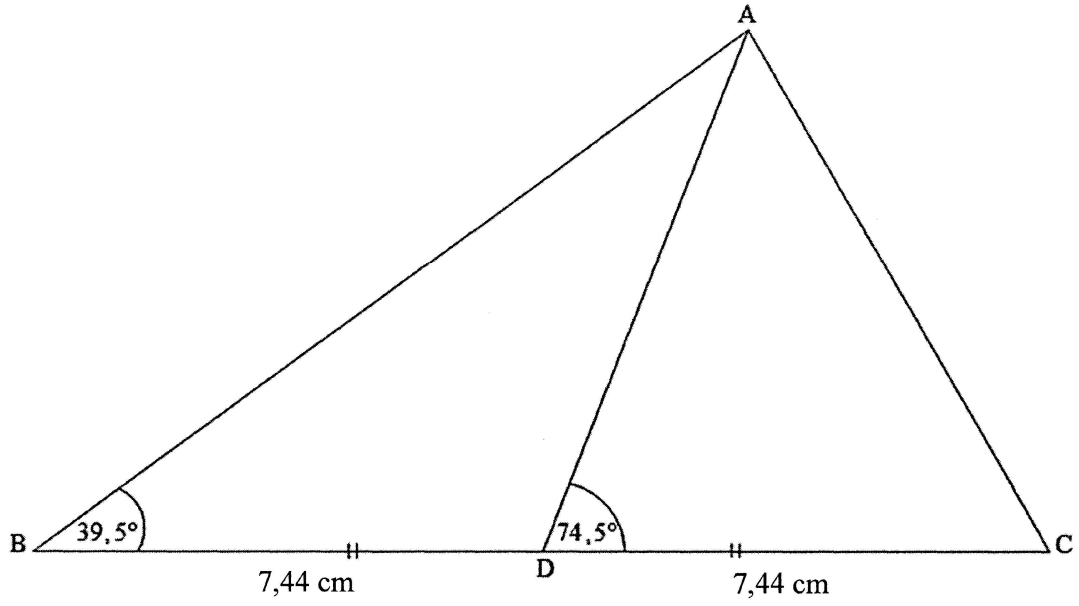
5.1.2 The amplitude of f (1)

5.2 Draw a sketch graph of f and g on the same set of axes on the grid provided in the ANSWER BOOK. Clearly indicate ALL turning points, end points and intercepts with the axes. (7)

5.3 Use the graph in QUESTION 5.2 to write down the value(s) of x for which the graph of g is increasing. (2)
[11]

QUESTION 6

In the diagram below, BDC is a straight line with BD = DC = 7,44 cm
$\hat{B} = 39{,}5°$ and $A\hat{D}C = 74{,}5°$



6.1 Determine the size of:

6.1.1 $B\hat{A}D$ (1)

6.1.2 $A\hat{D}B$ (1)

6.2 Consider ΔABD.

6.2.1 Complete the following with respect to ΔABD: $\dfrac{\cdots}{\sin B\hat{D}A} = \dfrac{BD}{\cdots}$ (1)

6.2.2 Hence, determine the length of AB. (2)

6.3 Hence, calculate the length of AC. (3)

6.4 Determine the area of ΔABC. (3)
[11]

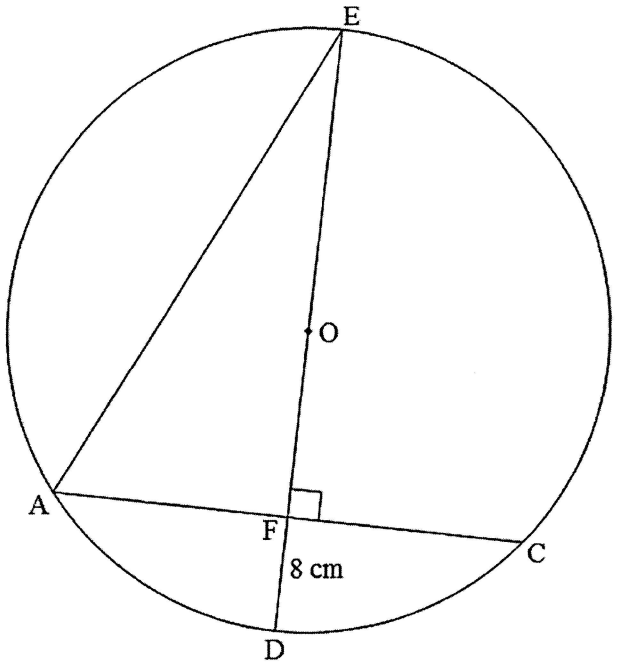
Give reasons for your statements in QUESTIONS 7, 8 and 9.

QUESTION 7

7.1 Fill in the missing word(s) in the following theorem statement:

The … of a chord passes through the centre of the circle. (1)

7.2 In the diagram below, AC and AE are chords of the circle with centre O.
Diameter ED is perpendicular to AC at F.
ED = 34 cm, FD = 8 cm and AC = 30 cm



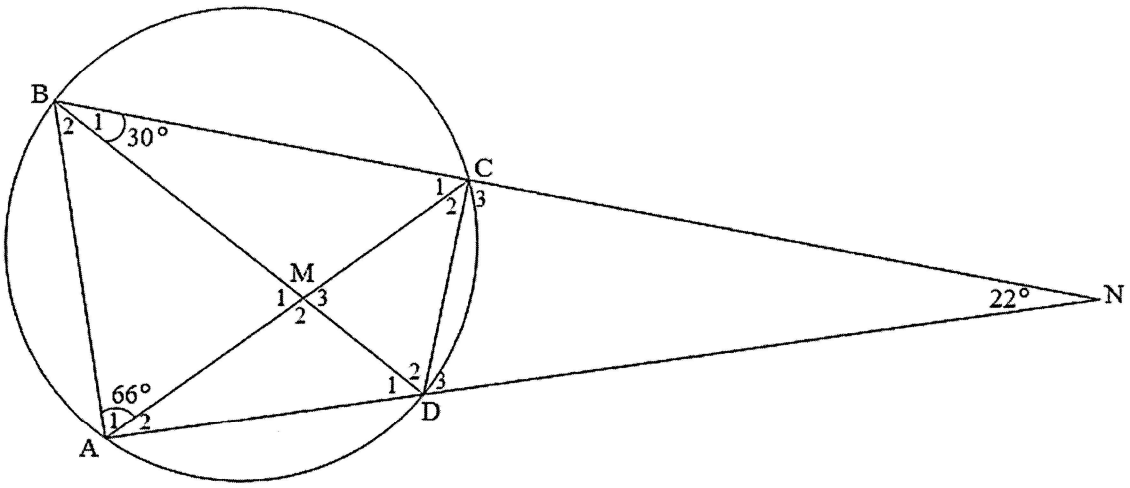
Determine, stating reasons, the length of AE. (5)
[6]

QUESTION 8

8.1 Complete the following theorem statement:

The exterior angle of a cyclic quadrilateral is equal to the … (1)

8.2 In the diagram below, ABCD is a cyclic quadrilateral. Diagonals BD and AC intersect at M. Chords BC and AD produced meet at N such that $\hat{N} = 22°$, $\hat{B}_1 = 30°$ and $\hat{A}_1 = 66°$.



8.2.1 Determine, stating reasons, the size of the following angles:

(a) $\hat{A}_2$ (2)

(b) $\hat{C}_1$ (2)

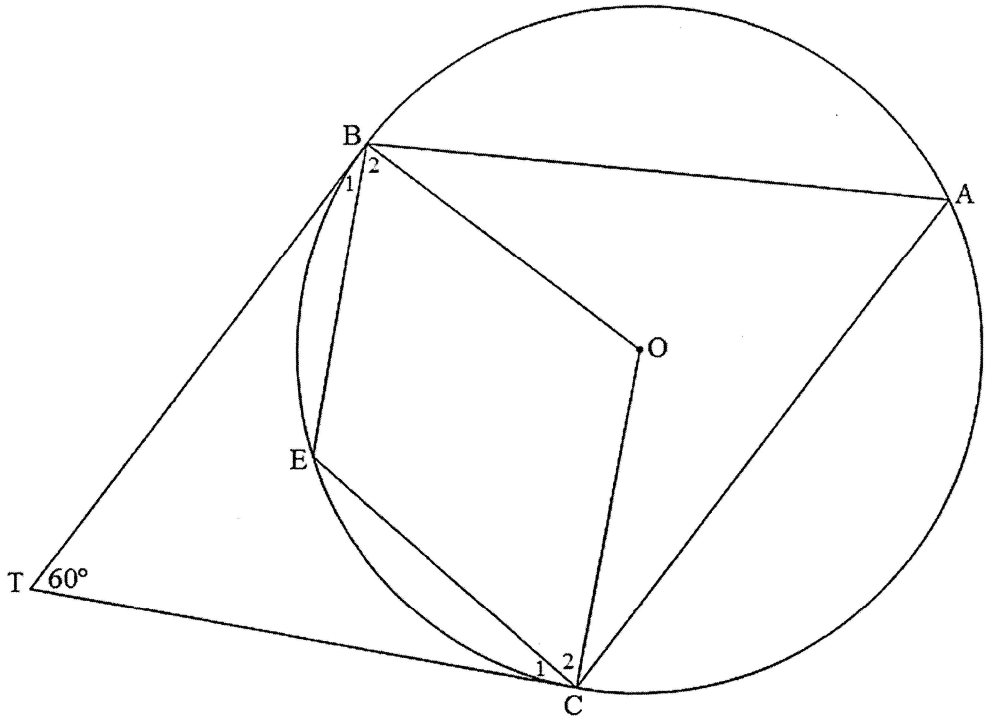
(c) $\hat{C}_3$ (2)

8.2.2 Show, stating reasons, that quadrilateral MCND is NOT a cyclic quadrilateral. (2)

8.3 In the diagram below, O is the centre of circle ABEC.

Tangents TB and TC touch the circle at B and C respectively, such that $\hat{T} = 60°$.

Radii OB and OC are drawn.



8.3.1 Write down, stating reasons, TWO angles each equal to $90°$. (3)

8.3.2 Determine, stating reasons, the size of the following angles:

(a) $\hat{A}$ (4)

(b) $\hat{E}$ (2)

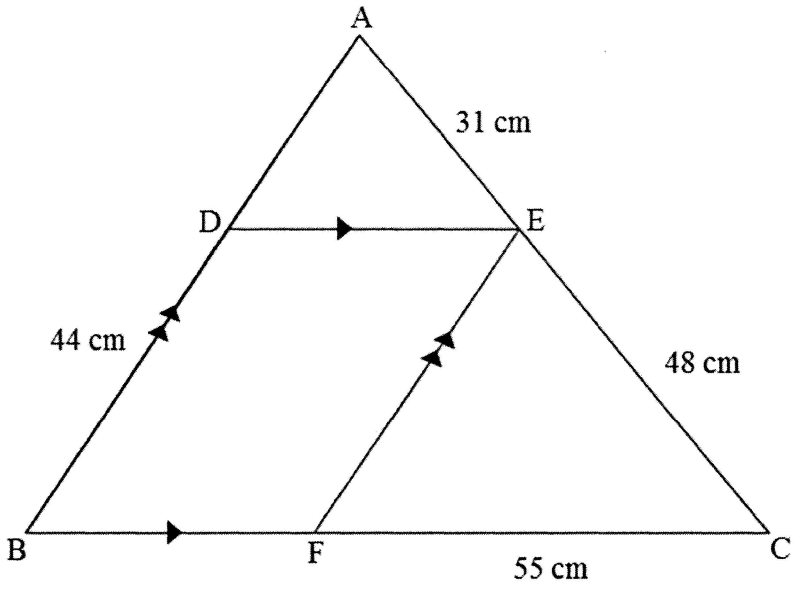
[18]

QUESTION 9

9.1 Complete the following theorem statement:

A line drawn parallel to one side of a triangle divides the other two sides … (1)

9.2 The diagram below shows ΔABC with points D, E and F on sides AB, AC and BC respectively. DEFB is a parallelogram.
AE = 31 cm, EC = 48 cm, BD = 44 cm and FC = 55 cm.

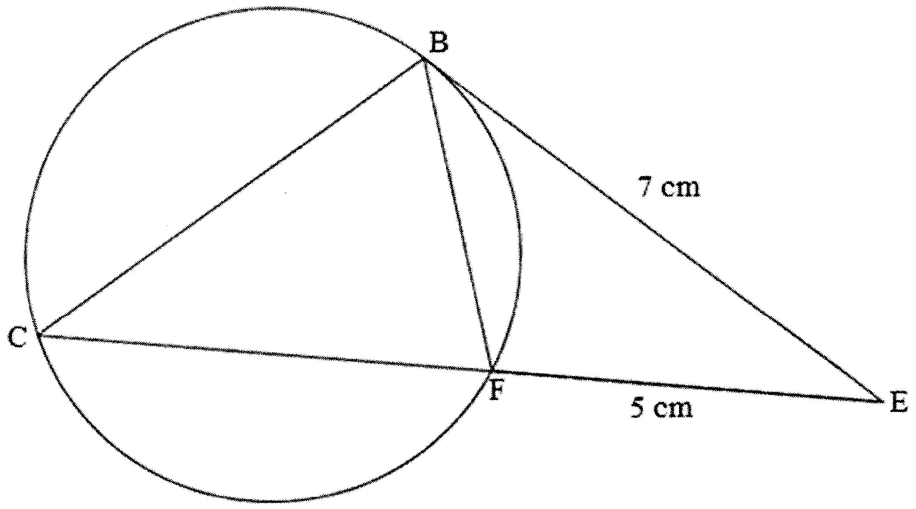


9.2.1 If DE || BF and BD || FE, state any TWO OTHER properties of the parallelogram. (2)

9.2.2 Determine, stating reasons, the length of AD. (3)

9.2.3 Determine the length of DE. (3)

9.3 In the diagram below, BE is a tangent to circle BCF at point B.
Chord CF produced meets BE at E such that BE = 7 cm and FE = 5 cm.



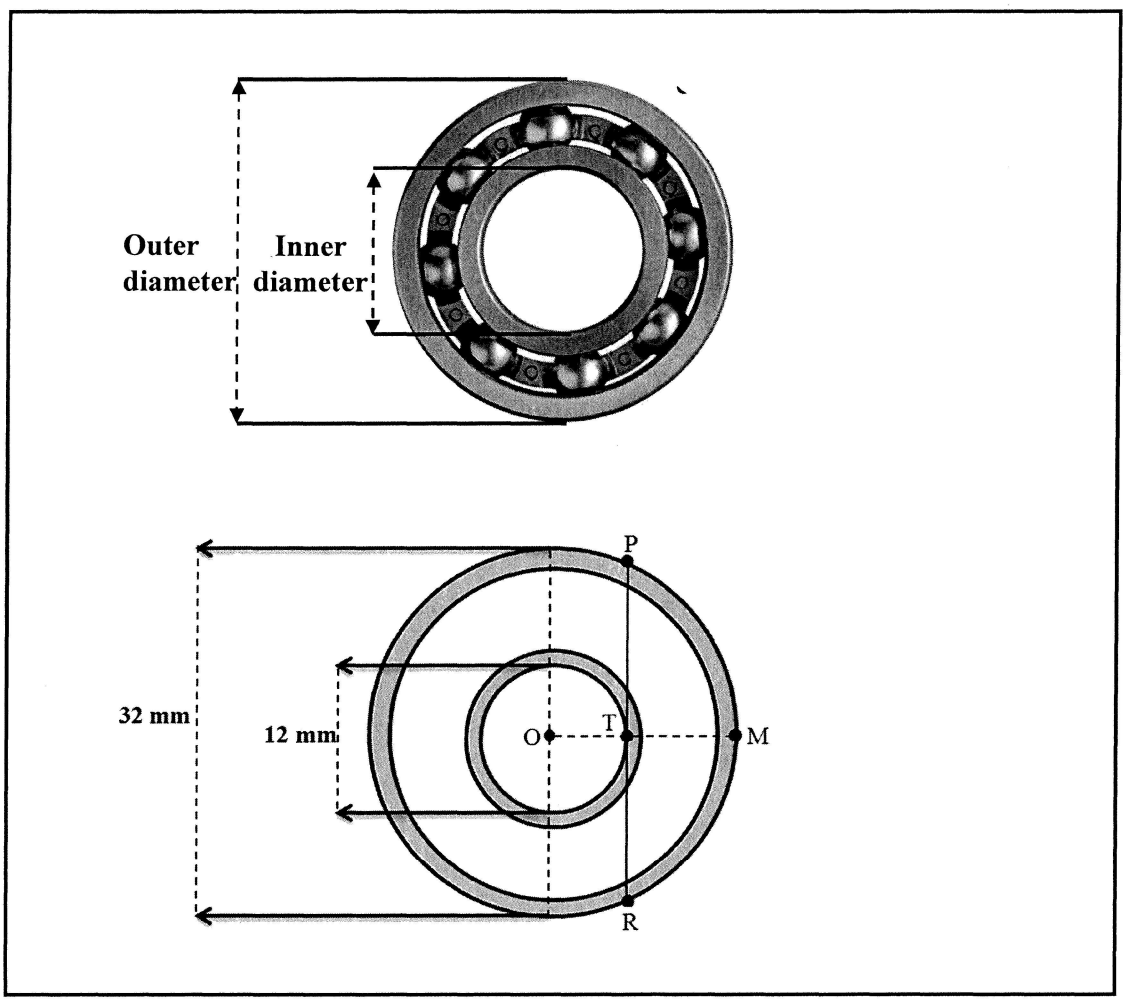
9.3.1 Prove that $\Delta EBF \;|||\; \Delta ECB$. (4)

9.3.2 Hence, deduce that $EB^2 = EC \times EF$ (1)

9.3.3 Determine the length of CF. (3)
[17]

QUESTION 10

10.1 Below is a picture of a bearing. The diagram below the picture models the bearing with an outer diameter of 32 mm and an inner diameter of 12 mm.
O is the centre of the circles.
P, M and R are points on the circumference of the bigger circle.
PR is a chord of the outer circle and is a tangent to the inner circle at point T.
TM is the height of segment PMR.



Determine:

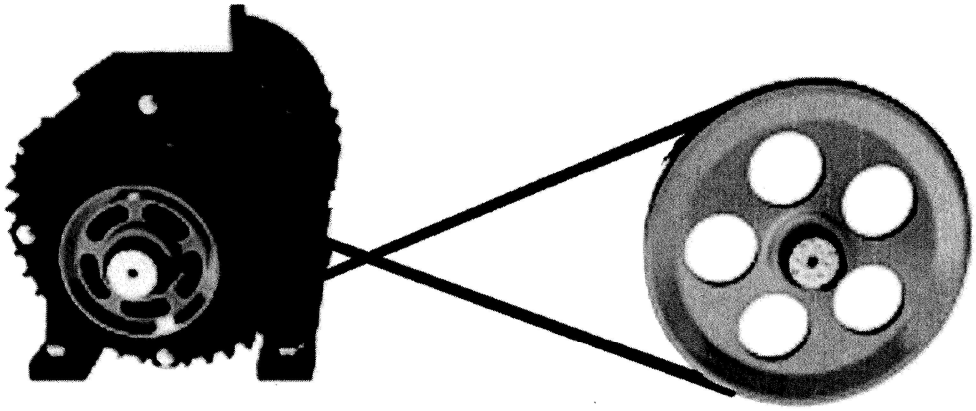
10.1.1 The circumferential velocity (in metres per second) of a particle at point M on the bearing when it rotates at 5 000 revolutions per minute (4)

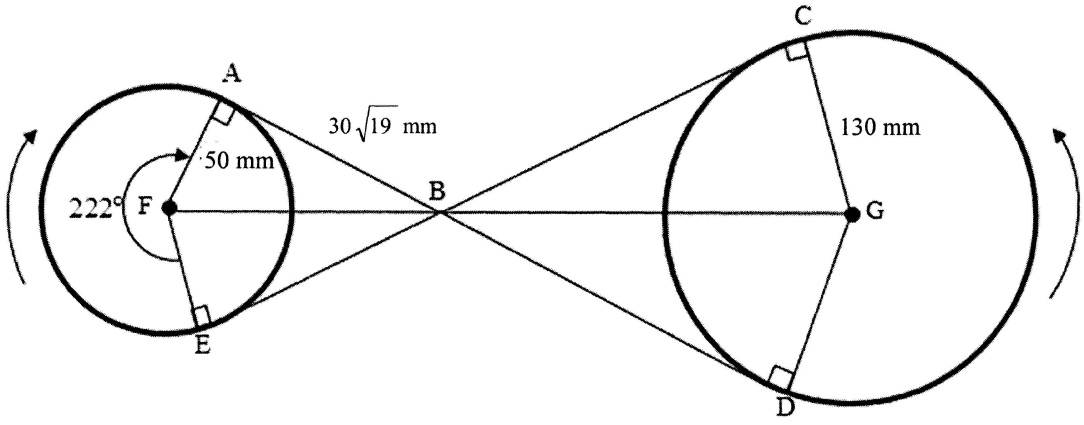
10.1.2 The length, in millimetres, of chord PR (5)

10.2

- The picture and the diagram below show a belt crossing at point B around two pulleys with centres F and G which are 500 mm apart.
- The smaller pulley is connected to a motor.
- As the smaller pulley, with centre F and a radius of 50 mm rotates, it causes the larger pulley, with centre G and a radius of 130 mm, to rotate in the opposite direction.
- Reflex $A\hat{F}E = 222°$
- $AB = 30\sqrt{19}$ mm

The diagram shows the two parallel pulleys.





10.2.1 Convert 222° to radians. (1)

10.2.2 Calculate the area of major sector AE. (3)

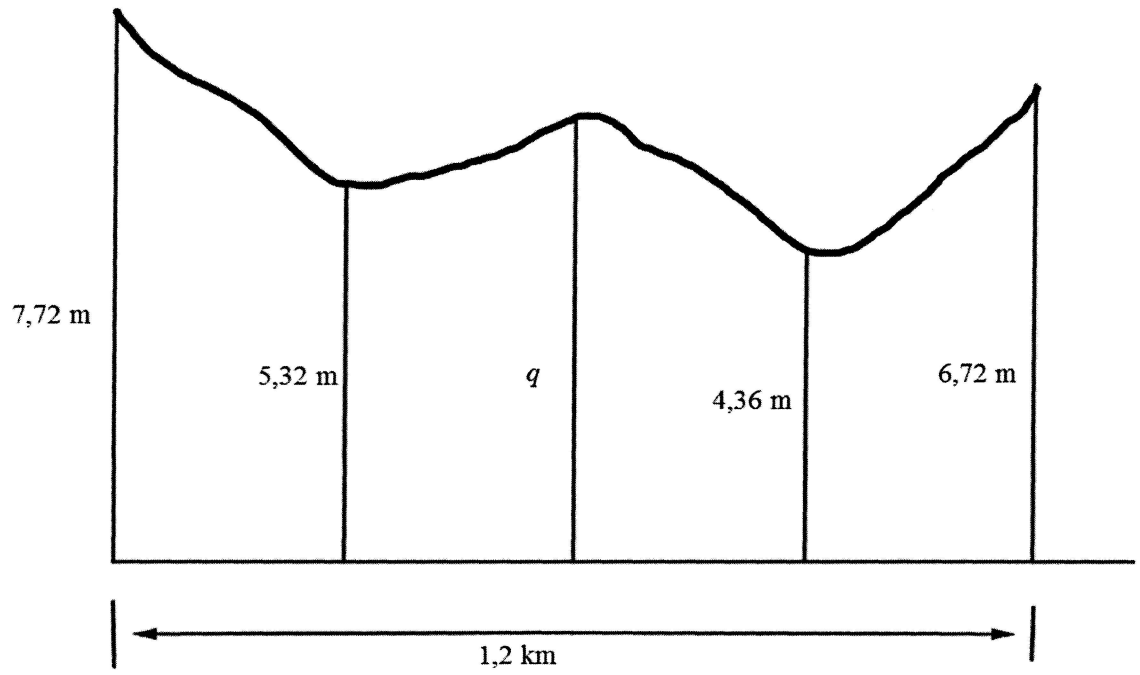
10.2.3 Determine the total length of the crossed belt if the length of the major arc CD is 503 mm. (7)

[20]

QUESTION 11

11.1 A farmer has an irregular piece of land on his farm that he wants to use. He determines that one straight side of the land is 1,2 km in length.

He divides this straight side of the land into four equal segments, resulting in five different ordinates of lengths, 7,72 m, 5,32 m, q, 4,36 m and 6,72 m, as shown in the diagram below.



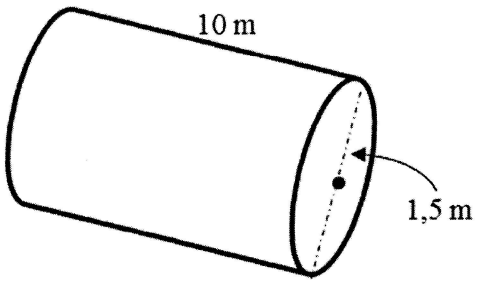
11.1.1 Convert 1,2 km to metres. (1)

11.1.2 If the area of the irregular piece of land is 6 948 m^2, determine the numerical value of q. (5)

11.2 A fuel station uses a horizontal right cylindrical tank to store fuel underground. The storage tank is filled to contain at most 68 m^3 of fuel (FIGURE A). The fuel tank of a car is filled to contain at most 52 litres of fuel (FIGURE B).

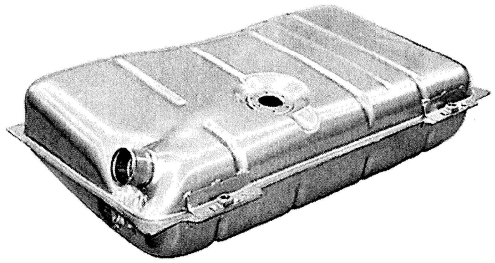
- The height of the cylindrical storage tank is 10 m and the radius is 1,5 m.
- The capacity of the right cylindrical storage tank is 70,69 m^3
- The capacity of the car fuel tank is 55 litres.

FIGURE A



$V = 70,69\ m^3$

FIGURE B



V = 55 litres

Total surface area of a closed right cylinder $= 2\pi r^2 + 2\pi r h$

Volume of a right cylinder $= \pi r^2 h$

11.2.1 The right cylindrical storage tank is covered with special material to prevent leakage. The material used costs R8,93 per square metre.

Show that the cost of material used to cover the right cylindrical tank will not exceed R1 000. (4)

11.2.2 The right cylindrical storage tank and the car fuel tank have air space when filled to the given capacity.

Determine which ONE of the tanks will have a bigger percentage air space. (5)

[15]

TOTAL: 150

INFORMATION SHEET: TECHNICAL MATHEMATICS

$$x = \frac{-b \pm \sqrt{b^2 - 4ac}}{2a} \qquad x = -\frac{b}{2a} \qquad y = \frac{4ac - b^2}{4a}$$

$$a^x = b \Leftrightarrow x = \log_a b, \quad a > 0, a \neq 1 \text{ and } b > 0$$

$$A = P(1 + ni) \qquad A = P(1 - ni) \qquad A = P(1 + i)^n \qquad A = P(1 - i)^n$$

$$i_{eff} = \left(1 + \frac{i}{m}\right)^m - 1$$

$$f'(x) = \lim_{h \to 0} \frac{f(x+h) - f(x)}{h}$$

$$\int k x^n dx = \frac{k x^{n+1}}{n+1} + C \quad , \; n, k \in \mathbb{R} \text{ with } n \neq -1 \text{ and } k \neq 0$$

$$\int \frac{k}{x} dx = k \ln x + C \quad , \; x > 0 \text{ and } k \in \mathbb{R} \; ; \; k \neq 0$$

$$\int k a^{nx} dx = \frac{k a^{nx}}{n \ln a} + C \quad , \; a > 0 \; ; \; a \neq 1 \text{ and } \; k, a \in \mathbb{R} \; ; \; k \neq 0$$

$$d = \sqrt{(x_2 - x_1)^2 + (y_2 - y_1)^2} \qquad M\left(\frac{x_2 + x_1}{2} ; \frac{y_2 + y_1}{2}\right)$$

$$y = mx + c \qquad y - y_1 = m(x - x_1) \qquad m = \frac{y_2 - y_1}{x_2 - x_1} \qquad \tan\theta = m$$

$$\frac{x^2}{a^2} + \frac{y^2}{b^2} = 1$$

In ΔABC: $\dfrac{a}{\sin A} = \dfrac{b}{\sin B} = \dfrac{c}{\sin C}$ $\qquad a^2 = b^2 + c^2 - 2bc \cdot \cos A$

$$\text{area of } \Delta ABC = \frac{1}{2} ab \cdot \sin C$$

$$\sin^2\theta + \cos^2\theta = 1 \qquad 1 + \tan^2\theta = \sec^2\theta \qquad 1 + \cot^2\theta = \text{cosec}^2\theta$$

$\pi\, rad = 180°$

Angular velocity $= \omega = 2\,\pi\, n$ where n = rotation frequency

Angular velocity $= \omega = 360°\, n$ where n = rotation frequency

Circumferential velocity $= v = \pi\, D\, n$ where D = diameter and n = rotation frequency

Circumferential velocity $= v = \omega\, r$ where ω = angular velocity and r = radius

Arc length $= s = r\theta$ where r = radius and θ = central angle in radians

Area of a sector $= \dfrac{r\,s}{2}$ where r = radius, s = arc length

Area of a sector $= \dfrac{r^2\,\theta}{2}$ where r = radius and θ = central angle in radians

$4h^2 - 4d\,h + x^2 = 0$ where h = height of segment, d = diameter of circle and x = length of chord

$A_T = a\left(m_1 + m_2 + m_3 + ... + m_n\right)$ where a = number of equal parts, $m_1 = \dfrac{o_1 + o_2}{2}$
$O_n = n^{th}$ ordinate and n = number of ordinates

OR

$A_T = a\left(\dfrac{o_1 + o_n}{2} + o_2 + o_3 + ... + o_{n-1}\right)$ where a = number of equal parts, $o_n = n^{th}$ ordinate and n = number of ordinates

basic education

Department:
Basic Education
REPUBLIC OF SOUTH AFRICA

NATIONAL SENIOR CERTIFICATE *NASIONALE SENIOR SERTIFIKAAT*

GRADE 12/*GRAAD* 12

TECHNICAL MATHEMATICS P2/*TEGNIESE WISKUNDE V2*

NOVEMBER 2022

FINAL MARKING GUIDELINES/*FINALE NASIENRIGLYNE*

MARKS/PUNTE: 150

CODE/*KODE*	EXPLANATION/*VERDUIDELIKING*
A	Accuracy/*Akkuraatheid*
AO	Answer only/*Slegs antwoord*
CA	Consistent accuracy/*Volgehoue akkuraatheid*
I	Identity/*Identiteit*
M	Method/*Metode*
NPR	No penalty for rounding/*Geen penalisering vir afronding nie*
NPU	No penalty for omitting units/*Geen penalisering vir eenhede weggelaat nie*
R	Rounding/*Afronding*
RE	Reason/*Rede*
S	Simplification/*Vereenvoudiging*
F	Formula/*Formule*
SF	Substitution in correct formula/*Vervanging in korrekte formule*
ST/RE	Statement with reason/*Bewering met rede*

These marking guidelines consist of 26 pages.*Hierdie nasienriglyne bestaan uit 26 bladsye.*

EXTERNAL/*EKSTERNE* MODERATORS	INTERNAL /*INTERNE* MODERATORS
M.A. HENDRICKS	N. TOM
MA HENDRICKS External Moderator UMALUSI	
B.J SHABANGU	N.S MUTHIGE
DATE APPROVED/DATUM GOEDGEKEUR	20 NOVEMBER 2022

Please turn over/*Blaai om asseblief*

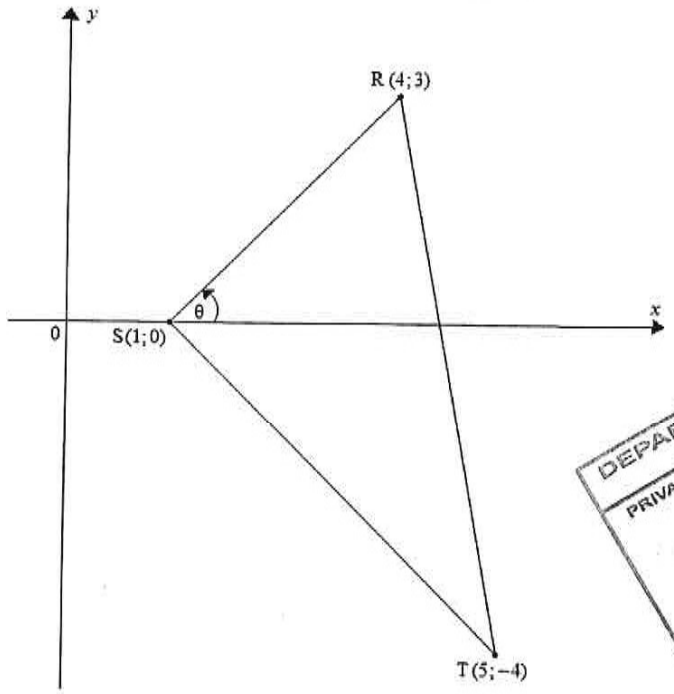
NOTE:

- If a candidate answers a question **TWICE**, only mark the **FIRST** attempt.
- Consistent Accuracy marking to be applied where indicated.
- Penalty for incorrect rounding only in QUESTION 10.2.3
- # Shows questions where Tolerance Range will be applied:. **Q 3.3; Q 4.1.3; Q 10.1.2; Q10.2.3**

LET WEL:

- *Indien 'n kandidaat 'n vraag **TWEE** keer beantwoord, sien slegs die **EERSTE** poging na.*
- *Volgehoue akkuraatheid-nasien moet toegepas word soos aangedui.*
- *Penalisering vir foutiewe afronding slegs in VRAAG 10.2.3*
- *# Toon vrae waar Toleransie wydte toegepas word:. **V 3.3; V 4.1.3; V10.1.2; V 10.2.3***

QUESTION/*VRAAG* 1



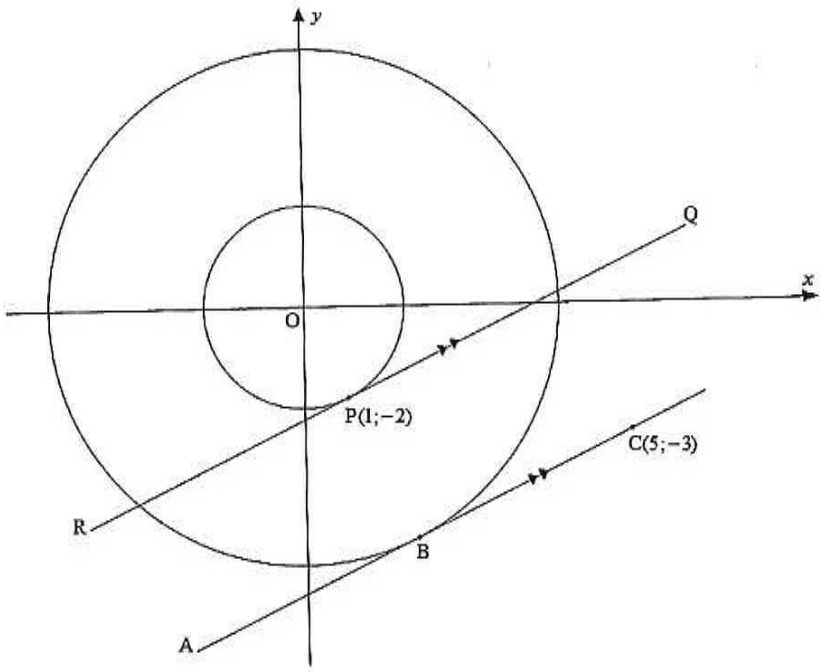
1.1	$m_{RS} = \frac{y_R - y_S}{x_R - x_S} = \frac{3-0}{4-1}$ $= 1$	✓ **SF** **A** ✓gradient **CA** **AO Full marks/ *Volpunte***	(2)
1.2.1	$m = \tan\theta$ **OR/*OF*** $\theta = \tan^{-1}(m)$	✓ **F** **A**	(1)

1.2.2	$\tan\theta = 1$ $\theta = 45°$	✓ **SF** ✓ value of / *waarde van* θ **AO Full marks/ *Volpunte***	**CA** **CA** (2)

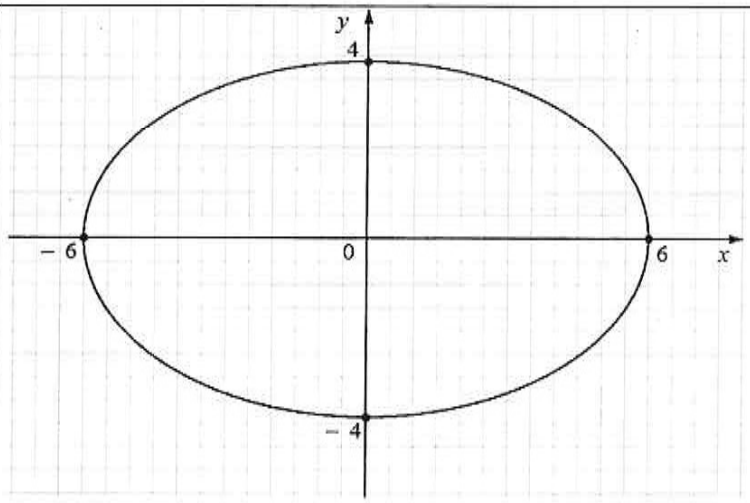
1.3	$RT = \sqrt{(x_T - x_R)^2 + (y_T - y_R)^2}$ $= \sqrt{(5-4)^2 + (-4-3)^2}$ $= \sqrt{50}$ *or / of* $5\sqrt{2}$	✓ **SF** ✓ **S** **AO Full marks/ *Volpunte***	**A** **CA** (2)
1.4	$M_{ST}\left(\frac{x_S + x_T}{2}; \frac{y_S + y_T}{2}\right)$ $= \left(\frac{5+1}{2}; \frac{-4+0}{2}\right)$ $= (3; -2)$	✓ x-value/ *waarde* ✓ y-value/ *waarde* **AO Full marks/ *Volpunte***	**A** **A** (2)
1.5.1	Equal/ the same / *gelyk / dieselfde*	✓ answer / *antwoord*	**A** (1)
1.5.2	$m_{\parallel line\ /\ lyn} = 1$ $y = 1x + c$ **OR/*OF*** $y + 2 = 1(x - 3)$ $-2 = 1\times 3 + c$ $y + 2 = x - 3$ $c = -5$ $y = x - 3 - 2$ $\therefore\ y = x - 5$	✓ gradient value/ *waarde* **CA From/*vanaf* Q1.1** ✓**SF** **CA From/*vanaf* Q1.4** ✓equation */vergelyking*	**CA** **CA** **CA** (3)
			[13]

Please turn over/*Blaai om asseblief*

QUESTION/*VRAAG* 2



2.1.1	$x^2 + y^2 = r^2$ $(1)^2 + (-2)^2 = r^2$ $r^2 = 5$ $\therefore x^2 + y^2 = 5$ **OR/*OF*** $x^2 + y^2 = (1)^2 + (-2)^2$ $= 5$ **OR/*OF*** $x^2 + y^2 = (1)^2 + (-2)^2$ $= 5$ $y = \pm\sqrt{5 - x^2}$ **OR/*OF*** $x = \pm\sqrt{5 - y^2}$	✓ **SF** **A** ✓ equation/*vergelyking* **CA** **OR/*OF*** ✓ **SF** **A** ✓ equation/*vergelyking* **CA** **OR/*OF*** ✓ **SF** **A** ✓ equation/*vergelyking* **CA** **AO Full marks/ *Volpunte*** **(2)**

2.1.2	$m_{OP} = -2$	✓ gradient of/*van* OP	**A** (1)
2.1.3	Radius/Diameter is perpendicular to tangent *Radius/middellyn is loodreg aan die raaklyn*	✓ **RE**	**A** (1)
2.1.4	$m_{RQ} = \frac{1}{2}$ $\therefore m_{AC} = \frac{1}{2}$ **OR/ *OF*** $m_{AC} \times m_{OP} = -1$ $m_{AC} \times (-2) = -1$ $m_{AC} = \frac{1}{2}$	✓ gradient of/*van* RQ ✓gradient of /*van* AC **AO Full marks/ *Volpunte***	**CA** **CA** (2)
2.1.5	$y - (-3) = \frac{1}{2}(x - 5)$ **OR**/*OF* $-3 = \frac{1}{2}(5) + c$ $y = \frac{1}{2}x - \frac{5}{2} - 3$ $c = -3 - \frac{5}{2}$ $y = \frac{1}{2}x - \frac{11}{2}$	✓ substitution/ *vervanging* ✓ **S** ✓equation/ *vergelyking*	**CA** **CA** **CA** (3)
2.2.1	$\frac{x^2}{6^2} + \frac{y^2}{4^2} = 1$	✓ standard form/*standaard vorm*	**A** (1)
2.2.2		✓*x* and *y* –intercepts/ *afsnitte* ✓elliptical shape/ *eliptiese vorm*	**CA** **CA** (2)
			[12]

QUESTION/*VRAAG* 3

3.1.1	$\frac{2}{7}\pi \text{ rad} = \frac{2}{7}\pi \times \frac{180°}{\pi} = \frac{360°}{7}$ **OR/*OF*** $\approx 51,43°$	✓ angle in degrees/*hoek in grade* **A** **NPR** (1)
3.1.2	$\text{cosec P} - \cos Q$ $= \text{cosec}\,(51,43°) - \cos(37°)$ $\approx 0,48$ **OR/*OF*** $\text{cosec P} - \cos Q$ $= \text{cosec}\left(\frac{2}{7}\pi\right) - \cos\left(37° \times \frac{\pi}{180°}\right)$ $\approx 0,48$	✓ substitution/*vervanging* **CA** ✓ **S** **CA** **OR/*OF*** ✓ substitution/*vervanging* **CA** ✓ **S** **CA** **NPR** **AO Full marks/ *Volpunte*** (2)
3.2.1	$\left(-\sqrt{5}\right)^2 + k^2 = 3^2$ $\therefore k^2 - 4 = 0$ **OR/*OF*** $k^2 = 4$ $(k-2)(k+2) = 0$ $k = \pm 2$ $k = 2$ or / *of* $k = -2$ $\therefore k = 2$	✓ substitution/*vervanging* **A** ✓ factors or square root /*faktore of vierkantswortel* **CA** ✓ correct value of/ *korrekte waarde van k* **CA** **AO Full marks/ *Volpunte*** (3)
3.2.2	$\sqrt{5}\cot\theta + 1$ *OR / OF* $\sqrt{5}\left(\frac{1}{\tan\theta}\right) + 1$ $= \sqrt{5}\left(-\frac{\sqrt{5}}{2}\right) + 1$ $\sqrt{5}\left(\frac{1}{-\frac{\sqrt{5}}{2}}\right) + 1$ $= -\frac{5}{2} + 1$ $= -\frac{3}{2}$	✓ substitution/*vervanging* **CA** ✓ **S** **CA** ✓ **S** **CA** (3)

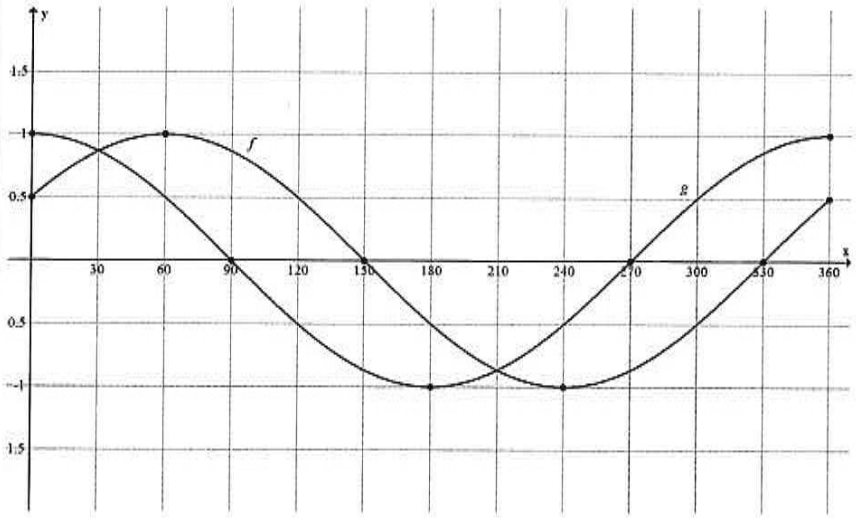
3.3	$3\tan x = -0,531$ $\tan x = -0,177$ Ref/ *verw.*$\angle \approx 10,04°$ $x \approx 180° - 10,04°$ or/*of* $x \approx 360° - 10,04°$ $\therefore x \approx 169,96°$ $\therefore x \approx 349,96°$ **OR / *OF*** $3\tan x = -0,531$ $\tan x = -0,177$ $x = 180° - \tan^{-1} 0,177$ **OR/*OF*** $x = 360° - \tan^{-1} 0,177$ $\therefore x \approx 169,96°$ $x \approx 349,96°$	✓ **S** **A** ✓ ref. Angle/*verw. hoek* **CA** ✓ both quadrants/beide kwadrante **A** ✓ both values of/*beide wrde van x* **CA** **OR / *OF*** ✓ **S** **A** ✓ S **CA** ✓ both quadrants/beide kwadrante **A** ✓ both values of/*beide wrde van x* **CA** (4)
		[13]

QUESTION/*VRAAG* 4

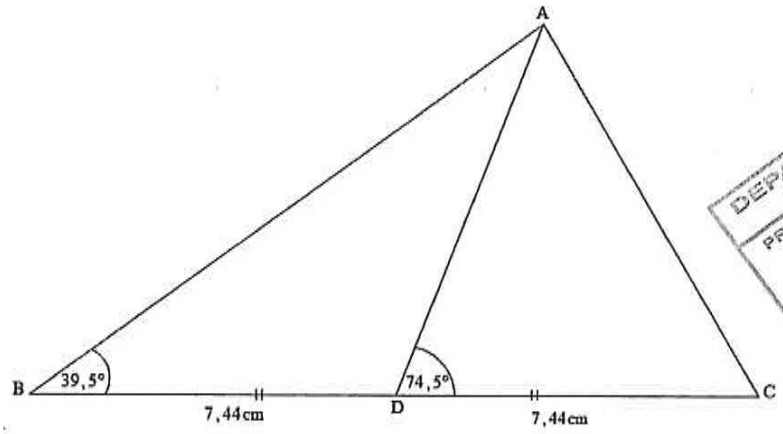
4.1.1	$-\sin\alpha$	✓ reduction/ *reduksie*	**A** (1)
4.1.2	$(-\tan\alpha)^2 = \tan^2\alpha$ **OR/*OF*** $\dfrac{\sin^2\alpha}{\cos^2\alpha}$	✓ $-\tan\alpha$ ✓ $\tan^2\alpha = \dfrac{\sin^2\alpha}{\cos^2\alpha}$ **AO Full marks/ *Volpunte***	**A** **A** (2)
4.1.3	$\dfrac{\sin(360°-\alpha)\cdot\tan(180°-\alpha)\cdot\text{cosec}(2\pi-\alpha)}{\cos(360°+\alpha)\cdot\text{cosec}(180°-\alpha)\cdot\tan^2(\pi-\alpha)}$ $= \dfrac{(-\sin\alpha)\cdot(-\tan\alpha)\cdot(-\text{cosec}\alpha)}{(\cos\alpha)\cdot(\text{cosec}\alpha)\cdot(\tan^2\alpha)}$ $= \dfrac{-\sin\alpha}{(\cos\alpha)\cdot\left(\dfrac{\sin\alpha}{\cos\alpha}\right)}$ $= \dfrac{-\sin\alpha}{\sin\alpha}$ $= -1$ **OR/*OF*** $\dfrac{\sin(360°-\alpha)\cdot\tan(180°-\alpha)\cdot\text{cosec}(2\pi-\alpha)}{\cos(360°+\alpha)\cdot\text{cosec}(180°-\alpha)\cdot\tan^2(\pi-\alpha)}$ $= \dfrac{(-\sin\alpha)\cdot(-\tan\alpha)\cdot(-\text{cosec}\alpha)}{(\cos\alpha)\cdot(\text{cosec}\alpha)\cdot(\tan^2\alpha)}$ $= \dfrac{(-\tan\alpha)(-\tan\alpha)(-1)}{(\tan^2\alpha)}$ $= -1$	**CA *From* Q4.1.1 *and* Q4.1.2** **CA *Vanuit* V4.1.1 *en* V4.1.2** ✓ $-\tan\alpha$ ✓ $-\text{cosec}\alpha$ ✓ $\cos\alpha$ ✓ $\text{cosec}\alpha$ ✓ **I** $\dfrac{\sin\alpha}{\cos\alpha}$ ✓ -1 **OR/*OF*** ✓ $-\tan\alpha$ ✓ $-\text{cosec}\alpha$ ✓ $\cos\alpha$ ✓ $\text{cosec}\alpha$ ✓ **I** $\tan\alpha$ ✓ -1	**A** **A** **A** **A** **A** **CA** **A** **A** **A** **A** **A** **CA** (6)

4.2	$\cos^2 x$	✓ **I**	**A** (1)
4.3	$\text{cosec } x - \sin x = \cot x \cdot \cos x$		
	LHS/*LK* $= \text{cosec } x - \sin x$		
	$= \frac{1}{\sin x} - \sin x$	✓ **I** $\frac{1}{\sin x}$	**A**
		✓ **S**	**CA**
	$= \frac{1-\sin^2 x}{\sin x}$		
	$= \frac{\cos^2 x}{\sin x}$	✓ **I** $\cos^2 x$	**A**
	$= \frac{\cos x}{\sin x} \cdot \cos x$	✓ **I** $\frac{\cos x}{\sin x}$	**A**
	$= \cot x \cdot \cos x = RHS \,/\, RK$		
	OR/ *OF*	**OR / *OF***	
	RHS/R*K* $= \cot x \cdot \cos x$		
	$= \frac{\cos x}{\sin x} \cdot \cos x$	✓ **I** $\frac{\cos x}{\sin x}$	**A**
	$= \frac{\cos^2 x}{\sin x}$		
		✓ **S**	**CA**
	$= \frac{1-\sin^2 x}{\sin x}$	✓ **I** $1-\sin^2 x$	**A**
	$= \frac{1}{\sin x} - \frac{\sin^2 x}{\sin x}$	✓ **I** $\frac{1}{\sin x}$	**A**
	$= \text{cosec } x - \sin x$		
	$= \text{LHS} \,/\, LK$		
	OR/ *OF*	**OR / *OF***	
	LHS/*LK* $= \text{cosec } x - \sin x$		
	$= \frac{1}{\sin x} - \sin x$	✓ **I** $\frac{1}{\sin x}$	**A**
	$= \frac{1-\sin^2 x}{\sin x}$	✓ **S**	**CA**
	$= \frac{\cos^2 x}{\sin x}$	✓ **I** $\cos^2 x$	**A**
	RHS/ *RK* $= \cot x \cdot \cos x$ $= \frac{\cos x}{\sin x} \cdot \cos x$	✓ **I** $\frac{\cos x}{\sin x}$	**A**
	$= \frac{\cos^2 x}{\sin x} = \text{LHS/ } LK$		(4)
			[14]

QUESTION/*VRAAG* 5

5.1.1	360°	✓ period/ *periode* **A** (1)
5.1.2	1	✓ amplitude **A** (1)
5.2		*f*: ✓ shape/*vorm* **A** ✓ *y*-intercept/ *afsnt* **A** ✓ *x*-intercepts/ *afsnt* **A** ✓ end point/ *eindpunt* $(360°;0,5)$ **A** *g*: ✓ shape/*vorm* **A** ✓ *y*-intercept/ *afsnt* **A** ✓ *x*-intercepts/ *afsnt* **A** (7)
5.3	$x \in (180° ; 360°)$ **OR/*OF*** $180° < x < 360°$	✓ Critical values/*kritiese waardes* **CA** ✓ correct notation/*korrekte notasie* **CA** (2)
		[11]

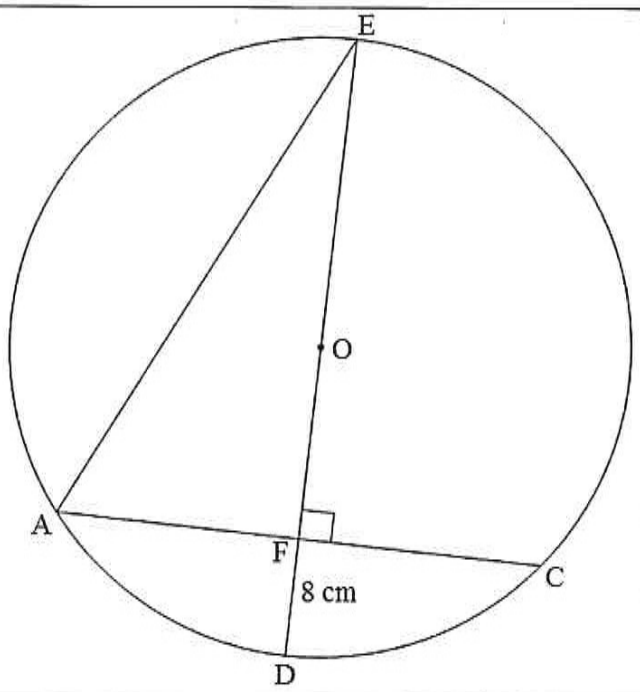
QUESTION/*VRAAG* 6



6.1.1	$B\hat{A}D = 74,5° - 39,5° = 35°$	✓ size/ *grootte* **A** (1)
6.1.2	$A\hat{D}B = 180° - 74,5° = 105,5°$ **OR/*OF*** $A\hat{D}B = 180° - 39,5° - 35° = 105,5°$	✓ size/ *grootte* **A** **OR/*OF*** ✓ size/ *grootte* **A** (1)
6.2.1	$\frac{AB}{\sin B\hat{D}A} = \frac{BD}{\sin B\hat{A}D}$ **OR/*OF*** $\frac{d}{\sin B\hat{D}A} = \frac{BD}{\sin A}$ **OR/*OF*** $\frac{d}{\sin B\hat{D}A} = \frac{BD}{\sin 35°}$	✓ Complete sine rule/*voltooi sinus- reël* **A** (1)
6.2.2	In ΔABD: $\frac{AB}{\sin 105,5°} = \frac{7,44}{\sin 35°}$ $AB = \frac{7,44\sin 105,5°}{\sin 35°}$ $\approx 12,5\,cm$ **OR/ *OF*** In ΔADB: $\frac{AD}{\sin 39,5°} = \frac{7,44}{\sin 35°}$ $AD = \frac{7,44\sin 39,5°}{\sin 35°} \approx 8,25\,cm$ $AB^2 = AD^2 + BD^2 - 2\times AD\times BD\times \cos D$ $AB^2 = 8,25^2 + 7,44^2 - 2\times 8,25\times 7,44\times \cos 105,5°$ $AB \approx 12,5$ cm	✓ **SF** **CA** ✓ length of/ *lengte van* AB **CA** **OR/*OF*** ✓ length of/ *lengte van* AD **CA** ✓ length of/ *lengte van* AB **CA**

		(2)
6.3	In ΔABC: $AC^2 = BC^2 + AB^2 - 2BC \times AB\cos B$ $= 14,88^2 + 12,5^2 - 2(14,88)(12,5)\cos 39,5°$ $= 90,62005498$ $\therefore AC \approx 9,52$ cm **OR / *OF*** In ΔABD: $\frac{AD}{\sin 39,5°} = \frac{7,44}{\sin 35°}$ $AD = \frac{7,44\sin 39,5°}{\sin 35°} \approx 8,25$ cm $\therefore$ In ΔADC: $AC^2 = AD^2 + DC^2 - 2AD \times DC \times \cos D$ $AC^2 = 8,25^2 + 7,44^2 - 2 \times 8,25 \times 7,44 \times \cos 74,5°$ $AC^2 = 90,60991695$ $\therefore AC \approx 9,52$ cm	✓ **M** cosine rule/*reël* **A** ✓ **SF** **CA** ✓ length of/ *lengte van* AC **CA** **OR / *OF*** ✓ length of AD **A** ✓ **M** cosine rule/*reël* **A** ✓ length of/ *lengte van* AC **CA** (3)
6.4	Area of/*Oppervlakte van* $\Delta ABC = \frac{1}{2}AB \times BC\sin B$ $= \frac{1}{2}(12,5)(14,88)\sin 39,5°$ $\approx 59,16$ cm^2 **OR/OF** Height/*Hoogte* $= 12,5\sin 39,5° \approx 7,95$ $= \frac{1}{2}bh = \frac{1}{2}AC \times h$ $\approx \frac{1}{2}(14,88)(7,95)$ $\approx 59,16$ cm	✓ **M** area rule/ *oppervlakte reël* **A** ✓ **SF** **CA** ✓ Area / *oppervlakte* **CA** **OR/OF** ✓ **M** area rule/ *oppervlakte reël* **A** ✓ **SF** **CA** ✓ Area / *oppervlakte* **CA** (3)
		[11]

QUESTION/*VRAAG* 7

7.1	Perpendicular bisector / *middelloodlyn*...	✓	**A** (1)
7.2			
	$AF = 15$ cm (line from centre ⊥ chord / *lyn vanaf middelpt* ⊥ *koord*)	✓ **ST** ✓ **RE**	**A** **A**
	$EF = 34\text{ cm} - 8\text{ cm} = 26\text{ cm}$	✓ **ST** length of / *lengte van* EF	**A**
	$AE = \sqrt{AF^2 + EF^2} = \sqrt{26^2 + 15^2}$ (Pythagoras) $= \sqrt{901}$ or / *of* $\approx 30,02$ cm	✓ **ST** ✓ **ST**	**CA** **CA**
	OR / *OF*	**OR / *OF***	
	$AF = 15$ cm (line from centre ⊥ chord / *lyn vanaf middelpt* ⊥ *koord*)	✓ **ST** ✓ **RE**	**A** **A**
	$EF = 34\text{ cm} - 8\text{ cm} = 26\text{ cm}$	✓ **ST** length of / *lengte van* EF	**A**
	$AE^2 = AF^2 + EF^2 - 2(AE)(EF)\cos 90°$ $= (15)^2 + (26)^2 - 2(15)(26)\cos 90°$	✓ **ST**	**CA**
	$AE = \sqrt{901}$ or / *of* $\approx 30,02\,cm$	✓ **ST**	**CA**
	OR / *OF*	**OR / *OF***	

	$AF = 15$ cm $\begin{pmatrix}\text{line from centre } \perp \text{ chord} / \\ \textit{lyn vanaf middelpt } \perp \textit{koord}\end{pmatrix}$	✓ **ST** ✓ **RE**	**A** **A**
	$EF = 34\text{ cm} - 8\text{ cm} = 26\text{ cm}$ $\tan E = \frac{AF}{FE} = \frac{15}{26}$	✓ **ST** length of / *lengte van* EF	**A**
	$\hat{E} \approx 29{,}98°$	✓ **ST**	**CA**
	$\sin E = \frac{AF}{AE}$ $AE \approx \frac{15}{\sin 29{,}98°}$ $\approx 30{,}02$ cm	✓ **ST**	**CA**
			(5)
			[6]

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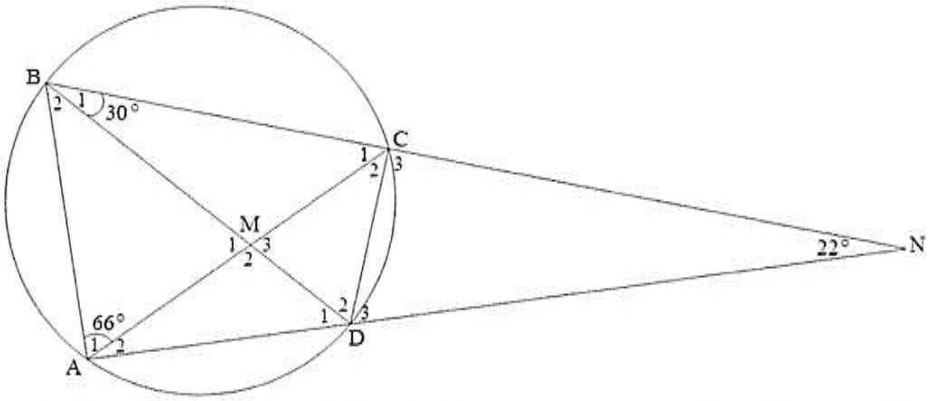
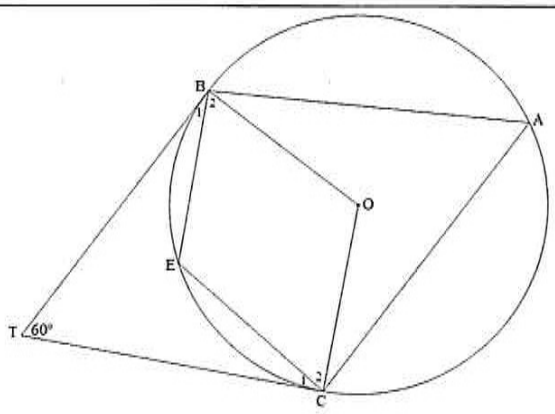
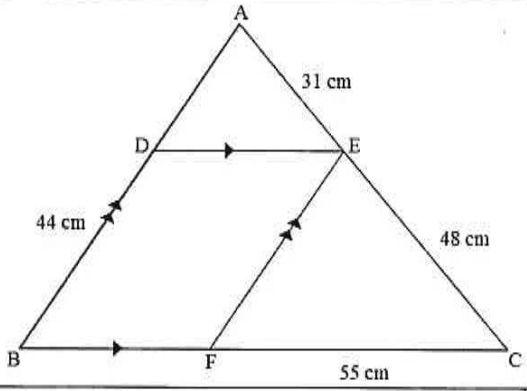
QUESTION/*VRAAG* 8

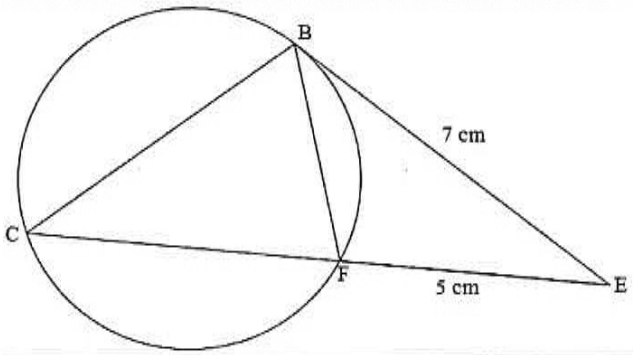
8.1	Interior opposite angle / *teenoorstaande binnehoek*	✓	**A** (1)
8.2			
8.2.1 a)	$\hat{A}_2 = 30°$ (*∠s in the same segment / ∠e in dieselfde segment*)	✓ **ST** **A** ✓ **RE** **A**	(2)
8.2.1 b)	$\hat{C}_1 = 52°$ (ext ∠ of Δ / *buite ∠ van Δ*)	✓ **ST** **CA** ✓ **RE** **A**	(2)
8.2.1 c)	$\hat{C}_3 = 96°$ (ext ∠ of cyclic quad / *buite ∠ van kdvh*)	✓ **ST** **CA** ✓ **RE** **A**	(2)
8.2.2	$\hat{M}_3 = 82°$ (ext ∠ of Δ / *buite ∠ van Δ*)	✓ **ST** **CA**	
	$\hat{M}_3 + \hat{N} = 82° + 22° = 104° \neq 180°$ ∴ MCND is NOT cyclic / *NIE siklies NIE* (opp∠s NOT suppl/ *teenoorst ∠e NIE suppl*)	✓ **RE** **A**	
	OR/ *OF*	**OR/ *OF***	
	From/*vanaf* Q/V8.2.1 b) $\hat{C}_1 = 52° \neq M\hat{D}N = 128°$ ∴ MCND is NOT cyclic / *NIEsiklies NIE* (ext ∠≠ int opp ∠/ *buite ∠e ≠ teen.binne ∠*)	✓ **ST** **CA** ✓ **RE** **A**	
	OR/ *OF*	**OR/ *OF***	
	$\hat{M}_2 = 98° \neq \hat{N} = 22°$ ∴ MCND is NIE cyclic / *NIE siklies NIE* (ext ∠≠ int opp ∠/ *buite ∠e ≠ teen.binne ∠*)	✓ **ST** **CA** ✓ **RE** **A**	(2)

8.3		
8.3.1	$O\hat{B}T = 90°$ $O\hat{C}T = 90°$ tan/*raaklyn* ⊥ rad	✓ $O\hat{B}T$ **A** ✓ $O\hat{C}T$ **A** ✓ **RE**..... ....**A** (3)
8.3.2 a)	$B\hat{O}C = 120°$ (sum of int ∠s of a quad / *som vd binne ∠e van 'n vhk*) $\hat{A} = 60°$ (∠at centre = 2 × ∠ at circum / *midpts ∠ = 2 × omtreks ∠*) **OR/ *OF*** TBOC is cyclic/*siklies* (CONVERSE opp ∠s of cyclic quad / *OMGEKEERDE teenoorst ∠e van kdvh*) $B\hat{O}C = 120°$ (opp ∠s of cyclic quad / *teenoorst ∠e van kdvh*) $\hat{A} = 60°$ (∠ at centre = 2 × ∠ at circum / *midpts ∠ = 2 × omtreks ∠*)	✓ **ST** **A** ✓ **RE** **A** ✓ **ST** **CA** ✓ **RE** **A** **OR/ *OF*** ✓ **RE** **A** ✓ **ST** **A** ✓ **ST** **CA** ✓ **RE** **A** (4)
8.3.2 b)	$\hat{E} = 120°$ (opp ∠s of cyclic quad / *teenoorst ∠e van kdvh*) **OR/ *OF*** reflex /*inspringend* $B\hat{O}C = 240°$ (Revolution /*omwenteling*) $\hat{E} = 120°$ (∠ at centre = 2 × ∠ at circm / *midpts∠ = 2 × omtreks∠*)	✓ **ST** **CA** ✓ **RE** **A** **OR/ *OF*** ✓ **RE** **A** ✓ **ST** **CA** (2)
		[18]

QUESTION/*VRAAG* 9

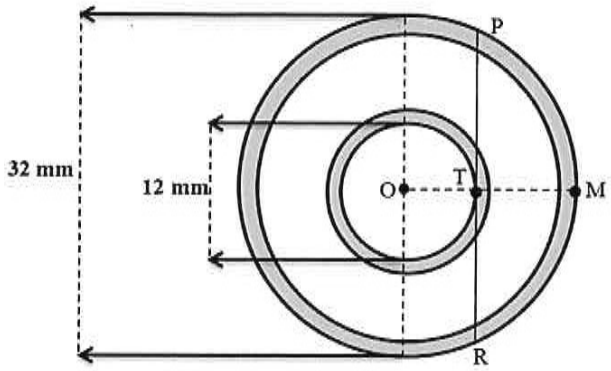
9.1	In proportion/ Proportionally / *In verhouding/ eweredig*	✓	**A** (1)
9.2			
9.2.1	• Both pairs of opposite sides are equal / *beide pare teenoorst sye is gelyk* • Both pairs of opposite angles are equal / *beide pare teenoorst hoeke is gelyk* • Diagonals bisect each other / *Hoeklyne halveer mekaar*	✓ **RE** ✓ **RE** Any two / *enige twee*	**A** **A** (2)
9.2.2	$\frac{AD}{DB}=\frac{AE}{EC}$ (prop th / *ewer st*; DE ‖ BC **OR** / *OF*; line ‖ one side of Δ / *lyn* ‖ *een sy van* Δ) $\therefore \frac{AD}{44}=\frac{31}{48}$ $\therefore AD=\frac{341}{12}$ or / *of* $\approx 28{,}42$ cm **OR/*OF*** $\frac{AD}{AB}=\frac{AE}{AC}$ (prop th /*ewer st*; DE ‖BC **OR/*OF***; line ‖ one side of Δ/ *lyn* ‖ *eensy van* Δ) $\frac{AD}{AD+44}=\frac{31}{79}$ $79\,AD = 31\,AD + 1364$ $48\,AD = 1364$ $AD=\frac{1364}{48}$ or / *of* $\approx 28{,}42$ cm	✓ **ST** ✓**RE** ✓ **ST** **OR/*OF*** ✓ **ST** ✓**RE** ✓ **ST**	**A** **A** **OR/*OF*** **A** **CA** (3)

9.2.3	$\frac{BF}{FC}=\frac{AE}{EC}$ (prop th / *ewer st*; FE ∥ AB **OR** / ***OF*** line ∥ one side of Δ / *lyn* ∥ *een sy van* Δ)	
	$\frac{BF}{55}=\frac{31}{48}$	✓ **ST** proportion/eweredig **A**
	$BF=\frac{31\times 55}{48}=\frac{1705}{48}$	✓ **ST** value of/ waarde van BF **CA**
	$DE=\frac{1705}{48}$ or / *of* $\approx 35{,}52\,\text{cm}$	✓ **ST** value of/ waarde van DE **CA**
	OR/*OF*	**OR/*OF***
	$\frac{BC}{FC}=\frac{AC}{EC}$ (prop th / *ewer st*; FE ∥ AB OR/OF line ∥ one side of Δ / *lyn* ∥ *een sy van* Δ)	✓ **ST** proportion/eweredig **A**
	$\frac{BC}{55}=\frac{79}{48}$	
	$\therefore BC=\frac{4345}{48}$	
	$\therefore BF=\frac{1705}{48}\,\text{cm}$	✓ **ST** value of/ waarde van BF **CA**
	$\therefore DE=\frac{1705}{48}$ or / *of* $\approx 35{,}52\,\text{cm}$	✓ **ST** value of/ waarde van DE **CA**
	OR / *OF*	**OR / *OF***
	$\frac{DE}{BC}=\frac{AE}{AC}$ (∥∥ Δ's/*e*)	✓ **ST** proportion/eweredig **A**
	$\frac{DE}{DE+55}=\frac{31}{79}$	✓ **SF** **CA**
	$79DE=31DE+1705$	
	$48DE=1705$	
	$DE=\frac{1705}{48}$ or / *of* $\approx 35{,}52\,\text{cm}$	✓ **ST** value of/ waarde van DE **CA** (3)

9.3			
9.3.1	In Δ EBF and/*en* Δ ECB : $\hat{E}$ is common / *gemeen* $E\hat{B}F = E\hat{C}B$ (tan - chord / *raaklyn - koord*) $\therefore B\hat{F}E = C\hat{B}E$ (int $\angle$s of Δ / *binne* $\angle$*e van* Δ) $\therefore \Delta EBF \;\|\|\| \Delta ECB$ $(\angle\angle\angle)$ **OR / *OF*** In Δ EBF and/*en* Δ ECB : $\hat{E}$ is common / *gemeen* $E\hat{B}F = E\hat{C}B$ (tan - chord / *raaklyn - koord*) $\therefore \Delta EBF \;\|\|\| \Delta ECB$ $(\angle\angle\angle)$	✓ **ST** **A** ✓ **ST** **A** ✓**RE** **A** ✓ **ST / RE** **A** **OR / *OF*** ✓ **ST** **A** ✓ **ST** **A** ✓**RE** **A** ✓ **ST/RE** **A** (4)	
9.3.2	$\frac{EB}{EC} = \frac{EF}{EB}$ $\therefore EB^2 = EF \times EC$	✓ **ST** proportion/ *eweredigheid* **A** (1)	
9.3.3	from/*vanuit* 9.3.2 $\therefore 7^2 = (CF+5).5$ $\therefore CF + 5 = \frac{49}{5}$ $\therefore CF = 4,8$ cm **OR/*OF*** $\therefore 7^2 = EC \times 5$ $\therefore EC = 9,8$ $\therefore CF = EC - 5$ $= 9,8 - 5 = 4,8$ cm **OR/*OF*** $\therefore 7^2 = (CF+5).5$ $\therefore 49 = 5CF + 25$ $\therefore 5CF = 24$ $\therefore CF = \frac{24}{5}$ $\therefore CF = 4,8$ cm	✓ **ST** EC = CF + 5 **A** ✓ **ST** substitution/ *vervanging* **CA** ✓ **ST** length of CF/ *lengte van CF* **CA** **OR / *OF*** ✓ **ST** $7^2 = EC \times 5$ **A** ✓ **ST** length of EC/ *lengte van EC* **CA** ✓ **ST** length of CF/ *lengte van CF* **CA** (3)	
			[17]

Please turn over/*Blaai om asseblief*

QUESTION/*VRAAG* 10

10.1		
10.1.1	$D = 32 \div 1000 = 0,032\,\text{m}$ $n = \frac{5000}{60} = \frac{250}{3}$ *or / of* 83,33 rev/s / *omw/s* $v = \pi D n$ $= \pi \times 0,032 \times \frac{250}{3}$ $= \frac{8}{3}\pi$ or / *of* $\approx 8,38$ m/s **OR/*OF*** $v = \pi D n$ $= \pi \times 32 \times 5000$ $= 160\,000\pi$ mm/ min $\approx \frac{160\,000}{60\,000}\pi = \frac{8}{3}\pi$ or / *of* $\approx 8,38$ m/s **OR/*OF*** $v = \omega r$ $= \frac{2\pi \times 5000 \times 0,016}{60}$ $= \frac{8}{3}\pi$ or / *of* $\approx 8,38$ m/s	✓**M** both Conversions/ *beide herleidings* **A** ✓**F** **A** ✓**SF** **CA** ✓circ.velocity/ *omtrksnelhd***CA** **OR/*OF*** ✓**F** **A** ✓**SF** **CA** ✓**M** both Conversion/ *beide herleidings* **A** ✓circ.velocity/ *omtrksnelhd***CA** **OR/*OF*** ✓**F** **A** ✓**SF** **CA** ✓**M** both Conversion/ *beide herleidings* **A** ✓circ.velocity/ *omtrksnelhd***CA** **NPU/NPR** (4)

Please turn over/*Blaai om asseblief*

10.1.2	Radius of the larger circle/ *Radius van die grootter sirkel* = 16mm Radius of the smaller circle/ *Radius van die kleiner sirkel* = 6 mm $TM = OM - OT$ $= 16 - 6$ **OR/*OF*** $TM = \frac{32-12}{2}$ $= 10\,mm$ $= 10\,mm$ $h = 10\,mm$ and/*en* $d = 32\,mm$ $4h^2 - 4dh + x^2 = 0$ $4(10)^2 - 4(32)(10) + x^2 = 0$ $400 - 1280 + x^2 = 0$ $x^2 = 880$ $x = 4\sqrt{55}$ or/*of* $\approx 29,66\,mm$ $PR = 4\sqrt{55}$ or/*of* $\approx 29,66\,mm$ **OR/*OF*** **Using the half chord of /** ***Gebruik halfkoord van*** **RQ** $OP^2 = OT^2 + PT^2$ $16^2 = (6)^2 + PT^2$ $PT^2 = 220$ $PT = \sqrt{220}$ $PR = 2 \times \sqrt{220}$ $PR \approx 29,66$ mm **OR/*OF*** Height of major segment = 22 mm $4h^2 - 4dh + x^2 = 0$ $4(22)^2 - 4(32)(22) + x^2 = 0$ $x^2 = 880$ $x = 4\sqrt{55}$ or/*of* $\approx 29,66\,mm$ $PR = 4\sqrt{55}$ or/*of* $\approx 29,66\,mm$ **OR/*OF***	✓both Radii/*beide radiusse* **A** ✓length/*lengte* **CA** ✓**F** **A** ✓**SF** **CA** ✓Length/*lengte* **CA** **OR/*OF*** ✓both Radii/*beide radiusse* **A** ✓ Pythagoras **A** ✓ **SF** **CA** ✓ Length of/*Lengte van* **PT** **CA** ✓ length/*lengte* **CA** **OR / *OF*** ✓ Height / *Hoogte* **A** ✓**F** **A** ✓**SF** **CA** ✓ **S** **CA** ✓Length/*lengte* **CA** **OR/*OF***

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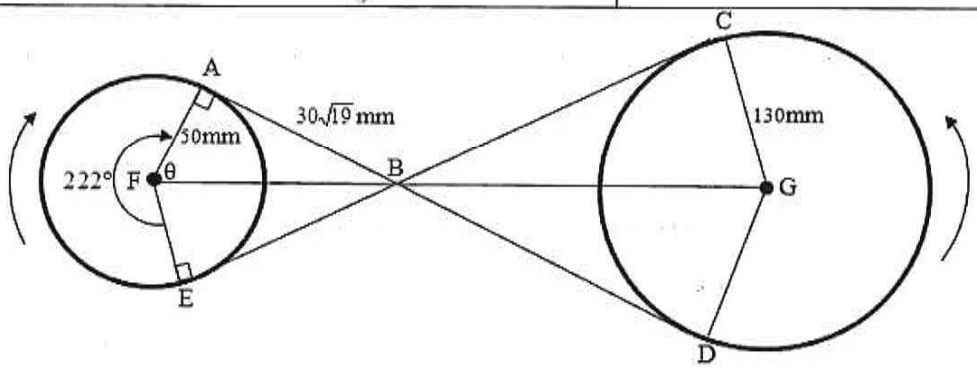
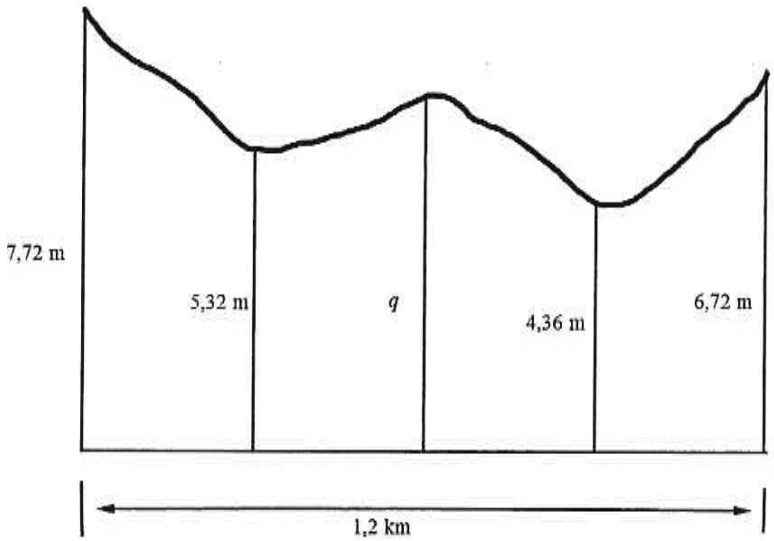
	In ΔOTP $\cos \hat{POT} = \frac{6}{16}$ $\hat{POT} = \cos^{-1}\left(\frac{6}{16}\right)$ $\approx 67,98°$ $\therefore \tan 67,98° = \frac{PT}{6}$ $PT = 6\tan 67,98°$ $\approx 14,83$ $PR \approx 2(14,83)$ $\approx 29,66\text{ mm}$	✓ trig ratio / *verh* **A** ✓ size of / *grootte van* ∠ **CA** ✓ trig ratio / *verh* **CA** ✓ Length/*lengte* PT **CA** ✓Length/*lengte* PR **CA** **NPU/NPR** (5)
10.2		
10.2.1	$222° \times \frac{\pi}{180°} = \frac{37}{30}\pi$ or / *of* $\approx 3,87$ rad.	✓Conversion/ *herleiding* **A** **NPR NPU** **AO Full marks/ *Volpunte*** (1)
10.2.2	$A = \frac{\theta}{360°}\pi r^2$ $= \frac{222°}{360°}\pi(50)^2$ $\approx 4843,29\text{ mm}^2$ **OR/*OF***	✓Formula/ *formule* **A** ✓**SF** **CA** ✓ Area of sector/ *oppervlakte van sektor* **CA** **OR/*OF***

	$A = \frac{r^2\theta}{2}$ $= \frac{(50)^2\left(\frac{37}{30}\pi\right)}{2}$ $= \frac{4625}{3}\pi$ or/*of* $\approx 4843,29\ mm^2$	✓Formula/ *formule* ✓**SF** ✓ Area of sector/ *oppervlakte van sektor*	A **CA** **CA**
	OR/*OF*	**OR/*OF***	
	$s = r\theta$ $= (50)\left(\frac{37}{30}\pi\right)$ $= \frac{185}{3}\pi$ or/*of* $\approx 193,73mm$	✓arc length/ *booglengte*	**CA**
	$A = \frac{rs}{2}$ $= \frac{(50)\left(\frac{185}{3}\pi\right)}{2}$ $= \frac{4625}{3}$ $\approx 4843,29mm^2$	✓Formula/ *formule* ✓Area of sector/ *oppervlakte van sektor*	**A** **CA** **NPU** (3)

10.2.3	$BG^2 = BC^2 + CG^2$ $BC^2 = BG^2 - CG^2$ $BC^2 = (360)^2 - (130)^2$ $BC = 70\sqrt{23} \approx 335,71\text{ mm}$	✓ length/ *lengte* BG **A** ✓ length / *lengte* BC **CA**
	$s = r\theta$ $= (50)\left(\frac{37}{30}\pi\right)$ $AE = \frac{185}{3}\pi$ or / *of* $\approx 193,73\text{ mm}$	✓**F** **A** ✓**SF** **CA** ✓$\frac{185}{3}\pi$ **CA**
	LENGTH OF BELT/ *LENGTE VAN BAND* $= AE + AB + BC + CD + BD + BE$ $= \frac{185}{3}\pi + 30\sqrt{19} + 70\sqrt{23} + 503 + 70\sqrt{23} + 30\sqrt{19}$ $\approx 1629,68\text{ mm}$	✓**M** **A** ✓ length/*lengte* **CA**
	OR/*OF* $\Delta ABF \,|||\, \Delta DBG$ $\frac{AB}{DB} = \frac{AF}{DG}$ $\frac{30\sqrt{19}}{DB} = \frac{50}{130}$ $DB \approx 339,99$	**OR / *OF*** ✓ Similarity/*gelykvormig* **A** ✓ Proportion/ *verhouding* **A** ✓ **SF** **A** ✓ length / *lengte* DB **CA**
	LENGTH OF BELT / *LENGTE VAN BAND* $= AE + 2\times AB + 2\times BC + CD$ $= \frac{185}{3}\pi + 2\times 30\sqrt{19} + 2\times 339,99 + 503$ $\approx 1638,25\text{ mm}$	✓**M** **A** ✓**SF** **CA** ✓ length/*lengte* **CA**
		Penalty for rounding/ ***Penalisering vir afronding***
		(7)
		[20]

QUESTION/*VRAAG* 11

11.1



11.1.1

$1,2 \text{ km} \times \frac{1000 \text{ m}}{1 \text{ km}} = 1200 \text{ m}$

✓ **A** (1)

11.1.2

$$A_T = a\left(\frac{o_1 + o_n}{2} + o_2 + o_3 + \ldots + o_{n-1}\right)$$

$$6948 = \frac{1200}{4}\left(\frac{7,72 + 6,72}{2} + 5,32 + q + 4,36\right)$$

$$= 300(16,9 + q)$$

$$16,9 + q = 23,16$$

$$q = 6,26 \text{ m}$$

OR/*OF*

$$A_T = a(m_1 + m_2 + m_3 + \ldots + m_n)$$

$$6948 = \frac{1200}{4}\left(\frac{7,72 + 5,32}{2} + \frac{5,32 + q}{2} + \frac{q + 4,36}{2} + \frac{4,36 + 6,72}{2}\right)$$

$$6948 = 300\left(6,52 + \frac{9,68 + 2q}{2} + 5,54\right)$$

$$23,16 = 12,06 + \frac{9,68 + 2q}{2}$$

$$\frac{9,68 + 2q}{2} = 11,1$$

$$9,68 + 2q = 22,2$$

$$2q = 12,52$$

$$q = 6,26 \text{ m}$$

✓ **F** **A**
✓ value of/*waarde van a* **CA**
✓ **SF** **CA**
✓ **S** **CA**
✓ value of/*waarde van q* **CA**

OR/*OF*

✓ **F** **A**
✓ value of/*waarde van a* **CA**
✓ **SF** **CA**
✓ **S** **CA**
✓ value of/*waarde van q* **CA**
(5)

11.2.1	$A_{cylinder/silinder} = 2\pi r^2 + 2\pi rh$ $= 2\pi(1{,}5\text{ m})^2 + 2\pi(1{,}5\text{ m})(10)$ $= 108{,}38\text{ m}^2$ cost/*koste* $= \text{R}\,8{,}93 \times 108{,}38\text{ m}^2$ $= \text{R}\,967{,}83 < \text{R}1000$	✓formula/*formule* **A** ✓**SF** **A** ✓ value of/ *waarde van* A **CA** ✓Cost not exceeding R1000/ *Koste* nie meer as R1000 **CA** **NPR** (4)
11.2.2	Airspace for cylindrical tank/ *lugruimte vir silindriese tenk* $= 70{,}69\text{ m}^3 - 68\text{ m}^3$ $= 2{,}69\text{ m}^3$ percentage/ persentasie $= \frac{2{,}69\text{ m}^3}{70{,}69\text{ m}^3} \times 100 = 3{,}81\%$ Airspace for car tank/ *lugruimte vir kar tenk* $= 55\,\ell - 52\,\ell$ $= 3\,\ell$ percentage / *persentasie* $= \frac{3}{55} \times 100 = 5{,}45\%$ The car fuel tank has a bigger percentage airspace/ *die kar brandstoftenk het 'n groter persentasie lugruimte.* **OR / *OF*** percentage/ persentasie $= \frac{68\text{ m}^3}{70{,}69\text{ m}^3} \times 100\% = 96{,}19\%$ percentage / *persentasie* $= \frac{52}{55} \times 100\% = 94{,}55\%$ The car fuel tank has a bigger percentage airspace/ *die kar brandstoftenk het n groter persentasie lugruimte.*	✓ **M** **A** ✓percentage/*persentasie* **CA** ✓ **M** **A** ✓percentage/*persentasie* **CA** ✓ conclusion/ *gvltgtreknng* **CA** **OR / *OF*** ✓ **M** **A** ✓percentage/*persentasie* **CA** ✓ **M** **A** ✓percentage/*persentasie* **CA** ✓ conclusion/ *gvltgtreknng* **CA** (5) **[15]**

TOTAL/*TOTAAL*: 150

MA HENDRICKS
External Moderator