**NATIONAL
SENIOR CERTIFICATE/**
*NASIONALE
SENIORSERTIFIKAAT*
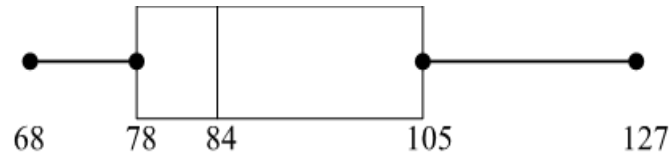
GRADE/*GRAAD* 11

NOVEMBER 2023

**MATHEMATICS P2/*WISKUNDE V2*
MARKING GUIDELINE/*NASIENRIGLYN***

MARKS/*PUNTE*:   150

This marking guideline consists of 13 pages.
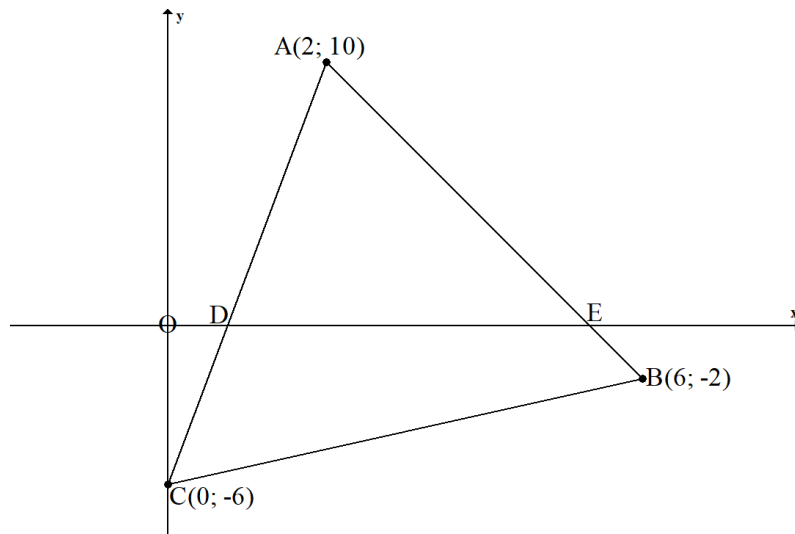*Hierdie nasienriglyn bestaan uit 13 bladsye.*

QUESTION 1/*VRAAG 1*

1.1	6 boys / *seuns*	✓ answer / *antwoord*	(1)
1.2	Skewed to the right / positively skewed *Skeef na regs / Positief skeef*	✓ answer / *antwoord*	(1)
1.3	$127 - 68 = 59$	✓ working out / *bewerking* ✓ answer / *antwoord*	(2)
1.4.1	Median/*Mediaan* = 84 / Remains unchanged / *Bly onveranderd*	✓ answer / *antwoord*	(1)
1.4.2	$(87 \times 24 + 20) \div 24 = 87{,}8$ **OR/*OF*** 88	✓ working out / *bewerking* ✓ answer / *antwoord*	(2)
			[7]

QUESTION 2/*VRAAG 2*

2.1					
	Time waiting / *Wagtyd* (minutes / *minute*)	Number of learners *Aantal leerders*	Cumulative frequency *Kumulatiewe frekwensie*	✓ for / *vir* 6; 16; 35 ✓ for / *vir* 50; 55	
	$0 < x \leq 4$	6	6		
	$4 < x \leq 8$	10	16		
	$8 < x \leq 12$	19	35		
	$12 < x \leq 16$	15	50		
	$16 < x \leq 20$	5	55		(2)

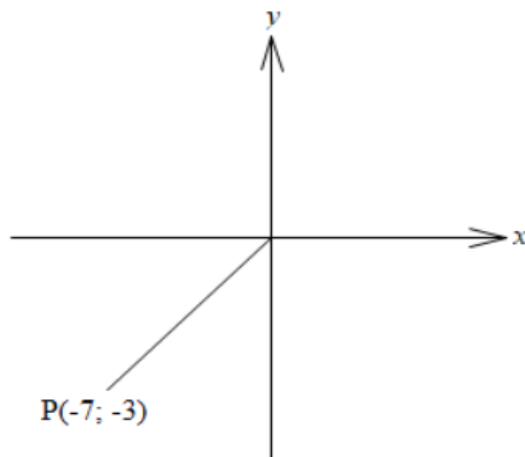| 2.2		✓ for grounding *vir anker* ✓ for endpoint *vir eindpunt* ✓✓ for any 2 correct points *vir enige 2 korrekte punte*	(4)
2.3	55 – 43 = 12 learners / *leerders* (accept/*aanvaar* 11 en 13)	✓ 55 ✓43 ✓12	(3)
2.4	$8 < x \leq 12$	✓ answer / *antwoord*	(1)
2.5	IQR / *IKW* = 14 – 7 = 7	✓ UQ ✓ LQ ✓ answer / *antwoord*	(3)
			[13]

QUESTION 3 / *VRAAG 3*

3.1	$m_{AB} = m_{BC}$ $\frac{-8-0}{x-5+8} = \frac{-14-0}{x+8}$ $\frac{-8}{x+3} = \frac{-14}{x+8}$ $-8x - 64 = -14x - 42$ $6x = 22$ $x = \frac{11}{3}$	✓ $\frac{-8-0}{x-5+8}$ ✓ $\frac{-14-0}{x+8}$ ✓ $-8x - 64$ ✓ $-14x - 42$ ✓ $x = \frac{11}{3}$	(5)
3.2.1	$M\left(\frac{6+0}{2}; \frac{-2-6}{2}\right)$ $= M(3; -4)$	✓ substitution / *vervanging* ✓ answer / *antwoord*	(2)
3.2.2	$AC = \sqrt{(2-0)^2 + (10-(-6))^2}$ $AC = \sqrt{4 + 256}$ $AC = 2\sqrt{65}$	✓ substitution / *vervanging* ✓ answer / *antwoord*	(2)
3.2.3	$F(-4; 6)$	✓ x ✓ y	(2)
3.2.4	$m_{AB} = \frac{10-(-2)}{2-6} = \frac{12}{-4} = -3$ $m_{BC} = \frac{-2+6}{6-0} = \frac{4}{6} = \frac{2}{3}$ $m_{AB} \times m_{BC} \neq -1$ ∴ABCF is not a rectangle / *is nie 'n reghoek nie* **OR/*OF*** $AC = \sqrt{260} = 2\sqrt{65}$ $AB = \sqrt{160} = 4\sqrt{10}$ $BC = \sqrt{50} = 5\sqrt{2}$ $260 \neq 160 + 50$ $AC^2 \neq AB^2 + BC^2$	✓ m_{AB} ✓ m_{BC} ✓ $m_{AB} \times m_{BC} \neq -1$ ✓ $AB = \sqrt{160} = 4\sqrt{10}$ ✓ $BC = \sqrt{50} = 5\sqrt{2}$ ✓ $260 \neq 160 + 50$	(3)

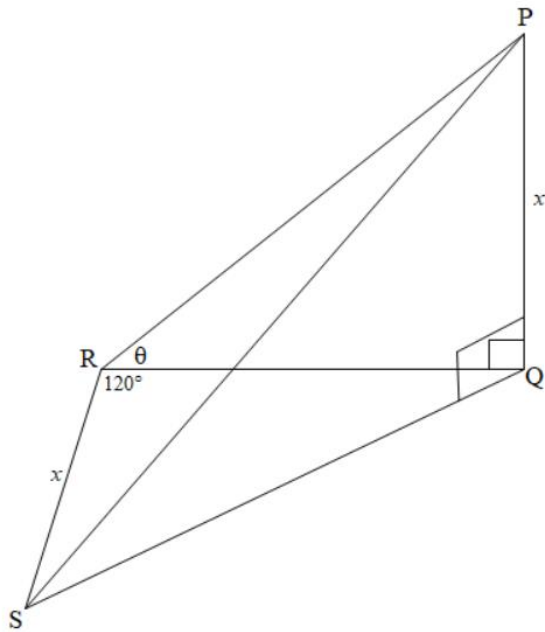| 3.2.5	$m_{AB} = -3$ $\therefore y = -3x + c$ $10 = -3(2) + c$ $c = 16$ $\therefore y = -3x + 16$ $m_{AC} = \frac{10-(-6)}{2-0} = 8$ $\therefore y = 8x - 6$	✓ m_{AB} ✓ substitution / *vervanging* ✓ equation of AB / *vergelyking van* AB ✓ gradient of AC/*gradiënt van* AC ✓ equation of /*vergelyking van* AC	(5)
3.2.6	$\tan A\hat{E}X = -3$ $\therefore A\hat{E}X = 108,43°$ $\tan A\hat{D}E = 8$ $\therefore A\hat{D}E = 82,87°$ $\hat{A} = 108,43° - 82,87° = 25,56°$	✓ $\tan A\hat{E}X = -3$ ✓ $A\hat{E}X$ ✓ $\tan A\hat{D}E = 8$ ✓ $A\hat{D}E$ ✓✓ answer / *antwoord*	(6)
3.2.7	$D\left(\frac{3}{4}; 0\right)$ $E\left(\frac{16}{3}; 0\right)$ $\therefore DE = \frac{55}{12}$ units / *eenhede* Perpendicular height / *Loodregte hoogte* $= 10$ units / *eenhede* $\therefore A = \frac{1}{2} \times \frac{55}{12} \times 10 = 22,92$ units²/ *eenhede²* **OR/*OF*** $AD = 10,08$ $AE = 10,54$ *Area of* $\Delta ADE = \frac{1}{2} AD.AE \sin A$ *Area of* $\Delta ADE = \frac{1}{2}(10,08)(10,54) \sin 25,56$ *Area of* $\Delta ADE = 22,92$ *units/eenhede* ²	✓ coordinates of D / *koördinate van* D ✓ coordinates of E / *koördinate van* E ✓ length of DE / *lengte van* DE ✓ height / *hoogte* ✓ answer / *antwoord* ✓ $AD = 10,08$ ✓$AE = 10,54$ ✓ Formula/*formule* ✓ substitution/*vervanging* ✓answer/*antwoord*	(5)
			[30]

QUESTION 4 / *VRAAG 4*

P(-7; -3)

4.1	$\tan\theta = \frac{3}{7}$ $r^2 = (-7)^2 + (-3)^2$ $r^2 = 58$ $r = \sqrt{58}$ $\dfrac{\sin\theta + \cos\theta}{2\sin\theta}$ $= \dfrac{\frac{-3}{\sqrt{58}} + \left(\frac{-7}{\sqrt{58}}\right)}{2\left(\frac{-3}{\sqrt{58}}\right)}$ $= \dfrac{-10}{\sqrt{58}} \times \dfrac{\sqrt{58}}{-6}$ $= \dfrac{5}{3}$	✓ correct quadrant / *korrekte kwadrant* ✓ $\tan\theta = \frac{3}{7}$ ✓ for/*vir* ✓ substitution / *vervanging* ✓ simplification / *vereenvoudiging* ✓ answer / *antwoord*	(6)

4.2.1	$\tan(-32°)$ $= -\tan 32°$ $= -\dfrac{p}{\sqrt{1-p^2}}$		✓ method used / triangle *metode gebruik / driehoek* ✓ $-\tan 32°$ ✓ answer / *antwoord*		(3)
4.2.2	$\sin 418°$ $= \sin 58°$ $= \sqrt{1-p^2}$	$= \sin(90° - 32)$ $= \cos 32°$ $= \sqrt{1-p^2}$	✓ $\sin 58°$ ✓ answer / *antwoord*	✓ $\cos 32°$ ✓ answer *antwoord*	(2)

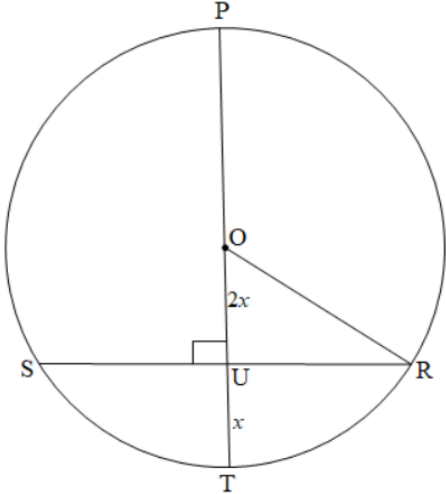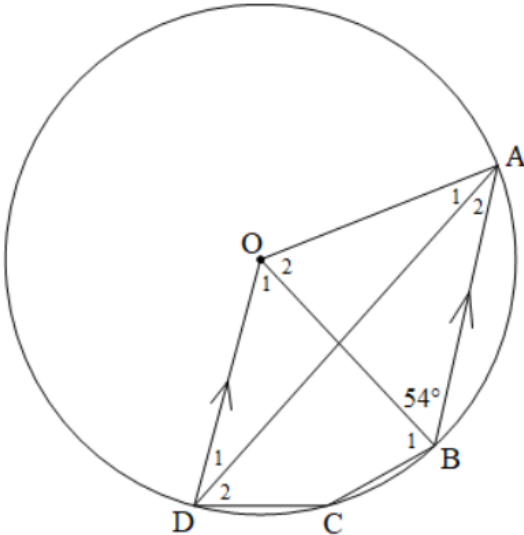| 4.3 | $\dfrac{\sin 120°.\tan 300°}{\cos(-60°).\tan 225°}$ | ✓ $\sin 60°$ | |
		✓ $-\tan 60°$	
	$= \dfrac{\sin 60° \ . \ -\tan 60°}{\cos 60°.\tan 45°}$	✓ $\cos 60°$	
		✓ $\tan 45°$	
	$= \dfrac{\frac{\sqrt{3}}{2}.\frac{-\sqrt{3}}{1}}{\frac{1}{2}.\ 1}$	✓ for numerator S/A / *vir teller*	
		✓ for denominator S/A / *vir noemer*	
		✓ answer / *antwoord*	
	$= -\dfrac{3}{2} \times \dfrac{2}{1} = -3$		(7)
4.4	$\dfrac{\cos^2 x - \cos x - \sin^2 x}{2\sin x \cos x + \sin x} = \dfrac{1}{\tan x} - \dfrac{1}{\sin x}$		
	LHS/LK $= \dfrac{\cos^2 x - \cos x - (1-\cos^2 x)}{2\sin x \cos x + \sin x}$	✓ $1-\cos^2 x$	
	LHS $= \dfrac{2\cos^2 x - \cos x - 1}{2\sin x \cos x + \sin x}$	✓ simplifying / *vereenvoudiging*	
		✓ factoring the top / *faktorisering teller*	
	LHS $= \dfrac{(2\cos x + 1)(\cos x - 1)}{\sin x (2\cos x + 1)}$	✓ factoring the bottom / *faktorisering noemer*	
	LHS $= \dfrac{\cos x - 1}{\sin x}$		
	LHS $= \dfrac{\cos x}{\sin x} - \dfrac{1}{\sin x}$	✓ $\dfrac{\cos x}{\sin x} - \dfrac{1}{\sin x}$	
	LHS $= \dfrac{1}{\tan x} - \dfrac{1}{\sin x} =$ RHS/*RK*		(5)
4.5	$\sin(5\theta) = \cos(\theta - 40°)$ $\sin(5\theta) = \sin[90° - (\theta - 40°)]$ $\sin(5\theta) = \sin(130° - \theta)$ $5\theta = 130° - \theta + 360°.k$ $6\theta = 130° + 360.k$ $\theta = 21{,}67° + 60°.k$	✓ co – function / *ko-funksie* ✓ $5\theta = 130° - \theta + 360°.k$ ✓ $\theta = 21{,}67° + 60°.k$	
	OR / *OF* $5\theta = 180° - (130° - \theta) + 360°.k$ $4\theta = 50° + 360.k$ $\theta = 12{,}5° + 90°.k$ where/*waar* $k\epsilon Z$	✓ $5\theta = 180° - (130° - \theta) + 360°.k$ ✓ $4\theta = 50° + 360.k$ ✓ $\theta = 12{,}5° + 90°.k$	
	OR / *OF* $0 - 5\theta = 360 - (\theta - 40) + 360k$ $-4\theta = 310 + 360k$ $\theta = -77{,}5 - 90k$	✓ co-function /*ko-funksie* ✓ $-6\theta = -130 + 360k$ ✓ $\theta = 21{,}67 - 60k$ ✓ $90 - 5\theta = 360 - (\theta - 40) + 360k$ ✓ $-4\theta = 310 + 360k$ ✓ $\theta = -77{,}5 - 90k$	(6)
			[29]

QUESTION 5/*VRAAG 5*

5.1	$\tan \theta = \frac{x}{QR}$ $QR = \frac{x}{\tan \theta}$	✓ $\tan \theta = \frac{x}{QR}$ ✓ answer / *antwoord*	(2)
5.2	$QS^2 = x^2 + \left(\frac{x}{\tan \theta}\right)^2 - 2x\left(\frac{x}{\tan \theta}\right)\cos 120°$ $QS^2 = x^2 + \frac{x^2}{tan^2\theta} - \frac{2x^2}{\tan \theta}\left(-\frac{1}{2}\right)$ $QS^2 = x^2 + \frac{x^2}{tan^2\theta} + \frac{x^2}{\tan \theta}$ $QS = \sqrt{x^2\left(1 + \frac{1}{tan^2\theta} + \frac{1}{\tan \theta}\right)}$ $QS = x.\sqrt{\frac{1}{tan^2\theta} + \frac{1}{\tan \theta} + 1}$	✓ for the cosine rule *vir die cosinusreël* ✓ $-\frac{1}{2}$ ✓ simplification *vereenvoudiging* ✓ for square rooting *vir vierkantswortel*	(4)
5.3.1	$QS = 15\sqrt{\frac{1}{tan^2 22°} + \frac{1}{\tan 22°} + 1}$ $QS = 46,48 \text{ m}$	✓ substitution / *vervanging* ✓ answer / *antwoord*	(2)
5.3.2	$\tan Q\hat{P}S = \frac{46,48}{15}$ $Q\hat{P}S = 72,11°$	✓ for using tan / *vir gebruik van* tan ✓ answer / *antwoord*	(2)
			[10]
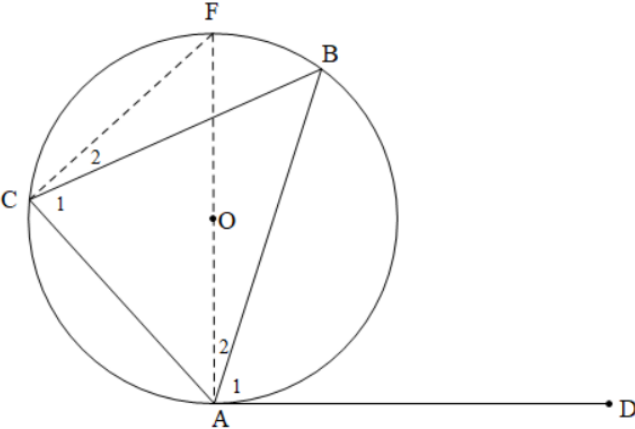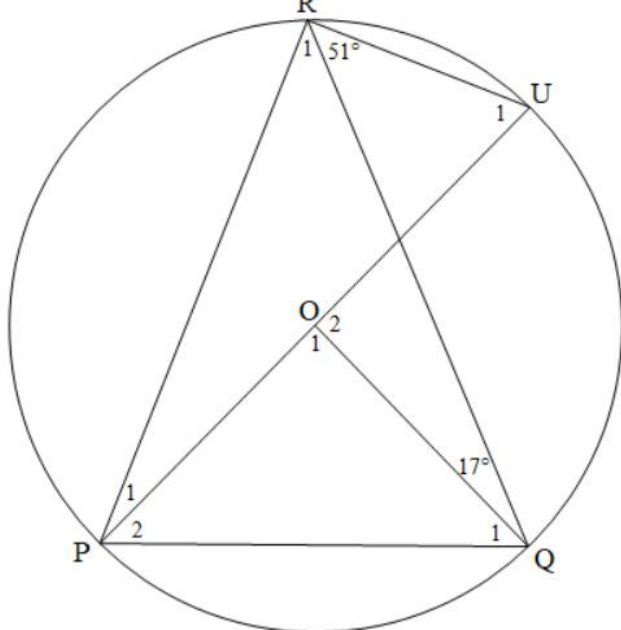
QUESTION 6 / *VRAAG 6*

6.1		
	For f: ✓ shape ✓ endpoints ✓ TP and/*en* x-intercepts *Vir f*: ✓ *vorm* ✓ *eindpunte* ✓ DP and/*en* x-*afsnitte* For g: ✓ asymptote ✓ shape ✓ y-intercept *Vir g*: ✓ *asimptoot* ✓ *vorm* ✓ y-*afsnit*	(6)
6.2.1	$0° < x < 90°$	✓ answer / *antwoord* (1)
6.2.2	$-45° \leq x \leq 0°$ **or/*of*** $45° \leq x < 90°$ **or/*of*** $90° < x \leq 180°$	✓✓✓ for each answer *vir elke antwoord* (3)
6.3	$1 \leq y \leq 3$ **or/*of*** $y\epsilon[1; 3]$	✓✓ for the answer *vir die antwoord* (2)
		[12]

QUESTION 7 / VRAAG 7

7.1			
7.1.1	$OR = 3x$	✓ answer / *antwoord*	(1)

7.1.2	UR = 20 cm (line from centre perpendicular…)
	           (*lyn vanaf middelpunt loodreg* …)
	 $(3x)^2 = (2x)^2 + 20^2$ (Pythagoras)
	 $9x^2 = 4x^2 + 400$
	 $5x^2 = 400$
	 $x^2 = 80$
	 $x = \sqrt{80} = 4\sqrt{5}$
		✓ S ✓ R
		 ✓ S and/*en* R
		
		 ✓ simplification
		   vereenvoudiging
		
		 ✓ answer / *antwoord*
			(5)

7.2			
7.2.1	$\hat{O}_1 = 54°$ (alt ∠s / *verw.* ∠*e*, OD ∥ AB) $\hat{A}_2 = 27°$ (∠ at centre) / (*middelpunts* ∠)	✓ S ✓ R ✓ S ✓ R	(4)
7.2.2	$\hat{C} = 153°$ (opp. ∠s of a cq) / (*teenoorst.* ∠*e van k.v*)	✓ S ✓ R	(2)
7.2.3	$\hat{A} = 54°$ (∠s opp = sides) / (∠*e teenoor = sye*) $\hat{A}_2 = 27°$ (proven) / (*reeds bewys*) ∴ $\hat{A}_1 = 27°$ ∴DA bisects O$\hat{A}$B / DA *halveer* O$\hat{A}$B	✓ S and/*en* R ✓ S ✓ S	(3)
			[15]

QUESTION 8 / VRAAG 8

8.1	 Let /*Laat* $\hat{A}_1 = x$ $\therefore \hat{A}_2 = 90° - x$ (tan $\perp$ rad) / (*raaklyn* $\perp$ *radius*) $F\hat{C}A = 90°$ ($\angle$ in a semi-circle) / ($\angle$ *in semi-sirkel*) $\hat{C}_2 = 90° - x$ ($\angle$s in the same segment) / ($\angle$e *in dies. segment*) $\therefore \hat{C}_1 = x$ $\therefore D\hat{A}B = B\hat{C}A$	✓ S ✓ R ✓ S and/*en* R ✓ S and/*en* R ✓ S	(5)
8.2			
8.2.1	$\hat{R}_1 = 39°$ ($\angle$ subtended by diameter)/ ($\angle$ *onderspan deur middellyn*)	✓ S ✓ R	(2)
8.2.2	$\hat{O}_1 = 78°$ ($\angle$ at centre) / (*Middelpunts* $\angle$)	✓ S ✓ R	(2)
8.2.3	$\hat{Q}_1 = 51°$ ($\angle$s opp = sides) / ($\angle$e *teenoor gelyke sye*)	✓ S ✓ R	(2)
8.2.4	$\hat{U}_1 = 68°$ ($\angle$s in the same segment) / ($\angle$e *in dieselfde segment*)	✓ S ✓ R	(2)
			[13]

QUESTION 9 / *VRAAG 9*

9.1.1	$A\hat{B}F = x$ (tan-chord theorem) / (*raaklyn-koord stelling*)	✓ S ✓ R	(2)
9.1.2	$B\hat{C}D = x$ (corresponding angles, BF∥CD) (*ooreenkomstige hoeke* ; BF ∥ CD)	✓ S ✓ R	(2)
9.2	Exterior angle = interior opp. angle *Buitehoek = teenoorstaande binnehoek.*	✓ R	(1)

9.3	$B\hat{D}C = x$ (angles in the same segment) /
	          (*hoeke in dieselfde segment*)
	
	 $F\hat{B}D = x$ (alternate angles BF∥CD)
	          (*verwisellende hoeke* BF ∥ CD)
		✓ S ✓ R
		
		 ✓ S ✓ R
			(4)

			[9]

QUESTION 10 / VRAAG 10

10.1	$\widehat{D} = \hat{C}_3$ ($\angle$s opp. = sides) / ($\angle$e *teenoor = sye*) $\widehat{D} = \hat{B}_1$ (ext. $\angle$ of a cq) / (*buite* $\angle$ *van k.v*) $\hat{C}_3 = \hat{B}_2$ ($\angle$s in the same segment) / ($\angle$e *in dieselfde segment*) $\hat{B}_1 = \hat{B}_2$	✓ S and/*en* R ✓ S ✓ R ✓ S and/*en* R (4)

10.2	$\hat{C}_1 + \hat{C}_2 = \widehat{D}$ (tan-chord theorem) / (*raaklyn-koord stelling*)
	 But/*Maar* $\widehat{D} = \hat{B}_2$ (proven) / (*reeds bewys*)
	 $\therefore \hat{B}_2 = \hat{C}_1 + \hat{C}_2$ (ext $\angle$ = int opp.angle) /
	       (*buite* $\angle$ = *teenoorst. binnehoek*)
	 BECH is a c.q. / BECH *is 'n kv* (converse. ext. $\angle$ of a cq) /
	       (*omgekeerde buite* $\angle$ *van kv*)
		✓ S and/*en* R
		
		 ✓ S and/*en* R
		 ✓ R
		
		 (3)

10.3	$\hat{B}_2 = \hat{A}_3 + \hat{T}_1$ (ext $\angle$ of a triangle) / (*buite* $\angle$ *van driehoek*)
	 But/*Maar* $\hat{B}_2 = \hat{C}_1 + \hat{C}_2$ (proven) / (*reeeds bewys*)
	 And/ *En* AT = CT (tangents from the same point)
	       (*raaklyne vanaf dieselfde punt*)
	 $\therefore \hat{C}_1 + \hat{C}_2 = \hat{A}_3 + \hat{A}_2$ ($\angle$s opp. = sides) / ($\angle$e *teenoor = sye*)
	 $\hat{A}_3 + \hat{A}_2 = \hat{A}_3 + \hat{T}_1$ (conv. tan-chord) /
	       (*omgekeerde raaklyn-koord*)
	 $\therefore \hat{A}_2 = \hat{T}_1$
	 CA is a tangent to circle ABT / CA *is 'n raaklyn aan sirkel* ABT)
		✓ S and/*en* R
		 ✓ S and/*en* R
		 ✓ S and/*en* R
		 ✓ S and/*en* R
		 ✓ S and/*en* R
		
		 (5)

		[12]

TOTAL/TOTAAL: **150**