



**NATIONAL SENIOR
CERTIFICATE/*NASIONALE
SENIORSERTIFIKAAT***

GRADE/*GRAAD* 12

NOVEMBER 2023

**TECHNICAL MATHEMATICS P2/*TEGNIESE WISKUNDE V2*
MARKING GUIDELINE/*NASIENRIGLYN***

MARKS/*PUNTE*: 150

This marking guideline consists of 16 pages./
Hierdie nasienriglyn bestaan uit 16 bladsye.

NOTE:

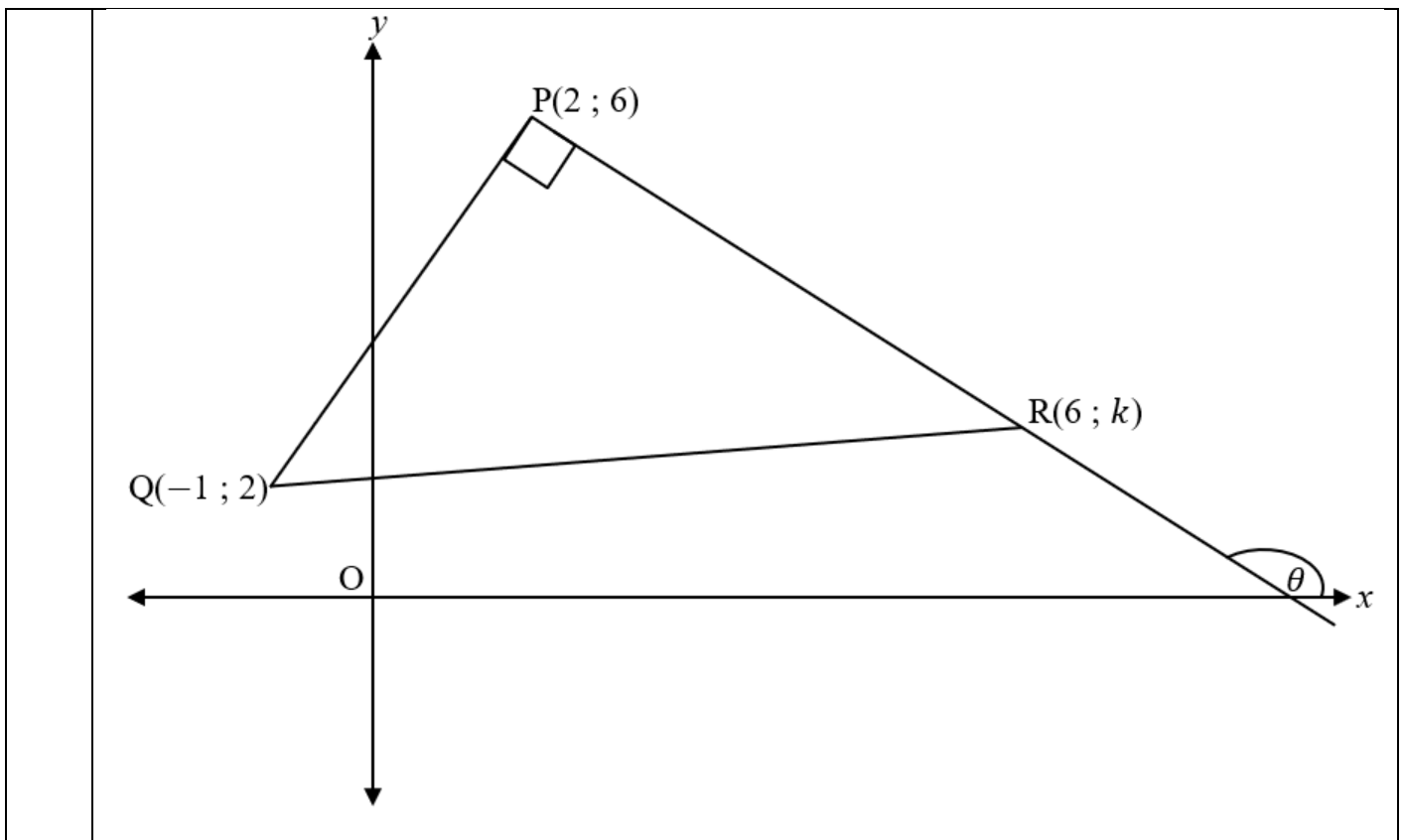
- Continuous accuracy (CA) applies only where indicated in this marking guideline.
- Assuming values/answers in order to solve a problem is unacceptable.

LET WEL:

- *Volgehoue akkuraatheid (CA) is slegs van toepassing soos aangedui in hierdie nasienriglyn.*
- *Aanvaarding van waardes/antwoorde om 'n probleem op te los, is onaanvaarbaar.*

MARKING CODES / NASIENKODES	
M	Method / Metode
A	Accuracy / Akkuraatheid
AO	Answer only / Slegs antwoord
CA	Consistent accuracy / Deurlopende akkuraatheid
F	Formula / Formule
I	Identity / Identiteit
R	Rounding / Afronding
S	Simplification / Vereenvoudiging
ST	Statement / Bewering
RE	Reason / Rede
ST RE	Statement and correct reason / Bewering en korrekte rede
SF	Substitution correctly in correct formula / Korrekte vervanging in die korrekte formule
NPU	No penalty for omitting units / Geen penalisering vir eenhede uitgelaat

QUESTION/VRAAG 1

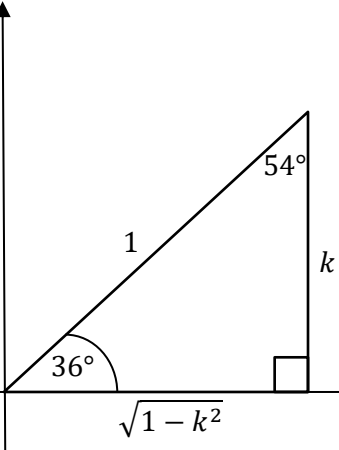


1.1	product / produk	✓	A	(1)
1.2	$m_{PQ} = \frac{y_2 - y_1}{x_2 - x_1}$ $m_{PQ} = \frac{6-2}{2-(-1)}$ $m_{PQ} = \frac{4}{3}$	✓ F ✓ SF	A A	✓ Ans. / Antw. CA (3)
1.3	$m_{PQ} \times m_{PR} = -1$ $\frac{4}{3} \times \frac{6-k}{2-6} = -1$ $\frac{4}{3} \times \frac{6-k}{-4} = -1$ $\frac{6-k}{-3} = -1$ $6-k = 3$ $-k = -3$ $k = 3$	✓ M ✓ SF	A A	✓ Simpl. / Vereenv. CA (3)
1.4	$M_{QR} \left(\frac{x_2 + x_1}{2}, \frac{y_2 + y_1}{2} \right)$ $M_{QR} \left(\frac{-1+6}{2}, \frac{2+3}{2} \right)$ $M_{QR} \left(\frac{5}{2}, \frac{5}{2} \right)$	✓ F ✓ SF	A A	✓ Ans. / Antw. CA (3)

1.5	$M_{QR} = M_{PS}$ $\left(\frac{5}{2}; \frac{5}{2}\right) = \left(\frac{2+x_S}{2}; \frac{6+y_S}{2}\right)$ $\therefore \frac{5}{2} = \frac{2+x_S}{2} \quad \text{and/en} \quad \therefore \frac{5}{2} = \frac{6+y_S}{2}$ $5 = 2 + x_S \quad \quad \quad 5 = 6 + y_S$ $3 = x_S \quad \quad \quad -1 = y_S$ $\therefore S(3; -1)$ <p style="text-align: center;">OR/OF</p> $P \rightarrow R = P(2; 6) \rightarrow R(6; 3)$ $\therefore P \rightarrow R = (x + 4; y - 3)$ $\therefore Q \rightarrow S = Q(-1; 2) \rightarrow S(-1 + 4; 2 - 3)$ $\therefore S(3; -1)$	<p>✓ $\frac{5}{2} = \frac{2+x_S}{2}$ CA</p> <p>✓ $\frac{5}{2} = \frac{6+y_S}{2}$ CA</p> <p>✓ value of/waarde van x CA</p> <p>✓ value of/waarde van y CA</p> <p>✓ M A</p> <p>✓ S CA</p> <p>✓ value of/waarde van x CA</p> <p>✓ value of/waarde van y CA</p> <p>(4)</p>
1.6	$m_{PR} = -\frac{3}{4}$ $\therefore y = -\frac{3}{4}x + c$ $(2; 6): 6 = -\frac{3}{4}(2) + c$ $\therefore 6 = -\frac{3}{2} + c$ $\therefore \frac{10}{3} = c$ $\therefore y = -\frac{3}{4}x + \frac{15}{2}$ <p style="text-align: center;">OR/OF</p> $m_{PR} = -\frac{3}{4}$ $\therefore y - y_1 = -\frac{3}{4}(x - x_1)$ $(2; 6): y - 6 = -\frac{3}{4}(x - 2)$ $\therefore y - 6 = -\frac{3}{4}x + \frac{3}{2}$ $\therefore y = -\frac{3}{4}x + \frac{3}{2} + 6$ $\therefore y = -\frac{3}{4}x + \frac{15}{2}$	<p>✓ Gradient/gradient CA</p> <p>✓ SF A</p> <p>✓ c-value / -waarde CA</p> <p>✓ Ans. /Antw. CA</p> <p>(4)</p>
1.7	$\tan\theta = m_{PR}$ $\tan\theta = \frac{3}{-4}$ $\therefore \text{ref/verw. } \angle = \tan^{-1}\left(\frac{3}{4}\right) \approx 36,87^\circ$ $\therefore \theta = 180^\circ - 36,87^\circ$ $\therefore \theta = 143,13^\circ$	<p>✓ SF CA</p> <p>✓ ref. \angle / verw. \angle CA</p> <p>✓ Ans. /Antw. CA</p> <p>(3)</p>

1.8	$RQ = \sqrt{(x_2 - x_1)^2 + (y_2 - y_1)^2}$ $= \sqrt{(6 + 1)^2 + (3 - 2)^2}$ $= \sqrt{50}$ $\therefore \cos Q = \frac{5}{\sqrt{50}}$ $\therefore \hat{Q} = 45^\circ$ <p style="text-align: center;">OR/OF</p> $PR = \sqrt{(x_2 - x_1)^2 + (y_2 - y_1)^2}$ $= \sqrt{(6 - 2)^2 + (3 - 6)^2}$ $= 5$ $\therefore \tan Q = \frac{5}{5}$ $\therefore \hat{Q} = 45^\circ$	\checkmark F A \checkmark SF A \checkmark length/lengte PQ CA \checkmark cos ratio/verh. CA \checkmark size of / grootte van Q CA OR/OF \checkmark F A \checkmark SF A \checkmark length/lengte PR CA \checkmark tan ratio/verh. CA \checkmark size of / grootte van Q CA (5)
		[26]

QUESTION/VRAAG 2

2.1.1	$\tan(30,5^\circ + 130,5^\circ) \approx -0,34$ AO: Full marks / Volpunte	✓ SF ✓ $-0,34$	A A (2)
2.1.2	$\operatorname{cosec}(130,5^\circ - 30,5^\circ)$ $= \frac{1}{\sin(100^\circ)}$ $\approx 1,02$ AO: Full marks / Volpunte	✓ SF ✓ reciprocal ratio <i>/resiprook verh.</i> ✓ 1,02	A A (3)
2.2.1	 <p> $(1)^2 = k^2 + x^2$ $1 - k^2 = x^2$ $\pm\sqrt{1 - k^2} = x$ $\therefore x = \sqrt{1 - k^2}$ </p> <p> $\therefore \cos 36^\circ = \sqrt{1 - k^2}$ </p>	✓ Pythagoras ✓ Diagram ✓ $x = \sqrt{1 - k^2}$ ✓ $\sqrt{1 - k^2}$	A A A CA (4)
2.2.2	$\sin(216^\circ) = \sin(180^\circ + 36^\circ)$ $\sin(216^\circ) = -\sin 36^\circ$ $\sin(216^\circ) = -k$	✓ Reduction/reduksie ✓ Ans. / Antw.	A CA (2)
2.3	$\tan\theta = 2\sin 38,1^\circ$ $\tan\theta = 1,234 \dots$ <i>ref/verw. $\angle \approx 50,98^\circ$</i> $\therefore Q1: \theta = 50,98^\circ$ AND/EN $\therefore Q3: \theta = 180^\circ + 50,98^\circ = 230,98^\circ$	✓ 1,234 ... ✓ <i>ref/verw. \angle</i> ✓ Quadrants/kwadrante ✓ Both answers / beide antwoorde	A A A CA (4)
			[15]

QUESTION/VRAAG 3

3.1	$\frac{\cos(360^\circ - \theta) \cdot \frac{1}{\cot(180^\circ + \theta)} \cdot \tan(360^\circ + \theta)}{\cos(180^\circ + \theta) \cdot \tan(180^\circ - \theta)}$ $= \frac{\cos\theta \cdot \frac{1}{\cot\theta} \cdot \tan\theta}{-\cos\theta \cdot -\tan\theta}$ $= \frac{1}{\cot\theta}$ $= \tan\theta$	$\checkmark \cos\theta$ A $\checkmark \frac{1}{\cot\theta}$ A $\checkmark \tan\theta$ A $\checkmark -\cos\theta$ A $\checkmark -\tan\theta$ A $\checkmark \tan\theta$ CA
3.2	$\left(\tan x + \frac{1}{\cos x}\right)^2 = \frac{1 + \sin x}{1 - \sin x}$ $\therefore LHS = \left(\tan x + \frac{1}{\cos x}\right)^2$ $\therefore LHS = \left(\frac{\sin x}{\cos x} + \frac{1}{\cos x}\right)^2$ $\therefore LHS = \left(\frac{\sin x + 1}{\cos x}\right)^2$ $\therefore LHS = \frac{(\sin x + 1)^2}{\cos^2 x}$ $\therefore LHS = \frac{(\sin x + 1)(\sin x + 1)}{1 - \sin^2 x}$ $\therefore LHS = \frac{(\sin x + 1)(\sin x + 1)}{(1 - \sin x)(1 + \sin x)}$ $\therefore LHS = \frac{1 + \sin x}{1 - \sin x} = RHS$	$\checkmark \frac{\sin x}{\cos x}$ A $\checkmark \frac{(\sin x + 1)^2}{\cos^2 x}$ CA $\checkmark 1 - \sin^2 x$ A $\checkmark LHS = RHS$
		[10]

QUESTION/VRAAG 4

4.1		<p>$f(x)$: ✓ both x-int. / <i>albei x-afsnitte</i> A ✓ y-int. / y-afsnit ✓ A Shape / vorm A</p> <p>$g(x)$: ✓ both x-int. / <i>albei x-afsnitte</i> A ✓ y-int. / y-afsnit A ✓ Shape / vorm A ✓ Turning points / <i>draaipunte</i> A</p>
4.2	$y \in [0 ; 2]$ <p style="text-align: center;">OR/OF</p> $0 \leq y \leq 2$	✓ Notation / <i>notasie</i> A ✓ start- and endpoints / <i>begin- en eindpunte</i> CA (2)
4.3	360°	✓ Ans. / <i>Antw.</i> A (1)
4.4	$90^\circ \leq x \leq 270^\circ$	✓ $90^\circ \leq$ CA ✓ $\leq 270^\circ$ CA (2)
		[12]

QUESTION/VRAAG 5

<p>5.1</p>	<p>$\widehat{BCA} = 180^\circ - 120^\circ - 40^\circ$ $\widehat{BCA} = 20^\circ$ (int. \angle of Δ / binne \angle van Δ)</p>	<p>✓ ST ✓ RE (2)</p>
<p>5.2</p>	<p>$\frac{AC}{\sin \widehat{ABC}} = \frac{AB}{\sin \widehat{BCA}}$ $\frac{AC}{\sin 120^\circ} = \frac{1600}{\sin 20^\circ}$ $AC = \frac{1600}{\sin 20^\circ} \times \sin 120^\circ$ $AC = 4\,051 \text{ m}$</p>	<p>✓ SF CA ✓ S CA ✓ Rounded Ans. / Afgeronde Antw. CA NPU (3)</p>
<p>5.3</p>	<p>Area $\Delta ABC = \frac{1}{2} \cdot a \cdot b \cdot \sin C$ OR/OF Area $\Delta ABC = \frac{1}{2} \cdot AB \cdot AC \cdot \sin \widehat{BAC}$ Area $\Delta ABC = \frac{1}{2} (1600)(4051) \sin 40^\circ$ Area $\Delta ABC = 2\,083\,146,085 \text{ m}^2$ $\widehat{CAD} = 50^\circ$ (compl. \angle's / kompl. \angle'e) Area $\Delta ABC = \frac{1}{2} \cdot a \cdot b \cdot \sin C$ OR/OF Area $\Delta ABC = \frac{1}{2} \cdot AC \cdot AD \cdot \sin \widehat{CAD}$ Area $\Delta ABC = \frac{1}{2} (2400)(4051) \sin 50^\circ$ Area $\Delta ABC = 3\,723\,895,247 \text{ m}^2$ \therefore Total Area = $2\,083\,146,085 + 3\,723\,895,247$ \therefore Totale Opperv. = $5\,807\,041,33 \text{ m}^2$</p>	<p>✓ F A ✓ SF CA ✓ Ans. / Antw. A ✓ Ans. / Antw. A ✓ M A ✓ Ans. / Antw. A NPU (6)</p>
		<p>[11]</p>

QUESTION/VRAAG 6

6.1	Double or Twice/ <i>Dubbel of Twee keer</i>	✓ Ans. / <i>Antw.</i> (1)
6.2		
6.2.1	$x = 48^\circ \times 2 = 96^\circ$ (\angle at centre = $2 \times \angle$ at circumf. / <i>middelpts \angle = $2 \times$ omtreks \angle) </i>	✓ ST ✓ RE (2)
6.2.2	OB = OC (radii) $\hat{C}_2 = \hat{B}_2 = y$ (\angle 's opp = sides / \angle 'e teenoor = sye) $\therefore y = \frac{180^\circ - 96^\circ}{2} = 42^\circ$ (int. \angle of Δ / <i>binne \angle van Δ) </i>	✓ ST ✓ RE (2)
6.3	Equal / <i>gelyk</i>	✓ Ans. / <i>Antw.</i> (1)
6.4		
6.4.1	$x = 120^\circ$ (ext. \angle of Δ / <i>buite \angle van kvh</i>)	✓ ST ✓ RE (2)
6.4.2	$y = 90^\circ$ (\angle in semi-circle / \angle in <i>semi-sirkel</i>)	✓ ST ✓ RE (2)
6.4.3	$z = 30^\circ$ (ext. \angle of Δ / <i>buite \angle van Δ)</i>	✓ ST ✓ RE (2)
		[12]

QUESTION/VRAAG 7

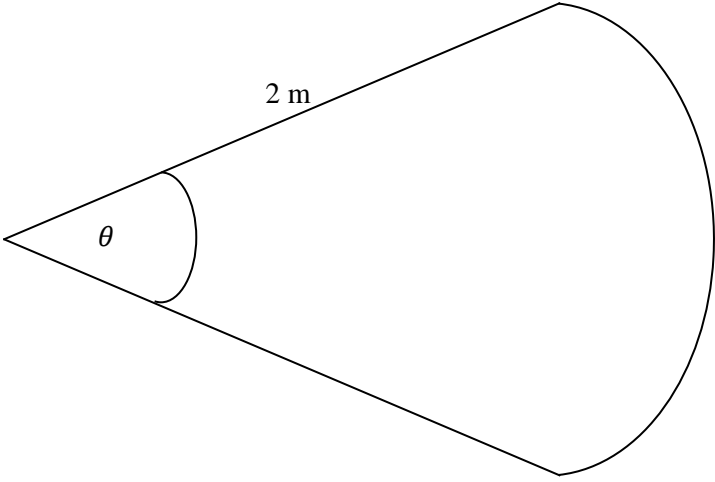
7.1	equal / gelyk	✓ Ans. / Antw. (1)
7.2		
7.2.1	$\hat{C}_4 = 25^\circ$ (tan-chord / raaklyn koord)	✓ ST ✓ RE (2)
7.2.2	$\hat{C}_3 = 42^\circ$ (\angle 's in same seg. / \angle 'e in dieselfde seg.)	✓ ST ✓ RE (2)
7.2.3	$B\hat{C}D = 90^\circ$ (\angle in semi-circle / sirkel) $\hat{D}_1 = 65^\circ$ (int. \angle of Δ / binne \angle van Δ)	✓ ST ✓ RE ✓ ST ✓ RE (4)
7.3	outside / buite	✓ Ans. / Antw. (1)
7.4		
7.4.1	$OQ = OR$ (radii) $\hat{S}_3 = 10^\circ$ (\angle 's opp = sides / \angle 'e teenoor = sye) $\therefore \hat{S}_2 = 40^\circ$ (\angle at centre = $2 \times \angle$ at circumf. / middelpts $\angle = 2 \times$ omtreks \angle)	✓ ST ✓ RE ✓ ST ✓ RE (4)
7.4.2	$\hat{S}_4 + \hat{S}_3 = 90^\circ$ $\therefore \hat{S}_4 = 80^\circ$ (tan \perp radius)	✓ ST ✓ RE (2)
7.4.3	$\hat{R}_1 = 80^\circ$ tan from same pt. / raaklyn uit dieselfde pt.) $\hat{P} = 20^\circ$ (int. \angle of Δ / binne \angle van Δ)	✓ ST ✓ RE ✓ ST ✓ RE (4)
		[20]

QUESTION/VRAAG 8

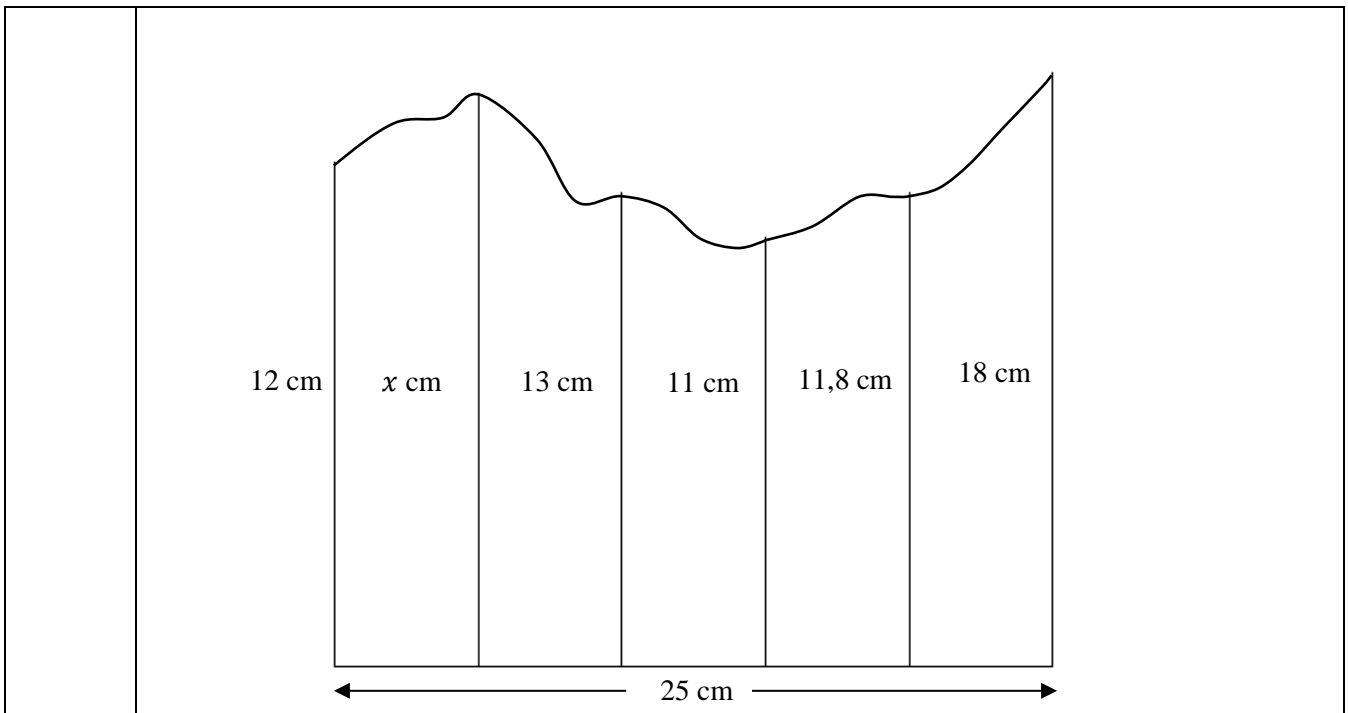
8.1	Perpendicular / loodreg	✓ Ans. / Antw. (1)
8.2		
8.2.1	$OE = x + 8$	✓ Ans. / Antw. (1)
8.2.2	$OA^2 = x^2 + (12)^2$ (Pyth) $OA^2 = x^2 + 144$ $OA = \pm\sqrt{x^2 + 144}$ $\therefore OA = \sqrt{x^2 + 144}$	✓ ST ✓ RE ✓ Answer / Antw. CA (3)
8.2.3	$x + 8 = \sqrt{x^2 + 144}$ $(x + 8)^2 = x^2 + 144$ $x^2 + 16x + 64 = x^2 + 144$ $16x = 80$ $x = 5$	✓ M A ✓ S A ✓ S CA ✓ Ans. / Antw. CA (4)
8.2.4	$r = 13$	✓ Ans. / Antw. (1)
		[10]

QUESTION/VRAAG 9

9.1	$\omega = 2\pi n$ $\omega = 2\pi(12)$ $\omega = 24\pi$ $\omega \approx 75,40 \text{ rad. / sec}$	✓ F A ✓ SF CA ✓ Answer/Antw. CA NPU (3)
9.2	$D = 80 \text{ mm}$ $V = \pi Dn$ $V = \pi(80)(20)$ $V = 1\,600\pi$ $V \approx 5\,026,55 \text{ mm/min}$	✓ diameter/middellyn A ✓ F A ✓ SF CA ✓ Answer/Antw. CA NPU (4)
9.3	$500 \text{ mm} = 50 \text{ cm}$ $4h^2 - 4dh + x^2 = 0$ $4h^2 - 4(56,6)h + (50)^2 = 0$ $4h^2 - 226,4h + 2500 = 0$ $\div 4: h^2 - 56,6h + 625 = 0$ $h = \frac{-b \pm \sqrt{b^2 - 4ac}}{2a}$ $h = \frac{-(-56,6) \pm \sqrt{(-56,6)^2 - 4(1)(625)}}{2(1)}$ $\therefore h \approx 41,56 \text{ or } h \approx 15,04$ $\therefore h = 15,04 \text{ cm}$	✓ conv. /herleid. A ✓ F A ✓ SF CA ✓ SF CA ✓ Both answers/ <i>beide antwoorde</i> CA ✓ Answer/Antw. CA NPU (6)

9.4		
9.4.1	$s = r\theta$ $2,5 = 2\theta$ $1,25 \text{ rad} = \theta$	✓ F A ✓ SF CA ✓ Answer/Antw. CA (3)
9.4.2	$\text{Area} = \frac{r^2\theta}{2}$ $\text{Area} = \frac{(2)^2(1,25)}{2}$ $\text{Area} = 2,5 \text{ cm}^2$ <p style="text-align: center;">OR/OF</p> $\text{Area} = \frac{rs}{2}$ $\text{Area} = \frac{(2)(2,5)}{2}$ $\text{Area} = 2,5 \text{ cm}^2$	✓ F A ✓ SF CA ✓ Ans./Antw. CA NPU (3) ✓ F A ✓ SF CA ✓ Ans./Antw. CA NPU (3)
9.4.3	$C = 2\pi r$ $2,5 = 2\pi r$ $0,40 = r$ $(2)^2 = h^2 + (0,40)^2$ $(2)^2 - (0,40)^2 = h^2$ $3,84 = h^2$ $1,96 \text{ m} = h$	✓ F A ✓ SF A ✓ Answer/Antw. CA ✓ M A ✓ Answer/Antw. CA NPU (5)
9.5	$\text{Vol}_{\text{Shell}} = \text{Vol}_{\text{outer}} - \text{Vol}_{\text{inner}}$ $\text{Vol}_{\text{Shell}} = \frac{4}{3}\pi(5,5)^3 - \frac{4}{3}\pi(3,5)^3$ $\text{Vol}_{\text{Shell}} = \frac{1331}{6}\pi - \frac{343}{6}\pi$ $\text{Vol}_{\text{Shell}} = \frac{494}{3}\pi \text{ cm}^3$ $\therefore \text{Mass/Gewig} = \frac{494}{3}\pi \times 30$ $\therefore \text{Mass/Gewig} = 4940\pi$ $\therefore \text{Mass/Gewig} = 15519,47 \text{ grams}$	✓ M A ✓ SF A ✓ Answer/Antw. CA ✓ M A ✓ Ans./Antw. CA NPU (5)
		[29]

QUESTION/VRAAG 10



$Area = a \left(\frac{O_1 + O_n}{2} + O_2 + O_3 + \dots + O_{n-1} \right)$ $329 = \frac{25}{5} \left(\frac{12+18}{2} + x + 13 + 11 + 11,8 \right)$ $329 = 5(x + 50,8)$ $65,8 = x + 50,8$ $15 = x$ <p style="text-align: center;">OR / OF</p> $Area = a(m_1 + m_2 + m_3 + \dots + m_{n-1})$ $329 = \frac{25}{5} \left(\frac{12+x}{2} + \frac{x+13}{2} + \frac{13+11}{2} + \frac{11+11,8}{2} + \frac{11,8+18}{2} \right)$ $329 = 5 \left(\frac{12+x}{2} + \frac{x+13}{2} + 12 + 11,4 + 14,9 \right)$ $329 = 5 \left(\frac{25+2x}{2} + 38,3 \right)$ $65,8 = \frac{25+2x}{2} + 38,3$ $27,5 = \frac{25+2x}{2}$ $55 = 25 + 2x$ $30 = 2x$ $15 = x$	$a = \frac{25}{5}$ $a = 5$	<ul style="list-style-type: none"> ✓ F A ✓ $a = 5$ A ✓ SF CA ✓ S CA ✓ $x = 15$ CA <p style="text-align: center;">OR / OF</p> <ul style="list-style-type: none"> ✓ F A ✓ $a = 5$ A ✓ SF CA ✓ S CA ✓ $x = 15$ CA
		(5)
		[5]
TOTAL/TOTAAL: 150		