Province of the EASTERN CAPE EDUCATION

NATIONAL SENIOR CERTIFICATE

GRADE 12

SEPTEMBER 2023

TECHNICAL MATHEMATICS P1 (DEAF)

MARKS: 150

TIME: 3 hours

This question paper has 14 pages, including a 2-page information sheet and 2 answer sheets.

INSTRUCTIONS AND INFORMATION

Read the **instructions**. **Answer** the **questions**.

1. This **question paper** has **NINE questions**.

2. **Answer ALL** the questions.

3. Answer **QUESTIONS 4.1.3** and **QUESTION** 7.4 on the **ANSWER SHEETS**.
   Write your **name** and **school's name** on the ANSWER SHEETS.
   Hand in the **ANSWER SHEETS** with your **ANSWER BOOK**.

4. **Number** the **answers** the **same** as the numbers on the **question paper**.

5. **Show** ALL **calculations**, **diagrams**, **graphs**, etc. that you have in your answers.

6. **Answers only** will **NOT** necessarily be awarded **full marks**.

7. You **may use** a prescribed **calculator**.
   Some questions will **tell** you **NOT** to use a **calculator**.

8. **Round off** answers to **TWO decimal places**.
   Some questions will **tell** you **how to round off**.

9. **Diagrams** are **NOT** always drawn to **scale**.
   Some questions will **tell** you to **use the scale**.

10. An **information sheet** with formulae is **added** at the **end** of the **question paper**.

11. Write **neatly**.
    Your **work** must be **easy** to **read**.

QUESTION 1

1.1 **Solve for** x:

1.1.1 $(x+17)(x-23)=0$ (2)

1.1.2 $\frac{x^2}{2}+x-\frac{1}{3}=0$ (Correct to **TWO decimal places**.) (3)

1.1.3 $x(2x+1)-3\leq 0$ (4)

1.2 **Solve** for x **and** y if:

$y=x+1$ and $y=3x^2-xy$ (5)

1.3 The **measure** of **Percentage Digestibility Coefficient (*D*)** of a **cow feed** is **measured** as the **difference** between the **amount of food eaten (*E*)** and the **food excreted(expelled)** in the **faeces (*F*), expressed** as a **percentage** of the **food ingested(eaten)**.

$D=100\left(\frac{E-F}{E}\right)$; **where:**

D **= Percentage Digestibility Coefficient (%)**

E **= Food eaten (kg)**

F **= Food excreted (kg)**

1.3.1 Make ***E***, the **food eaten**, the **subject** of the **formula**. (4)

1.3.2 **Calculate** the **amount** of **food eaten** by the **cow** if **percentage digestibility coefficient** is 80% and 3,75 kg of **food** was **excreted(expelled)** in the **faeces**. (2)

1.3.3 Hence or otherwise, **express**, in **grams**, the **amount** of **food eaten** by the cow in **QUESTION 1.3.**2, if 1 000 g = 1 kg. (1)

1.3.4 **Express** the **answer** in QUESTION 1.3.3 in ***scientific notation***. (1)

1.4 **Simplify** the **binary operation**, **without using a calculator**:

$1\,000_2 - 110_2$ (2)

[24]

QUESTION 2

Given: $f(x) = ax^2 - 3x + 2$

2.1 **Determine** the **value** of a if the **discriminant** of $f(x)$ is 6. (3)

2.2 Hence, **without solving** the **equation**, **describe** the **nature** of **roots** of $f(x)$. (1)

2.3 **Determine** the **numerical value** of a for **which** the **roots** of $f(x)$ are **equal**. (3)

[7]

QUESTION 3

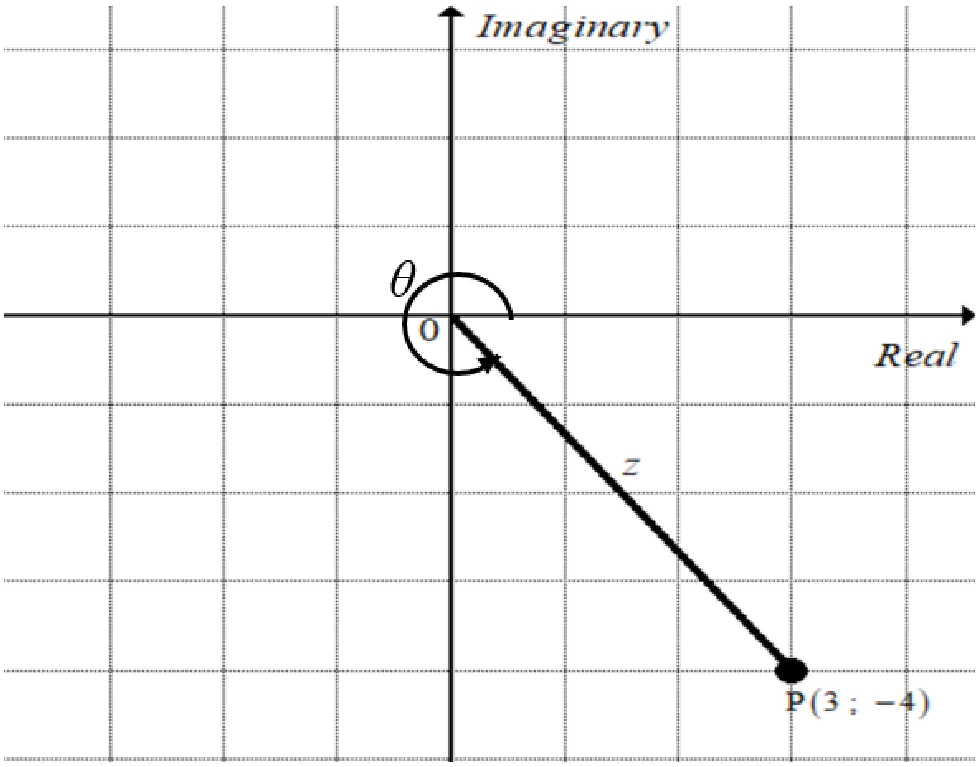
3.1 **Do not use a calculator.**
Simplify:

3.1.1 $\log_x \left(\frac{1}{x} \right)$ (2)

3.1.2 $4^x - 2^{2x-1}$ (3)

3.2 Show that: $\frac{\sqrt{3x^2} \times \sqrt[3]{12x^3}}{2x^2} = \frac{\sqrt[6]{243}}{\sqrt[3]{2}}$ (4)

3.3 **Drawn** below **is** an **Argand diagram** of **complex number** z with point P $(3 ; -4)$:



3.3.1 **Write** the **complex number** z in **rectangular form**. (1)

3.3.2 **Calculate** the **modulus** of z. (2)

3.3.3 **Determine** the **size** of θ. (3)

3.3.4 Hence **express** z in **polar form** (where θ is in degrees). (1)

3.4 **Solve** for x and y if: $\frac{x-i}{2i+1} = y + 3i$ (5)

[21]

QUESTION 4

4.1 **Given** the **function** f, defined by $f(x) = -\frac{2}{x} - 1$

4.1.1 **Write** down the **equation** of the **horizontal asymptote** of f. (1)

4.1.2 **Determine** the x-intercept of f. (3)

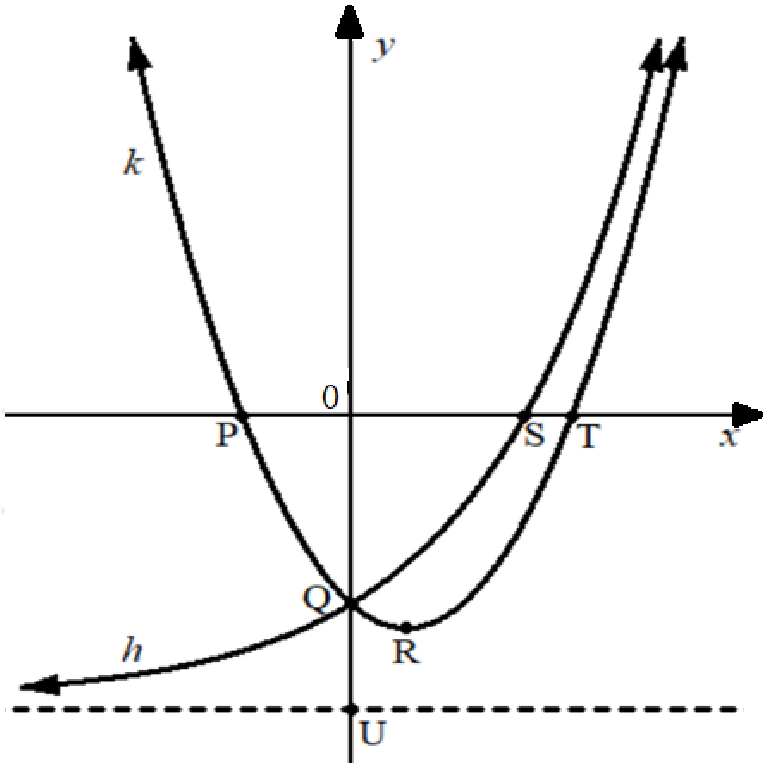
4.1.3 Hence, **sketch** the **graph** of f on the **ANSWER SHEET**. **Show** the **intercepts** with the **axes** and any **asymptotes**. (3)

4.1.4 **Write** down the **range** of f. (1)

4.2 The **diagram shows sketch graphs** of **functions** defined by:

$k(x) = x^2 - x - 2$ and $h(x) = 2^x - 3$

- Points P and T are the x-intercepts of k and S is the x intercept of h.
- Q is a common y-intercept for **both graphs**.
- R is the **turning point** of k.
- The **asymptote** of h cuts the y-axis at U.



Determine:

4.2.1 The y-coordinate of Q (1)

4.2.2 The **equation** of the **asymptote** of h (1)

4.2.3 The x-intercepts of k (3)

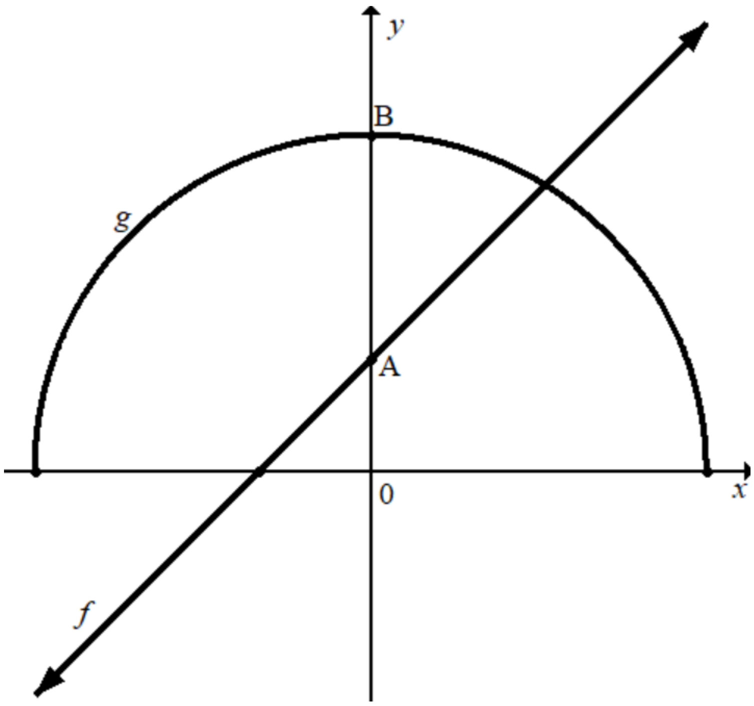
4.2.4 The **coordinates** of S (3)

4.2.5 The **coordinates** of R (4)

4.2.6 The **domain** of h (1)

4.2.7 The **value** of x for which $k(x) - h(x) = 0$ (1)

4.3 The diagram below shows sketch graphs of functions defined by:
$f(x) = x + 1$ and $g(x) = \sqrt{r^2 - x^2}$
Points A and B are the y-intercepts of f and g, respectively.



4.3.1 **Determine** the x-intercept of f. (2)

4.3.2 If f is shifted 2 units upwards, point **A coincides** with point **B** of function g. Hence, write down:

(a) The **coordinates** of point **B**. (2)

(b) The **equation** of a new function $h(x)$, the **result** of the **translation** of f (1)

(c) The **equation** of g. (1)

[28]

QUESTION 5

5.1 An **asset valued** at **R15 000 depreciates** at a **rate** of **3%** per **annum(yearly) compounded quarterly**.
Determine the **amount** to which the **asset depreciates** at the **end** of **5 years**. (3)

5.2 The **price** of **brown bread increased** from **R3,80 in 2004** to **R18,80 in 2023**.

5.2.1 **Determine** the **amount** by which the **price** of **brown bread increased** from **2004 to 2023**. (1)

5.2.2 **Determine** the **inflation rate** from **2004 to 2023**. (5)

5.3 An **artisan deposited** a sum of **R350 000** into an **investment account** that **generates 7% per annum** for **8 years**.

- At the **end** of **4 years** the **artisan deposited** an **amount R*x*** into the **investment account**.
- He **withdrew R100 000** at the **beginning** of the **6th year** and **invested** the **remaining amount** at a **rate** of **7% per annum**.
- It is **compounded monthly** for the **remainde**r of the **investment period**.

Determine the **amount, R*x*,** the **artisan deposited** at the **end** of the **4th year**, if at the **end** of the **8-year investment period** the **investment yielded(produced) R620 000**. (6)
[15]

QUESTION 6

6.1 **Determine** $f'(x)$ by using FIRST PRINCIPLES if $f(x) = 2 - 5x$ (5)

6.2 **Determine**:

6.2.1 $D_x\left(\frac{1}{\sqrt{x}} - 3kx\right)$ (4)

6.2.2 $\frac{dy}{dx}$ if: $y = \frac{2x^3 - 8x}{x - 2}$ (4)

6.3 **Determine** the **coordinates** of the **point** on the **curve** $h(x) = 3x^2 - 4x$ where the **gradient** of the **tangent** is **equal** to 2. (4)
[17]

QUESTION 7

Given: $f(x) = -x^3 - 4x^2 - 3x$

7.1 **Determine** the x-intercepts of f. (4)

7.2 **Write** down y-intercept of f. (1)

7.3 **Determine** the **coordinates** of the **turning points** of f. (5)

7.4 **Sketch** the **graph** of f on the **ANSWER SHEET**.
Show all the **coordinates** of the **turning points** and **intercepts** with the **axis**. (4)

7.5 **Determine** the **average gradient** of f between $x = -2$ and $x = -1$. (4)
[18]

QUESTION 8

The **second Newton's Law of Motion** of a **body falling due** to **gravity** is given by:

$s = ut + \frac{1}{2}gt^2$ where:

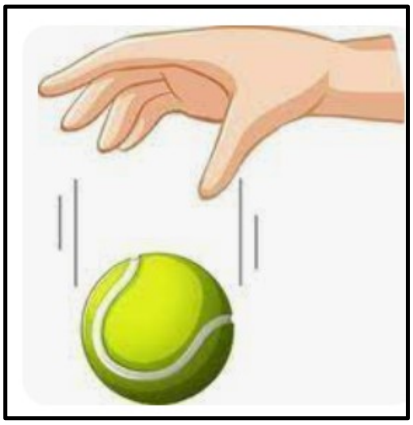
s is the **displacement** of an **object**, in meters

u is the **initial velocity** of an **object**, in m/s

t is the **time taken** for the **fall**, in seconds

g is the **gravitational acceleration** = 10 m/s^2

The picture **shows** a **ball dropped** from a **hand** and **falling** to the **ground**.



8.1 **Determine** the **displacemen**t of the **ball** after **4 seconds** if its **initial velocity** was 5 m/s. (2)

8.2 **Write** down the **equation** of **final velocity** of this **object**, as a **function** of **time**. (2)

8.3 Hence or otherwise, **calculate** the **final velocity** after **4 seconds**. (2)

8.4 **Determine** the **average rate** of **change** over the **period of 4 seconds**. (2)
[8]

QUESTION 9

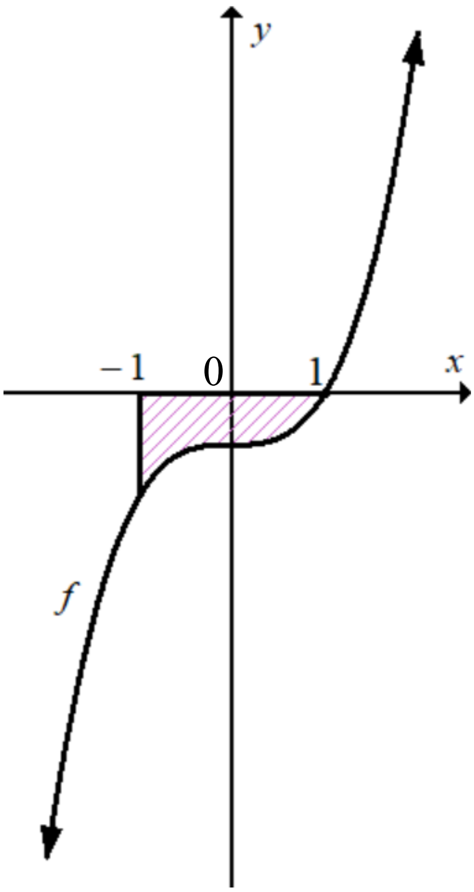
9.1 **Given** $f(x) = x^3 - 8$

Simplify the **integrals**:

9.1.1 $\int f(x)\,dx$ (3)

9.1.2 $\int\left(\frac{f(x)}{x^2+2x+4} - 2^{3x}\right)dx$ (4)

9.2 The **diagram** shows the **shaded area bounded** by the **function** f defined by $f(x) = x^3 - 1$ and the x-axis between the points where $x = -1$ and $x = 1$.



Determine the **area** of the **shaded region bounded** by f and the x-axis between the **point** where $x = -1$ and $x = 1$. (5)

[12]

TOTAL: 150

INFORMATION SHEET: TECHNICAL MATHEMATICS

$$x = \frac{-b \pm \sqrt{b^2 - 4ac}}{2a} \qquad x = -\frac{b}{2a} \qquad y = \frac{4ac - b^2}{4a}$$

$a^x = b \Leftrightarrow x = \log_a b$, $\quad a > 0,\ a \neq 1$ and $b > 0$

$$A = P(1 + ni) \qquad A = P(1 - ni) \qquad A = P(1 + i)^n \qquad A = P(1 - i)^n$$

$$i_{eff} = \left(1 + \frac{i}{m}\right)^m - 1$$

$$f^{/}(x) = \lim_{h \to 0} \frac{f(x+h) - f(x)}{h}$$

$\int k x^n dx = \frac{k x^{n+1}}{n+1} + C$, $n, k \in \mathbb{R}$ with $n \neq -1$ and $k \neq 0$

$\int \frac{k}{x} dx = k \ln x + C$, $x > 0$ and $k \in \mathbb{R}$; $k \neq 0$

$\int k a^{nx} dx = \frac{k a^{nx}}{n \ln a} + C$, $a > 0$; $a \neq 1$ and $k, a \in \mathbb{R}$; $k \neq 0$

$$d = \sqrt{(x_2 - x_1)^2 + (y_2 - y_1)^2} \qquad M\left(\frac{x_2 + x_1}{2} ; \frac{y_2 + y_1}{2}\right)$$

$$y = mx + c \qquad y - y_1 = m(x - x_1) \qquad m = \frac{y_2 - y_1}{x_2 - x_1} \qquad \tan\theta = m$$

$$\frac{x^2}{a^2} + \frac{y^2}{b^2} = 1$$

In ΔABC: $\frac{a}{\sin A} = \frac{b}{\sin B} = \frac{c}{\sin C}$ $\qquad a^2 = b^2 + c^2 - 2bc \cdot \cos A$

area of Δ ABC $= \frac{1}{2} ab \cdot \sin C$

$$\sin^2\theta + \cos^2\theta = 1 \qquad 1 + \tan^2\theta = \sec^2\theta \qquad 1 + \cot^2\theta = \text{cosec}^2\theta$$

$\pi\, rad = 180°$

Angular velocity $= \omega = 2\,\pi\,n$ where n = rotation frequency

Angular velocity $= \omega = 360°\,n$ where n = rotation frequency

Circumferential velocity $= v = \pi D n$ where D = diameter and n = rotation frequency

Circumferential velocity $= v = \omega\, r$ where ω = angular velocity and r = radius

Arc length $= s = r\theta$ where r = radius and θ = central angle in radians

Area of a sector $= \dfrac{r\,s}{2}$ where r = radius, s = arc length

Area of a sector $= \dfrac{r^2\,\theta}{2}$ where r = radius and θ = central angle in radians

$4h^2 - 4dh + x^2 = 0$ where h = height of segment, d = diameter of circle and x = length of chord

$A_T = a\left(m_1 + m_2 + m_3 + ... + m_n\right)$ where $a =$ width of equal parts, $m_1 = \dfrac{o_1 + o_2}{2}$
$o_n = n^{th}$ ordinate and n = number of ordinates

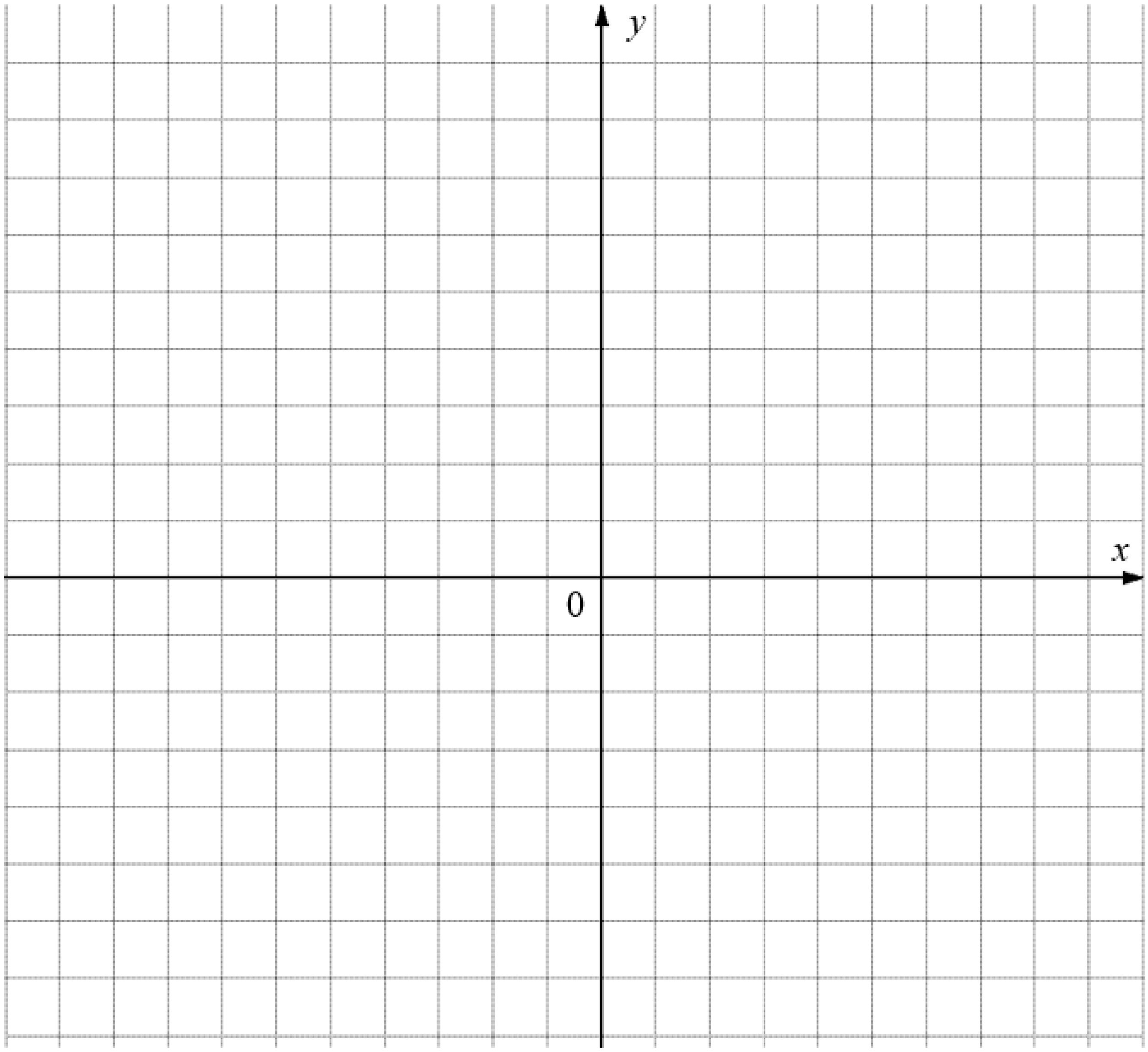
OR

$A_T = a\left(\dfrac{o_1 + o_n}{2} + o_2 + o_3 + ... + o_{n-1}\right)$ where $a =$ width of equal parts, $o_n = n^{th}$ ordinate and $n =$ number of ordinates

DIAGRAM SHEET

SURNAME AND NAME	
SCHOOL	

QUESTION 4.1.3



QUESTION 7.4