



**NATIONAL
SENIOR CERTIFICATE
*NASIONALE
SENIORSERTIFIKAAT***

GRADE/*GRAAD* 12

SEPTEMBER 2023

**MATHEMATICS P2/*WISKUNDE V2*
MARKING GUIDELINE/*NASIENRIGLYN***

MARKS/*PUNTE*: 150

This marking guideline consists of 14 pages.
Hierdie nasienriglyn bestaan uit 14 bladsye.

QUESTION 1/VRAAG 1

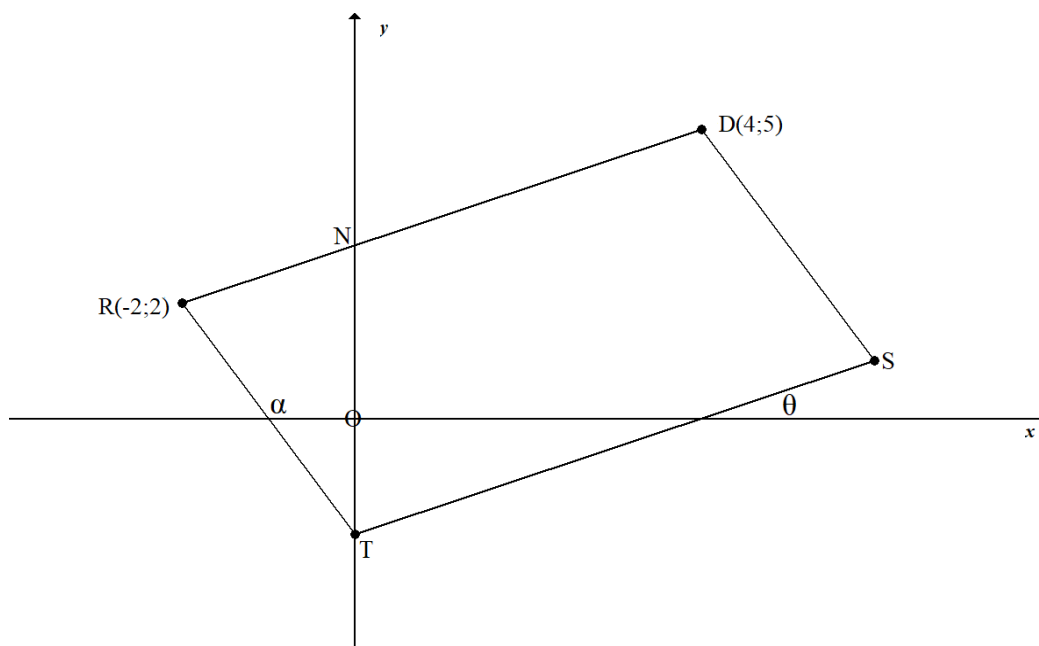
Data Set / <i>Datastel</i> :		29	27	24	31	22	19	30	
1.1.1(a)	Mean / <i>Gemiddelde</i> = $\frac{182}{7} = 26$	✓✓ mean / <i>gemiddelde</i>						(2)	
1.1.1(b)	Standard deviation / <i>standaardafwyking</i> = 4,14	✓ SD / <i>standaardafwyking</i>						(1)	
1.1.2	{26 – 4,14; 26 + 4,14} = {21, 86; 30, 14} 5 players / <i>spelers</i>	✓✓ calculations <i>berekeninge</i> ✓ answer / <i>antwoord</i>						(3)	
1.1.3	Rugby team has the same average number of push-ups. The rugby team results were clustered around the mean because of the smaller standard deviation. <i>Rugbyspan het dieselfde gemiddelde aantal opstote. Die rugbyspan se uitslae was rondom die gemiddelde gegroepeer agv die kleiner standaardafwyking.</i>	✓✓ for any two valid comments using SD and the mean <i>Vir enige twee geldige opmerkings in gebruik van standaardafwyking en die gemiddelde</i>						(2)	
1.2.1	50%	✓ answer / <i>antwoord</i>						(1)	
1.2.2	Mean / <i>Gemiddelde</i> Distribution skewed to the right. (positively skewed) <i>Verspreiding is skeef na regs (positief skeef)</i>	✓ answer / <i>antwoord</i> ✓ reason / <i>rede</i>						(2)	
								[11]	

QUESTION 2/VRAAG 2

Third Term % Derde Kw. %	71	80	59	38	41	98	80	88	91	94	64	94	70	42	64
Final Term % Finale Kw. %	74	77	58	41	42	98	78	92	85	92	68	96	73	52	71

2.1	a = 9, 035 b = 0, 895 $\hat{y} = 9,035 + 0,895x$	✓ for a / <i>vir a</i> ✓ for b / <i>vir b</i> ✓ for equation / <i>vir vergelyking</i>	(3)
2.2	r = 0, 98	✓ answer / <i>antwoord</i>	(1)
2.3.1	$y = 9,035 + 0,895(48)$ $y \approx 52$	✓ substitution / <i>vervanging</i> ✓ answer / <i>antwoord</i>	(2)
2.3.2	correlation is very strong OR 48 is within domain of regression line. <i>korrelasie is baie sterk OF 48 is binne die gebied van die regressie-lyn.</i>	✓ answer / <i>antwoord</i>	(1)
2.4.1	50% is outside the domain of the line (data set) OR (50 ; 80) is an outlier. <i>50% is buite die gebied van die lyn (datastel) OF (50 ; 80) is 'n uitskieter</i>	✓ answer / <i>antwoord</i>	(1)
2.4.2	Increase the gradient / <i>Vermeerder die gradiënt</i>	✓ answer / <i>antwoord</i>	(1)
			[9]

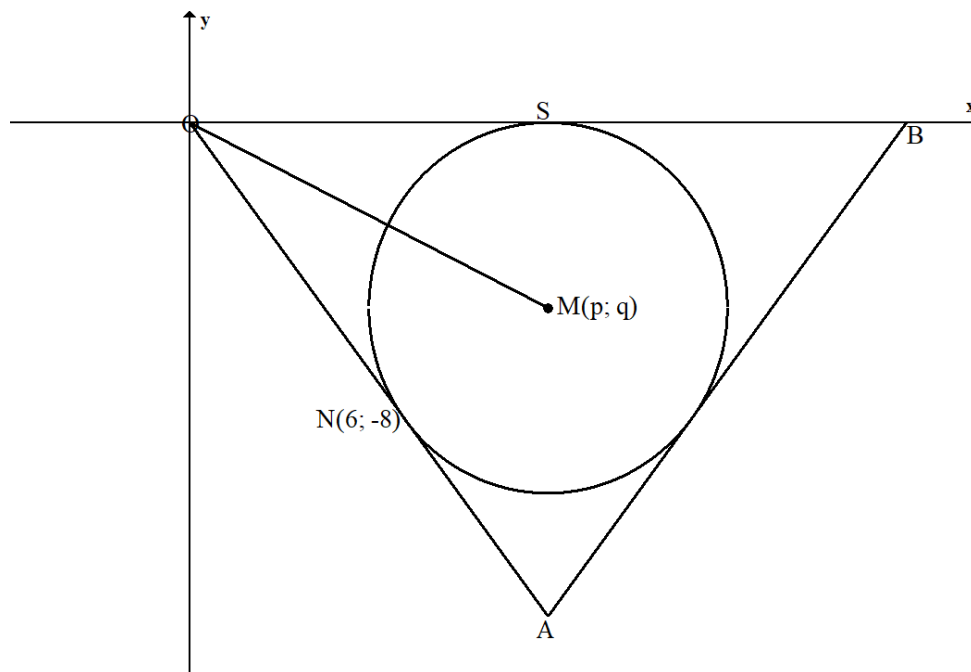
QUESTION 3 / VRAAG 3



3.1	T(0; -2)	✓ answer / antwoord	(1)
3.2.1	$m_{RT} = \frac{2 - (-2)}{-2 - 0} = -\frac{4}{2} = -2$	✓ substitution / vervanging ✓ answer / antwoord	(2)
3.2.2	$\tan \theta = \frac{1}{2}$ $\therefore \theta = 26,57^\circ$ $\tan \alpha = -2$ $\therefore \alpha = 116,57^\circ$ $\therefore \hat{R\hat{T}S} = 116,57^\circ - 26,57^\circ = 90^\circ$ OR / OF $m_{TS} = \frac{1}{2}$ $m_{RT} = -2$ $m_{TS} \times m_{RT} = -1$ $\hat{R\hat{T}S} = 90^\circ$	✓ for / vir $\tan \theta = \frac{1}{2}$ ✓ for / vir $\theta = 26,57^\circ$ ✓ for / vir $\tan \alpha = -2$ ✓ for / vir $\alpha = 116,57^\circ$ ✓ $\hat{R\hat{T}S} = 90^\circ$ OR / OF ✓ $m_{TS} = \frac{1}{2}$ ✓ $m_{RT} = -2$ ✓✓ product of gradients <i>produk van gradiënte</i> ✓ $\hat{R\hat{T}S} = 90^\circ$	(5)
3.3	$y = \frac{1}{2}x + c$ $2 = \frac{1}{2}(-2) + c$ $c = 3$ $\therefore y = \frac{1}{2}x + 3$	✓ gradient / gradiënt ✓ substitution of point R or D <i>vervang van punt R of D</i> ✓ answer / antwoord	(3)

3.4	RDST is a parallelogram (opposite sides parallel) RDST is 'n parallelogram (teenoorst. sye ewewydig) Midpoint of DT / Middelpunt van DT: $\left(\frac{4+0}{2}; \frac{5-2}{2}\right) = \left(2; \frac{3}{2}\right)$ Midpoint of RS is the same as midpoint of DT (diagonals bisect each other.) Middelpunt van RS is dieselfde as die middelpunt van DT (hoeklyne halveer mekaar) Midpoint of RS / Middelpunt van RS: $\left(2; \frac{3}{2}\right)$ OR / OF $S(6; 1)$ Midpoint of RS / Middelpunt van RS: $\left(\frac{6-2}{2}; \frac{1+2}{2}\right) = \left(2; \frac{3}{2}\right)$	✓ substitution in the MP formula vervang in die MP formule ✓ S/R ✓ answer / antwoord OR / OF ✓✓ $S(6; 1)$ ✓ answer / antwoord	(3)
3.5	$N(0; 3)$ $RN = \sqrt{2^2 + 1^2} = \sqrt{5}$ $RT = \sqrt{2^2 + 4^2} = \sqrt{20}$ Area / Oppervlakte = $\frac{1}{2} \times \sqrt{20} \times \sqrt{5}$ = 5 square units / eenhede² OR / OF $TN = 5$ units / eenhede Height / Hoogte = 2 units / eenhede Area / Oppervlakte = $\frac{1}{2} \times 5 \times 2$ Area / Oppervlakte = 5 square units / eenhede²	✓ coordinates of N koördinate van N ✓ for / vir RN ✓ for / vir RT ✓ for the answer / vir die antwoord OR / OF ✓ $TN = 5$ units / eenhede ✓ Height/hoogte = 2 units / eenhede ✓ sub. into formula vervang in formule ✓ answer / antwoord	(4)
			[18]

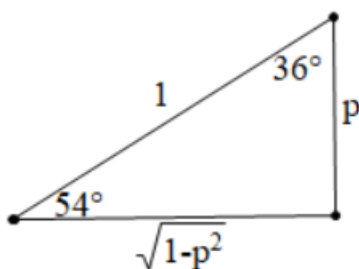
QUESTION 4 / VRAAG 4



4.1.1	$ON = \sqrt{(6-0)^2 + (-8)^2}$ $= \sqrt{100} = 10 \text{ units / eenhede}$	✓ substitution in correct formula <i>vervanging in korrekte formule</i> ✓ answer / antwoord	(2)
4.1.2	$ON = OS$ (tangents from the same point) <i>(raaklyne vanaf dieselfde punt)</i> $\therefore p = 10 \text{ units / eenhede}$	✓ S and/en R ✓ answer / antwoord	(2)
4.1.3	$ON \perp NM$ (tan – radius) / <i>(raaklyn – radius)</i> $m_{ON} = \frac{-8}{6} = \frac{-4}{3}$ $m_{NM} = \frac{3}{4}$	✓ S and/en R ✓ gradient of ON / <i>gradiënt van ON</i> ✓ gradient of NM <i>gradiënt van NM</i>	(3)
4.1.4	$m_{NM} = \frac{q+8}{10-6} = \frac{3}{4}$ $\frac{q+8}{4} = \frac{3}{4}$ $q = -5$	✓ for subs and equating <i>vir vervanging en gelyk stel</i> ✓ answer / antwoord	(2)
4.2	$MS = r = 5 \text{ units / eenhede}$ $(x-10)^2 + (y+5)^2 = 25$	✓ radius / radius ✓ centre sub/ vervang middelpunt ✓ answer / antwoord	(3)
4.3	$k = 5$ OR/OF $k = 15$	✓ $k = 5$ OR/OF ✓ $k = 15$	(2)

4.4	Coordinates of the point directly opposite N is C. <i>Koördinate van die punt regoor N is C.</i> $C\left(\frac{x+6}{2} = 10; \frac{y-8}{2} = -5\right)$ $C(14; -2)$ Equation of the tangent at C: <i>Vergelyking van die raaklyn by C:</i> $y + 2 = -\frac{4}{3}(x - 14)$ $y = -\frac{4}{3}x + \frac{50}{3}$ $\therefore 0 < t < \frac{50}{3}$	✓ formula and sub / <i>formule en vervanging</i> ✓ for x-coordinate <i>vir x-koördinaat</i> ✓ for y-coordinate <i>vir y-koördinaat</i> ✓ substitution / vervanging ✓ for the answer / <i>vir die antwoord</i> ✓ for the value of t. <i>vir die waarde van t</i>	(6)
4.5	They will not touch. The new circle is the old circle shifted up by 11. <i>Hulle sal nie raak nie.</i> <i>Die nuwe sirkel is die ou sirkel 11 eenhede opwaarts geskuif.</i>	✓ answer / <i>antwoord</i> ✓ any valid reason / <i>enige geldige antwoord</i>	(2)
			[22]

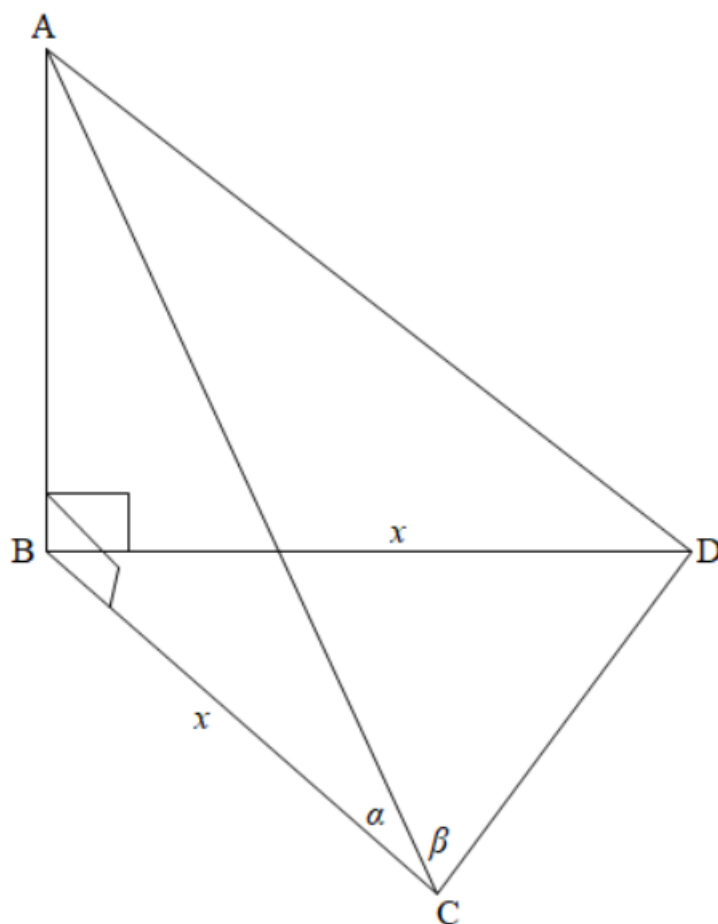
QUESTION 5/VRAAG 5



5.1.1	$\sin 594^\circ$ $= \sin 234^\circ$ $= -\sin 54^\circ$ $= -p$	$\checkmark -\sin 54^\circ$ $\checkmark -p$	(2)
5.1.2	$\cos 36^\circ$ $= p$	$\checkmark \checkmark$ answer / <i>antwoord</i>	(2)
5.1.3	$\cos 18^\circ$ $= \sin 72^\circ$ $= \sin 2(36^\circ)$ $= 2 \sin 36^\circ \cos 36^\circ$ $= 2 \left(\frac{\sqrt{1-p^2}}{1} \right) \left(\frac{p}{1} \right)$ $= 2p(\sqrt{1-p^2})$	$\checkmark$ for / <i>vir</i> $\sin 72^\circ$ $\checkmark$ for / <i>vir</i> $\sin 2(36^\circ)$ $\checkmark$ for / <i>vir</i> $2 \sin 36^\circ \cos 36^\circ$ $\checkmark$ for / <i>vir</i> $2p(\sqrt{1-p^2})$	(4)
5.2	$\frac{\cos 140^\circ - \sin(90^\circ - \theta)}{\sin 410^\circ + \cos(-\theta)}$ $= \frac{\cos(90^\circ + 50^\circ) - \cos \theta}{\sin 50^\circ + \cos \theta}$ $= \frac{-\sin 50^\circ - \cos \theta}{\sin 50^\circ + \cos \theta}$ $= \frac{-(\sin 50^\circ + \cos \theta)}{(\sin 50^\circ + \cos \theta)}$ $= -1$	$\checkmark -\sin 50^\circ$ $\checkmark \cos \theta$ $\checkmark \sin 50^\circ$ $\checkmark \cos \theta$ $\checkmark$ for the common factor <i>vir die gemene faktor</i> $\checkmark$ for the answer <i>vir die antwoord</i>	(6)

5.3	$\cos(x + 65^\circ) \cdot \cos(x + 20^\circ) - \sin(x + 245^\circ) \cdot \sin(x + 20^\circ)$ $= \cos(x + 65^\circ) \cdot \cos(x + 20^\circ) + \sin(x + 65^\circ) \cdot \sin(x + 20^\circ)$ $= \cos[(x + 65^\circ) - (x + 20^\circ)]$ $= \cos 45^\circ$ $= \frac{1}{\sqrt{2}}$	✓ reduction / <i>reduksie</i> ✓ compound angle <i>saamgestelde hoek</i> ✓ $\cos 45^\circ$ ✓ answer / <i>antwoord</i>	(4)
5.4	$\cos^2 x - \sin^2 x = \frac{1}{2}$ $\cos 2x = \frac{1}{2}$ $2x = 60^\circ + 360^\circ \cdot k$ or/of $2x = 300^\circ + 360^\circ \cdot k$ $x = 30^\circ + 180^\circ \cdot k$ or/of $x = 150^\circ + 180^\circ \cdot k \quad k \in \mathbb{Z}$ OR / OF $2 \cos^2 x - 2 \sin^2 x = 1$ $2 \cos^2 x - 2 \sin^2 x = \sin^2 x + \cos^2 x$ $3 \sin^2 x = \cos^2 x$ $\tan^2 x = \frac{1}{3}$ $\tan x = \frac{1}{\sqrt{3}}$ $x = 30^\circ$ (reference angle / <i>verwysingshoek</i>) $x = 30^\circ + 180^\circ \cdot k, k \in \mathbb{Z}$	✓ $\cos 2x = \frac{1}{2}$ ✓ for / <i>vir</i> $2x$ in both quadrants / <i>in beide kwadrante</i> ✓ $x = 30^\circ + 180^\circ \cdot k$ ✓ $x = 150^\circ + 180^\circ \cdot k$ OR / OF ✓ multiplying by 2 and using identity / <i>vermenigvuldig met 2 en gebruik van identiteit</i> ✓ $3 \sin^2 x = \cos^2 x$ ✓ $\tan x = \frac{1}{\sqrt{3}}$ ✓ answer / <i>antwoord</i>	(4)
5.5.1	$\text{LHS} = \frac{\sin 2\theta \cdot \tan \theta}{\cos 2\theta + 1}$ $= \frac{2 \sin \theta \cos \theta \cdot \frac{\sin \theta}{\cos \theta}}{2 \cos^2 \theta - 1 + 1}$ $= \frac{2 \sin^2 \theta}{2 \cos^2 \theta}$ $= \tan^2 \theta$	✓ $2 \sin \theta \cos \theta$ ✓ $\frac{\sin \theta}{\cos \theta}$ ✓ $2 \cos^2 \theta - 1$ ✓ $\frac{2 \sin^2 \theta}{2 \cos^2 \theta}$	(4)
5.5.2	$\cos 2\theta + 1 = 0$ $\cos 2\theta = -1$ $2\theta = 180^\circ$ $\theta = 90^\circ$	✓ $\cos 2\theta + 1 = 0$ ✓ $\cos 2\theta = -1$ ✓ $2\theta = 180^\circ$ ✓ $\theta = 90^\circ$	(4)
			[30]

QUESTION 6 / VRAAG 6



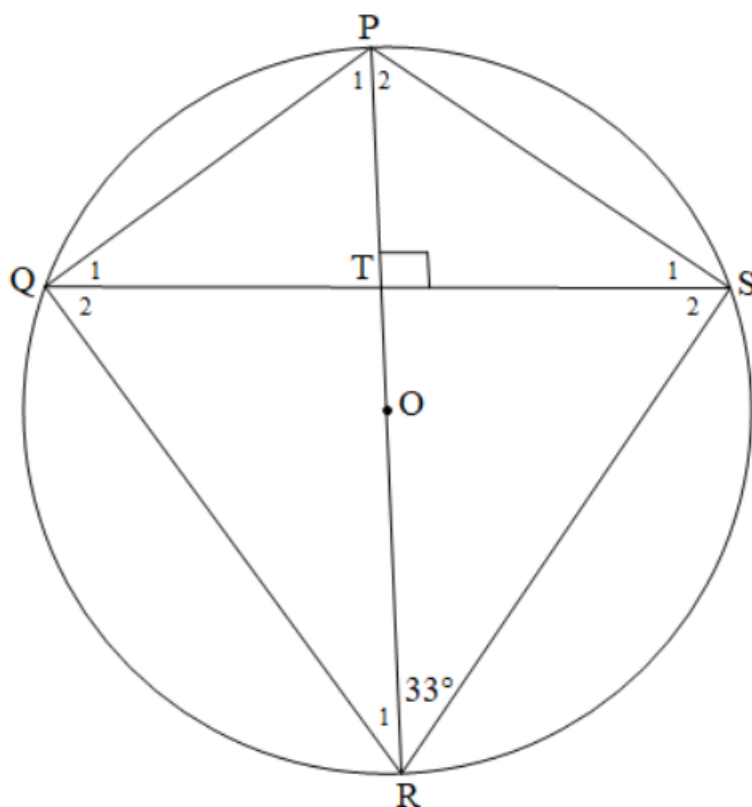
6.1	$\triangle ABC \equiv \triangle ABD$ (SAS) / (SHS)	✓ S and/en R	(1)
6.2	$\cos \alpha = \frac{x}{AC}$ $AC = \frac{x}{\cos \alpha}$	✓ S ✓ answer / <i>antwoord</i>	(2)
6.3	$CD^2 = \left(\frac{x}{\cos \alpha}\right)^2 + \left(\frac{x}{\cos \alpha}\right)^2 - 2\left(\frac{x}{\cos \alpha}\right)\left(\frac{x}{\cos \alpha}\right)\cos(180^\circ - 2\beta)$ $CD^2 = \frac{x^2}{\cos^2 \alpha} + \frac{x^2}{\cos^2 \alpha} + 2\left(\frac{x^2}{\cos^2 \alpha}\right)\cos 2\beta$ $CD^2 = \frac{2x^2}{\cos^2 \alpha}(1 + \cos 2\beta)$ $CD^2 = \frac{2x^2}{\cos^2 \alpha}(1 + (1 - 2\sin^2 \beta))$ $CD^2 = \frac{2x^2}{\cos^2 \alpha} \times 2\cos^2 \beta$ $CD^2 = \frac{4x^2 \cos^2 \beta}{\cos^2 \alpha}$ $\therefore CD = \frac{2x \cos \beta}{\cos \alpha}$	✓ cos rule / <i>cos-reël</i> ✓ simplification <i>vereenvoudiging</i> ✓ double angles expansion <i>dubbelhoeke uitbreiding</i> ✓ simplification <i>vereenvoudiging</i>	
	OR / OF	OR / OF	

	$\widehat{ADB} = \beta$ $\widehat{CAD} = 180^\circ - 2\beta$ $\frac{CD}{\sin(180^\circ - 2\beta)} = \frac{AC}{\sin \beta}$ $CD = \frac{AC \sin 2\beta}{\sin \beta}$ $CD = \frac{x \cdot 2 \sin \beta \cos \beta}{\cos \alpha \sin \beta}$ $CD = \frac{2x \cos \beta}{\cos \alpha}$	✓ for /vir $\widehat{ADB} = \beta$ and/en $\widehat{CAD} = 180^\circ - 2\beta$ ✓ use of sin rule <i>gebruik van sinus-reël</i> ✓ substitution of AC <i>vervangings van AC</i> ✓ simplification / <i>vereenvoudiging</i>	(4)
6.4	$CD = \frac{2x \cos \beta}{\cos \alpha}$ $CD = \frac{2(25) \cos 65,62^\circ}{\cos 30^\circ}$ $CD = 23,83 \text{ cm}$	✓ substitution / <i>vereenvoudiging</i> ✓ answer / <i>antwoord</i>	(2)
[9]			

QUESTION 7 / VRAAG 7

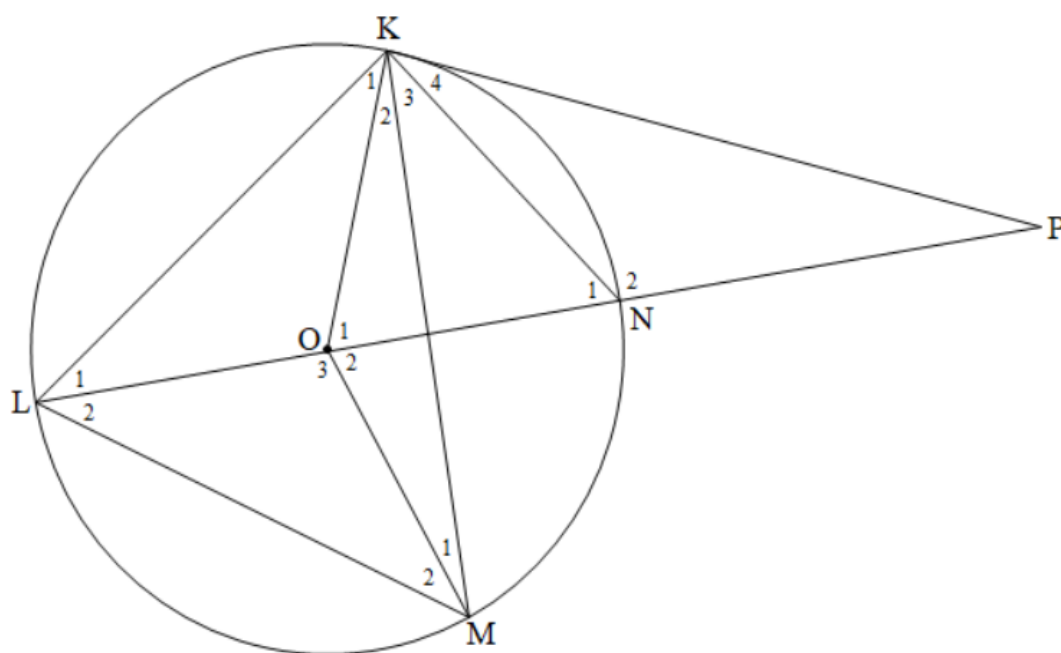
7.1	$f(180^\circ) = -0,71$ $\therefore$ Range: <i>Terrein: Waardeversameling:</i> $-0,71 \leq y \leq 1$ OR / OF $[-0,71; 1]$	✓ $f(180^\circ)$ ✓ answer / <i>antwoord</i>	(2)
7.2	✓ intercepts / <i>afsnitte</i> ✓ shape / <i>vorm</i> ✓ turning points / <i>draaipunte</i>		(3)
7.3	Period / <i>Periode</i> = 180°	✓ answer / <i>antwoord</i>	(1)
7.4	$-45^\circ < x < 45^\circ$	✓✓ answer / <i>antwoord</i>	(2)
7.5	$x = -45^\circ$ or / <i>of</i> $x = 135^\circ$	✓ $x = -45^\circ$ ✓ $x = 135^\circ$	(2)
7.6	$g(x) = \cos(x + 15^\circ)$	✓ answer / <i>antwoord</i>	(1)
[11]			

QUESTION 8 / VRAAG 8



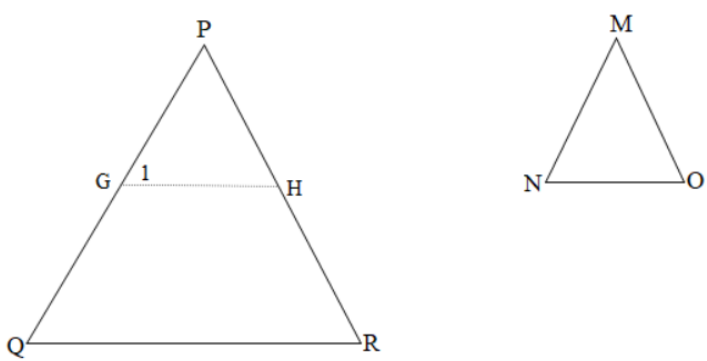
8.1.1	$\hat{Q}_1 = 33^\circ$ ($\angle$ s in the same segment) ($\angle$ e in dieselfde segment)	✓ S ✓ R	(3)
	$\hat{P}_1 = 57^\circ$ ($\angle$ s of a triangle) / ($\angle$ e van 'n driehoek)	✓ S and/en R	
8.1.2	$\hat{Q} = 90^\circ$ ($\angle$ subtended by the diameter) ($\angle$ onderspan deur middellyn)	✓ S and/en R	(2)
	$\hat{Q}_2 = 57^\circ$ (complementary $\angle$ s / $\angle$ s of a triangle) (komplementêre $\angle$ e / $\angle$ e van 'n driehoek)	✓ S and/en R	
8.2	$QT = TS = 8 \text{ cm}$ (line from centre perp to chord) (lyn vanaf middelpunt loodreg op koord)	✓ S and/en R	(4)
	$OQ = OS = 10 \text{ cm}$ (radii) / (radiusse)	✓ S and/en R	
	$OQ^2 = TO^2 + QT^2$ (Pythagoras) / (Pythagoras) $10^2 = OT^2 + 8^2$	✓ S and/en R	
	$TO = \sqrt{100 - 64} = 6 \text{ cm}$	✓ answer / antwoord	
			[9]

QUESTION 9 / VRAAG 9



9.1	$\hat{LKN} = 90^\circ$ $\hat{OKP} = 90^\circ$	✓ answer / antwoord ✓ answer / antwoord	(2)
9.2.1	$\hat{L}_1 = x$ (tan – chord theorem) / (raaklyn-koord stelling)	✓ S ✓ R	(2)
9.2.2	$\hat{K}_1 = x$ ($\angle$ s opp. = sides) / ($\angle$ e teenoor = sye)	✓ S ✓ R	(2)
9.2.3	In $\triangle LKP$; $\hat{LKP} = 90^\circ + x$ and/en $\hat{L}_1 = x$ $\therefore \hat{P} = 90^\circ - 2x$ ($\angle$ s of a triangle) ($\angle$ e van 'n driehoek) OR / OF $\hat{N}_2 = 90^\circ + x$ (ext $\angle$ of Δ) / (buite $\angle$ van Δ) $\therefore \hat{P} = 90^\circ - 2x$ ($\angle$ s of a triangle) ($\angle$ e van 'n driehoek)	✓ S ✓ R OR / OF ✓ S/R ✓ S/R	(2)
9.3	$\hat{OMP} = 90^\circ$ (tan $\perp$ rad) / (raaklyn $\perp$ radius) $\hat{OKP} = 90^\circ$ (proven) / (bewys) $\therefore$ KOMP is a c.q. (opp. $\angle$ s suppl.) KOMP is 'n kv (teenoorst. $\angle$ e supplementêr)	✓ S & R ✓ S ✓ R	(3)
			[11]

QUESTION 10 / VRAAG 10

10.1	 On PQ, mark PG = MN and on PR, mark off PH = MO Join GH In $\triangle PGH$ and $\triangle MNO$ (1) PG = MN (construction) (2) $\hat{P} = \hat{M}$ (given) (3) PH = MO (construction) $\therefore \triangle PGH \equiv \triangle MNO$ (SAS) $\therefore \hat{G}_1 = \hat{N}$ (Congruency) But $\hat{Q} = \hat{N}$ (given) $\therefore \hat{G}_1 = \hat{Q}$ $\therefore GH \parallel QR$ (corresponding angles formed =) $\therefore \frac{PG}{PQ} = \frac{PH}{PR}$ (prop.int; $GH \parallel QR$) But PG = MN and PH = MO $\therefore \frac{MN}{PQ} = \frac{MO}{PR}$	✓ construction ✓ congruency proof ✓ S ✓ R ✓ S and/en R ✓ S	
	Afrikaans Op PQ, merk af PG = MN en op PR, merk af PH = MO Verbind GH In $\triangle PGH$ en $\triangle MNO$ (1) PG = MN (konstruksie) (2) $\hat{P} = \hat{M}$ (gegee) (3) PH = MO (konstruksie) $\therefore \triangle PGH \equiv \triangle MNO$ (SHS) $\therefore \hat{G}_1 = \hat{N}$ (Kongruensie) Maar, $\hat{Q} = \hat{N}$ (gegee) $\therefore \hat{G}_1 = \hat{Q}$ $\therefore GH \parallel QR$ (ooreenkomstige hoeke gevorm =) $\therefore \frac{PG}{PQ} = \frac{PH}{PR}$ (eweredigheid; $GH \parallel QR$) Maar, PG = MN en PH = MO $\therefore \frac{MN}{PQ} = \frac{MO}{PR}$	✓ konstruksie ✓ kongruensie bewys ✓ S ✓ R ✓ S and/en R ✓ S	(6)

