



**NATIONAL
SENIOR CERTIFICATE /
*NASIONALE
SENIORSERTIFIKAAT***

GRADE/GRAAD 12

SEPTEMBER 2023

**TECHNICAL MATHEMATICS P1
MARKING GUIDELINE
*TEGNIESE WISKUNDE V1
NASIENRIGLYN***

MARKS/ PUNTE: 150

MARKING CODES/NASIENKODES	
A	Accuracy/Akkuraatheid
AO	Answer only/slegs antwoord
CA	Consistent accuracy/Volgehoue akkuraatheid
M	Method/Metode
R	Rounding/Afronding
NPR	No penalty for rounding/Geen penalisering vir afronding nie
NPU	No penalty for units omitted/Geen penalisering vir eenhede weggelaat nie
S	Simplification/Vereenvoudiging
SF	Substitution in the correct formula/Vervanging in die korrekte formule

These marking guidelines consist of 18 pages.
Hierdie nasienriglyne bestaan uit 18 bladsye.

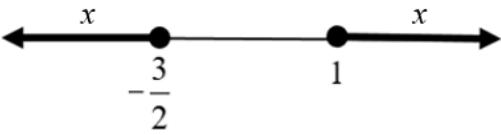
NOTE:

- If a candidate answers a question TWICE, only mark the FIRST attempt.
- The method of consistent accuracy marking must be applied to all aspects of the marking guideline where applicable as indicated with the marking code CA.
- If a candidate strikes off a response to a question and does not attempt the question again, then the struck off question should be marked.

LET WEL:

- Indien 'n kandidaat 'n vraag TWEE keer beantwoord, sien slegs die EERSTE poging na.
- Die metode van volgehoue akkuraatheid-nasien moet waar moontlik op alle aspekte van die nasienriglyne toegepas word soos aangedui deur die nasienkode CA.
- Indien 'n kandidaat 'n antwoord deurhaal en nie poog om die vraag weer te beantwoord dan moet die deurgehaalde antwoord gemerk word.

QUESTION/VRAAG 1

1.1.1	$(x+17)(x-23)=0$ $x = -17 \text{ or/of } x = 23$	✓ $x = -17$ ✓ $x = 23$	A A (2)
1.1.2	$\frac{x^2}{2} + x - \frac{1}{3} = 0$ $x = \frac{-(1) \pm \sqrt{(1)^2 - 4\left(\frac{1}{2}\right)\left(-\frac{1}{3}\right)}}{2\left(\frac{1}{2}\right)}$ $\therefore x = -2,29 \text{ or/of } x = 0,29$	✓ Substitution / vervanging ✓ $x = -2,29$ ✓ $x = 0,29$	A CA CA (3)
1.1.3	$x(2x+1)-3 \leq 0$ $2x^2 + x - 3 \leq 0$ $(2x+3)(x-1) \leq 0$ CVs/KWs: $-\frac{3}{2}$ and/en 1 $\therefore x \leq -\frac{3}{2} \text{ or/of } x \geq 1$ OR/OF 	✓ Standard form /standaardvorm ✓ Factorisation/faktorisering / ✓ Both critical values/beide kritiese waardes ✓ correct notation/korrekte notasie OR/OF ✓ Correct number line /korrekte getallelyn	A SF CA CA CA OR/OF CA (4)

1.3.1	$D = 100 \left(\frac{E - F}{E} \right)$ $D.E = 100(E - F)$ $D.E = 100E - 100F$ $100F = 100E - D.E$ $100F = (100 - D)E$ $\therefore E = \frac{100F}{100 - D}$ <p style="text-align: center;">OR/OF</p> $D = 100 \left(\frac{E - F}{E} \right)$ $D.E = 100(E - F)$ $\frac{D.E}{100} = E - F$ $\frac{D.E}{100} - E = F$ $E \left(\frac{D}{100} - 1 \right) = F$ $E = \frac{F}{\frac{D}{100} - 1}$	✓ D.E subject /onderwerp A ✓ $100E - 100F$ A ✓ Common factor/gemene faktor CA ✓ E subject/onderwerp CA <p style="text-align: center;">OR/OF</p> ✓ D.E subject /onderwerp A ✓ $E - F$ A ✓ Common factor/gemene faktor CA ✓ E subject /onderwerp CA (4)
1.3.2	$E = \frac{100F}{100 - D}$ $E = \frac{100(80)}{100 - 3,75}$ $\therefore E = 83,17 \text{ kg}$	✓ Substitution/vervanging A ✓ $E = 83,17$ CA (2)
1.3.3	$E = 83,17 \times 1000\text{g}$ $E = 83170\text{g}$	✓ 83170 CA (1)

1.3.4	$E = 83170\text{g} = 8,3170 \times 10^4\text{g}$	$\checkmark 8,3170 \times 10^4$ (1)	CA
1.4	$ \begin{array}{r} 1000_2 \\ -110_2 \\ \hline 10_2 \end{array} $ <p style="text-align: center;">OR/OF</p> $ \begin{aligned} & 2^3 - (2^2 + 2^1) \\ &= 2 \\ \therefore & 1000_2 - 110_2 = 10_2 \end{aligned} $	\checkmark Method/metode $\checkmark 10_2$ <p style="text-align: center;">OR/OF</p> \checkmark Method/metode $\checkmark 10_2$	A A A A (2)

QUESTION/VRAAG 2

2.1	$\Delta = b^2 - 4ac$ $6 = (-3)^2 - 4a(2)$ $6 - 9 = -8a$ $-3 = -8a$ $a = \frac{3}{8}$	✓ Formula /formule A ✓ Substitution/vervanging CA ✓ $a = \frac{3}{8}$ CA (3)
2.2	Roots are Real, irrational and unequal. / Wortels is Reël, irrasionaal en ongelyk	✓ Irrational/irrasionaal A (1)
	Accept Real and Unequal / Aanvaar Reël en ongelyk	
2.3	$b^2 - 4ac = 0$ $(-3)^2 - 4a(2) = 0$ $9 - 8a = 0$ $a = \frac{9}{8}$	✓ $\Delta = 0$ A ✓ Substitution/vervanging CA ✓ $a = \frac{9}{8}$ CA (3) [7]

QUESTION/VRAAG 3

3.1.1	$\log_x\left(\frac{1}{x}\right)$ $= \log_x x^{-1}$ $= -1$	✓ Exponential form / eksponensiële vorm A ✓ Log property/eienskap CA (2)
3.1.2	$4^x - 2^{2x-1}$ $= (2^2)^x - 2^{2x-1}$ $= 2^{2x} - 2^{2x} \cdot 2^{-1}$ $= 2^{2x}(1 - 2^{-1})$ $= 2^{2x} \cdot 2^{-1}$ $= 2^{2x-1}$ <p style="text-align: center;">OR/OF</p>	✓ Prime factorisation /priem faktorisering A ✓ Factorisation/faktorisering CA ✓ 2^{2x-1} CA <p style="text-align: center;">OR/OF</p>

	$ \begin{aligned} & 4^x - 2^{2x-1} \\ &= 4^x - 2^{2x} \cdot 2^{-1} \\ &= 4^x - 4^x \cdot 2^{-1} \\ &= 4^x(1 - 2^{-1}) \\ &= 4^x \cdot 2^{-1} \end{aligned} $	<ul style="list-style-type: none"> ✓ Exponential law/eksponensiële wet A ✓ Factorisation/faktorisering CA ✓ $4^x \cdot 2^{-1}$ CA
3.2	$ \begin{aligned} \frac{\sqrt{3x^2} \times \sqrt[3]{12x^3}}{2x^2} &= \frac{\sqrt[6]{243}}{\sqrt[3]{2}} \\ \text{LHS/LK} &= \frac{\sqrt{3x^2} \times \sqrt[3]{12x^3}}{2x^2} \\ &= \frac{x\sqrt{3} \times x\sqrt[3]{2^2 \cdot 3}}{2x^2} \\ &= \frac{\sqrt{3} \times \sqrt[3]{2^2 \cdot 3}}{2} \\ &= \frac{\frac{1}{3^2} \times \frac{2}{2^3} \cdot \frac{1}{3^3}}{2} \\ &= \frac{\frac{1+\frac{1}{3}}{2-\frac{2}{3}}}{2} \\ &= \frac{\frac{5}{3^6}}{\frac{1}{2^3}} = \frac{\sqrt[6]{3^5}}{\sqrt[3]{2}} \\ &= \frac{\sqrt[6]{243}}{\sqrt[3]{2}} = \text{RHS/RK} \end{aligned} $	<ul style="list-style-type: none"> ✓ Prime factorisation/priem faktorisering A ✓ Simplification/vereenvoudiging CA ✓ Same base law/dieselfde basis wet CA ✓ Exponential property/eksponensiële eienskap CA
3.3.1	$z = 3 - 4i$	<ul style="list-style-type: none"> ✓ $z = 3 - 4i$ A
3.3.2	$ \begin{aligned} z &= \sqrt{(3)^2 + (-4)^2} \\ z &= 5 \end{aligned} $	<ul style="list-style-type: none"> ✓ Substitution /vervanging CA ✓ $z = 5$ CA

3.3.3	$\text{RA/VH: } \tan \theta = \frac{4}{3}$ $\text{RA/VH: } \theta = 53,13^\circ$ $\theta = 360^\circ - 53,13^\circ$ $\theta = 306,87^\circ$	✓ tan ratio /verhouding ✓ Reference angle / verwysingshoek ✓ 306,87° (3)	CA
3.3.4	$z = 5\text{cis}306,87^\circ$	✓ $z = 5\text{cis}306,87^\circ$	CA
3.4	$\frac{x-i}{2i+1} = y+3i$ $x-i = (y+3i)(2i+1)$ $x-i = 2yi + y + 6i^2 + 3i$ $x-i = (2y+3)i + y - 6$ $2y+3 = -1 \text{ and/en } x = y-6$ $y = -2 \text{ and/en } x = -8$	✓ Simplification / vereenvoudiging ✓ $i^2 = -1$ ✓ Simplification / vereenvoudiging ✓ $x = -8$ ✓ $y = -2$	A
	OR /OF $\frac{x-i}{2i+1} = y+3i$ $\frac{(x-i)(2i-1)}{(2i+1)(2i-1)} = y+3i$ $\frac{2xi - x - 2i^2 + i}{4i^2 - 1} = y+3i$ $\frac{(2x+1)i - x + 2}{-4-1} = y+3i$ $(2x+1)i - x + 2 = -5y - 15i$ $2x+1 = -15 \text{ and/en } -x+2 = -5y$ $\therefore x = -8 \text{ and/en } -(-8) + 2 = -5y$ $x = -8 \text{ and/en } y = -2$	OR/OF ✓ Simplification / vereenvoudiging ✓ $i^2 = -1$ ✓ Simplification / vereenvoudiging ✓ $x = -8$ ✓ $y = -2$	CA

(5)
[21]

QUESTION/VRAAG 4

4.1.1	$y = -1$	✓ $y = -1$ A (1)
4.1.2	$0 = -\frac{2}{x} - 1$ $1 = -\frac{2}{x}$ $x = -2$	✓ $y = 0$ A ✓ Simplification/ vereenvoudiging A ✓ $x = -2$ CA (1)
4.1.3		✓ Shape/vorm CA ✓ x -intercept/afsnit CA ✓ Asymptote/asimptoot CA (3)
4.1.4	$y \neq -1$ OR / OF $-\infty < y < -1$ or / of $-1 < y < \infty$ OR / OF $y \in (-\infty; -1)$ or / of $y \in (-1; \infty)$	✓ $y \neq -1$ A (1)

4.2.1	$y = -2$	✓ $y = -2$ A (1)
4.2.2	$y = -3$	✓ $y = -3$ A (1)
4.2.3	$0 = x^2 - x - 2$ $0 = (x-2)(x+1)$ $x = -1 \text{ or/of } x = 2$	✓ $y = 0$ A ✓ Factorization/ Substitution /Faktorisering / Vervanging A ✓ $x = -1 \text{ or/of } x = 2$ CA (3)
4.2.4	$0 = 2^x - 3$ $3 = 2^x$ $x = \log_2 3$ $x = 1,58$	✓ $y = 0$ A ✓ log function/funksie A ✓ $x = 1,58$ A (3)
4.2.5	$k(x) = x^2 - x - 2$ $k'(x) = 2x - 1 = 0 \text{ OR/OF } x = -\frac{(-1)}{2(1)}$ OR/OF $x = \frac{-1+2}{2}$ $x = \frac{1}{2}$ $k\left(\frac{1}{2}\right) = \left(\frac{1}{2}\right)^2 - \frac{1}{2} - 2$ $k\left(\frac{1}{2}\right) = -\frac{9}{4}$	✓ $k'(x)$ A ✓ $k'(x) = 0$ A ✓ $x = \frac{1}{2}$ CA ✓ $k\left(\frac{1}{2}\right) = -\frac{9}{4}$ CA (4)
4.2.6	$x \in R$ OR/OF $x \in (-\infty; \infty)$ OR/OF $-\infty < x < \infty$	✓ $x \in R$ A (1)
4.2.7	$x = 0$	✓ $x = 0$ A (1)
4.3.1	$0 = x + 1$ $x = -1$	✓ $y = 0$ A ✓ $x = -1$ A (2)
4.3.2(a)	$B(0 ; 3)$	✓ $x = 0$ A ✓ $y = 3$ A (2)
4.3.2(b)	$h(x) = x + 3$	✓ $h(x) = x + 3$ A (1)
4.3.2(c)	$g(x) = \sqrt{9 - x^2}$	✓ $g(x) = \sqrt{9 - x^2}$ CA (1) [28]

QUESTION/VRAAG 5

5.1	$A = P(1-i)^n$ $A = 15000 \left(1 - \frac{0,03}{4}\right)^{5 \times 4}$ $= R12\,903,32$	✓ Formula /formule A ✓ Substitution /vervanging A ✓ R12 903,32 CA (3)
5.2.1	R15,00	✓ R15,00 A (1)
5.2.2	$A = P(1+i)^n$ $18,80 = 3,80(1+i)^{20}$ $\frac{94}{19} = (1+i)^{20}$ $\sqrt[20]{\frac{94}{19}} = 1+i$ $i = 0,0832$ <p>Rate/Koers = 8,32%</p>	✓ Substitute/vervang A and/en P A ✓ Substitute/vervang n A ✓ Simplification /vereenvoudiging CA ✓ Simplification /vereenvoudiging CA ✓ 8,32 CA (5)

<p>5.3</p> $A = P(1+i)^n$ $A_4 = R350\ 000(1+0,07)^4 + x$ $A_4 = 458778,6035 + x$ $A_6 = (458778,6035 + x)(1+0,07)^2 - 100\ 000$ $A_6 = 425\ 255,6231 + 1,1449x$ $A_8 = (425\ 255,6231 + 1,1449x) \left(1 + \frac{0,07}{12}\right)^{2 \times 12} = 620\ 000$ $1,1449x = \frac{620\ 000}{\left(1 + \frac{0,07}{12}\right)^{2 \times 12}} - 425\ 255,6231$ $\therefore x = R99\ 542,12$	<p>✓ R350 000$(1+0,07)^4$ A</p> <p>✓ M adding/tel by x A</p> <p>✓ $(458778,6035 + x)(1+0,07)^2$ CA</p> <p>✓ M subtracting/trek af 100 000 A</p> <p>✓ Substitution/vervanging CA</p> <p>✓ R99 542,12 CA</p>
<p>OR/OF</p> $[R350\ 000(1,07)^6 + x(1,07)^2 - R100\ 000] \left(\frac{1207}{1200}\right)^{24} = R620\ 000$ $425\ 255,6231 + x(1,07)^2 = 539221,3909$ $x = R99\ 542,12$	<p>OR/OF</p> <p>✓ R350 000$(1,07)^6$</p> <p>✓ $+x(1,07)^2$</p> <p>✓ M subtracting/trek af 100 000 A</p> <p>✓ $\left(\frac{1207}{1200}\right)^{24}$</p> <p>✓ = R620 000</p> <p>✓ $x = R99\ 542,12$</p>

(6)
[15]

QUESTION/VRAAG 6

6.1	$f(x) = 2 - 5x$ $f'(x) = \lim_{h \rightarrow 0} \frac{f(x+h) - f(x)}{h}$ $= \lim_{h \rightarrow 0} \frac{2 - 5(x+h) - (2 - 5x)}{h}$ $= \lim_{h \rightarrow 0} \frac{-5x - 5h + 5x}{h}$ $= \lim_{h \rightarrow 0} \frac{-5h}{h}$ $= \lim_{h \rightarrow 0} -5$ $\therefore f'(x) = -5$	<p style="text-align: center;">Penalty of one mark for incorrect notation <i>Penaliseer een punt indien notasie foutief is.</i></p> <p style="text-align: right;">(5)</p>
6.2.1	$D_x \left(\frac{1}{\sqrt{x}} - 3kx \right)$ $= D_x \left(\frac{1}{x^{\frac{1}{2}}} - 3kx \right)$ $= D_x \left(x^{-\frac{1}{2}} - 3kx \right)$ $= -\frac{1}{2} x^{-\frac{3}{2}} - 3k$	<p style="text-align: center;"> ✓ Exponential form /eksponensiële vorm A ✓ Simplification/ vereenvoudiging CA ✓ $-\frac{1}{2} x^{-\frac{3}{2}}$ CA ✓ $-3k$ A </p> <p style="text-align: right;">(4)</p>

6.2.2	$\frac{dy}{dx}$ if: $y = \frac{2x^3 - 8x}{x - 2}$ $y = \frac{2x(x^2 - 4)}{x - 2}$ $y = \frac{2x(x-2)(x+2)}{x-2}$ $y = 2x(x+2)$ $y = 2x^2 + 4x$ $\frac{dy}{dx} = 4x + 4$	✓ Factorisation/faktorisering A ✓ $2x^2 + 4x$ CA ✓ $4x$ CA ✓ 4 CA (4)
6.3	$h(x) = 3x^2 - 4x$ $h'(x) = 6x - 4 = 2$ $x = 1$ $h(1) = 3(1)^2 - 4(1)$ $h(1) = -1$	✓ $h'(x)$ A ✓ $h'(x) = 2$ A ✓ $x = 1$ CA ✓ $h(1) = -1$ CA (4)
		[17]

QUESTION/VRAAG 7

7.1	$0 = -x^3 - 4x^2 - 3x$ $0 = -x(x^2 + 4x + 3)$ $0 = -x(x + 1)(x + 3)$ $x = 0 \text{ or/of } x = -1 \text{ or/of } x = -3$	✓ $y = 0$ ✓ Factorisation/faktorisering ✓ $x = 0$ ✓ $x = -1 \text{ or/of } x = -3$ (4)	A A CA CA
7.2	$y = 0$	✓ $y = 0$	A (1)
7.3	$f(x) = -x^3 - 4x^2 - 3x$ $f'(x) = -3x^2 - 8x - 3 = 0$ $x = \frac{-(-8) \pm \sqrt{(-8)^2 - 4(-3)(-3)}}{2(-3)}$ $\therefore x = -2,22 \text{ or/of } x = -0,45$ $y = -(-2,22)^3 - 4(-2,22)^2 - 3(-2,22)$ $y = -2,11$ or/of $y = -(-0,45)^3 - 4(-0,45)^2 - 3(-0,45)$ $y = 0,63$	✓ $f'(x)$ ✓ $f'(x) = 0$ ✓ Substitution/vervanging ✓ Both x -values/beide x -waardes CA ✓ Both y -values/beide y -waardes CA (5)	A A CA CA

7.4	<p>f</p> <p>$(-0,45 ; 0,63)$</p> <p>$(-2,22 ; -2,11)$</p> <p>-3</p> <p>1</p> <p>0</p>	✓ Shape /vorm ✓ Negative x -intercepts/negatiewe x -afsnitte ✓ y -intercept/afsnit ✓ Both turning points/beide draaipunte (4)
7.5	$f(-2) = -(-2)^3 - 4(-2)^2 - 3(-2) = -2$ $f(-1) = -(-1)^3 - 4(-1)^2 - 3(-1) = 0$ $\text{Average gradient} = \frac{0 - (-2)}{-1 - (-2)}$ $\therefore \text{Gemiddelde gradiënt} = 2$	✓ -2 ✓ 0 ✓ SF ✓ Ave/Gemid grad = 2 (4) [18]

QUESTION/VRAAG 8

8.1 $s = ut + \frac{1}{2}gt^2$. $s = 5 \times 4 + \frac{1}{2}(10)(4)^2$ $s = 100 \text{ m}$	NPU	<ul style="list-style-type: none"> ✓ Substitution/vervanging A ✓ $s = 100 \text{ m}$ CA <p style="text-align: right;">(2)</p>
8.2 $\frac{ds}{dt} = v = u + gt$ OR/OF $\frac{ds}{dt} = 5 + 10t$		<ul style="list-style-type: none"> ✓ u A ✓ gt A <p style="text-align: center;">OR/OF</p> <ul style="list-style-type: none"> ✓ 5 ✓ $10t$ <p style="text-align: right;">(2)</p>
8.3 $v = u + gt$ $v = 5 + 10(4)$ $v = 45 \text{ m/s}$	NPU	<ul style="list-style-type: none"> ✓ Substitution/vervanging CA ✓ 45 m/s CA <p style="text-align: right;">(2)</p>
8.4 Average rate of change = $\frac{45 - 5}{4 - 0}$ $\therefore (\text{Gemid tempo verandering}) = 10 \text{ m/s}^2$	NPU	<ul style="list-style-type: none"> ✓ Substitution/vervanging CA ✓ 10 m/s CA <p style="text-align: right;">(2)</p> <p style="text-align: right;">[8]</p>

QUESTION/VRAAG 9

9.1.1	$\int (x^3 - 8) dx$ $= \frac{x^4}{4} - 8x + c$	✓ $\frac{x^4}{4}$ ✓ $-8x$ ✓ c (3)
9.1.2	$\int \left(\frac{f(x)}{x^2 + 2x + 4} - 2^{3x} \right) dx$ $= \int \left(\frac{x^3 - 8}{x^2 + 2x + 4} - 2^{3x} \right) dx$ $= \int \left(\frac{(x-2)(x^2 + 2x + 4)}{x^2 + 2x + 4} - 2^{3x} \right) dx$ $= \int (x-2 - 2^{3x}) dx$ $= \frac{x^2}{2} - 2x - \frac{2^{3x}}{3\ln 2} + c$	✓ Factors/faktore ✓ Simplification/vereenvoudiging ✓ $\frac{x^2}{2} - 2x$ ✓ $-\frac{2^{3x}}{3\ln 2} + c$ (4)
9.2	Area = $-\int_{-1}^1 (x^3 - 1) dx$ $= - \left[\frac{x^4}{4} - x \right]_{-1}^1$ $= - \left[\left(\frac{(1)^4}{4} - (1) \right) - \left(\frac{(-1)^4}{4} - (-1) \right) \right]$ $= - \left(\frac{(1)^4}{4} - (1) \right) + \left(\frac{(-1)^4}{4} - (-1) \right)$ $= \frac{3}{4} + \frac{5}{4}$ $= 2 \text{ sq units/vk eenhede}$	✓ Area notation/notasie ✓ Integral/integraal ✓✓ Substitution/vervanging ✓ Area = 2 (5) [12]

TOTAL/TOTAAL: 150