NATIONAL SENIOR CERTIFICATE/*NASIONALE SENIORSERTIFIKAAT*

GRADE/*GRAAD* 12

SEPTEMBER 2023

TECHNICAL MATHEMATICS P2/*TEGNIESE WISKUNDE V2* MARKING GUIDELINE/*NASIENRIGLYN*

MARKS/*PUNTE*:   150

This marking guideline consists of 16 pages./
Hierdie nasienriglyn bestaan uit 16 bladsye.

NOTE:
- Continuous accuracy (CA) applies only where indicated in this marking guideline.
- Assuming values/answers in order to solve a problem is unacceptable.

LET WEL:
- *Volgehoue akkuraatheid (CA) is slegs van toepassing soos aangedui in hierdie nasienriglyn.*
- *Aanvaarding van waardes/antwoorde om 'n probleem op te los, is onaanvaarbaar.*
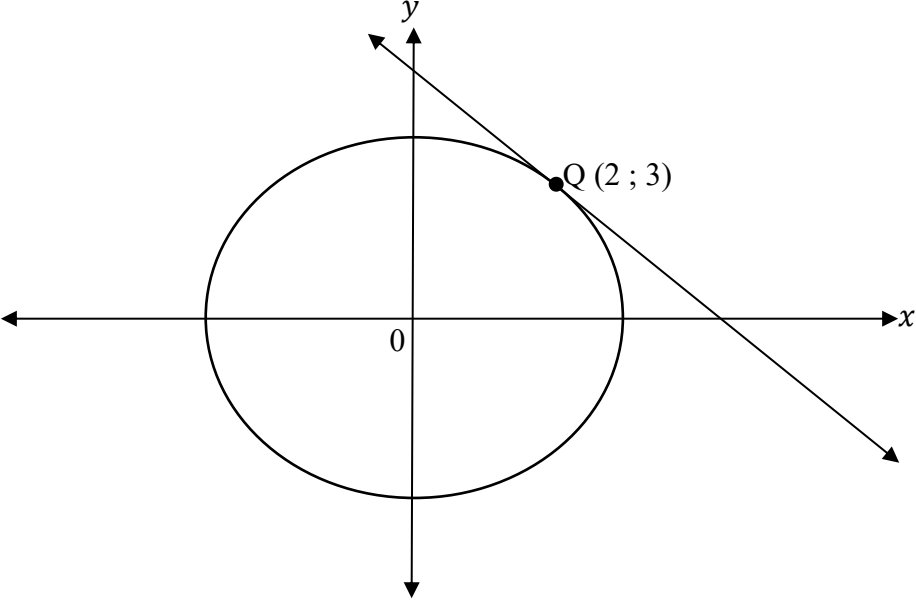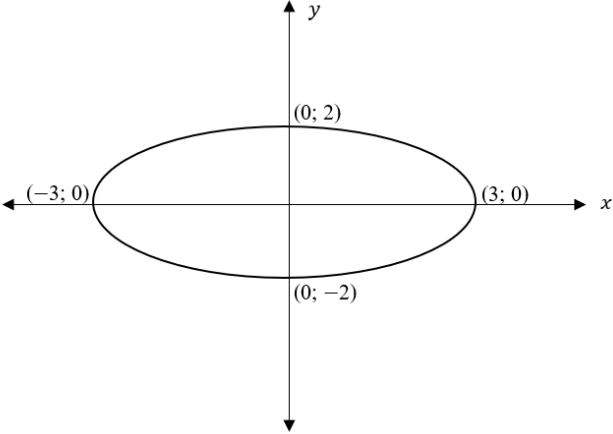
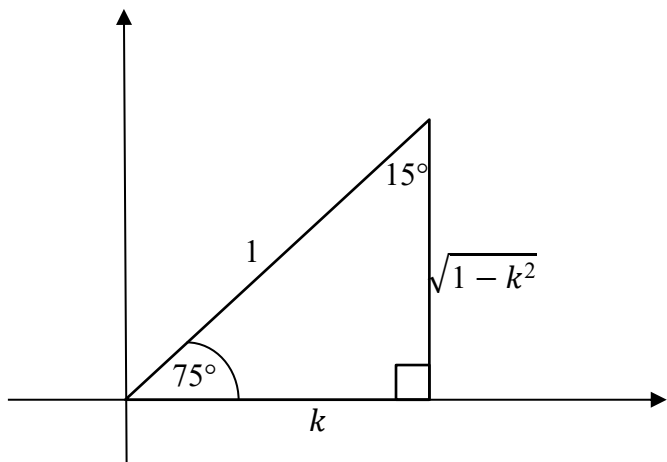| MARKING CODES / *NASIENKODES*	
M	**Method / *Metode***
A	**Accuracy / *Akkuraatheid***
AO	**Answer only / *Slegs antwoord***
CA	**Consistent accuracy / *Deurlopende akkuraatheid***
F	**Formula / *Formule***
I	**Identity / *Identiteit***
R	**Rounding / *Afronding***
S	**Simplification / *Vereenvoudiging***
ST	**Statement / *Bewering***
RE	**Reason / *Rede***
ST RE	**Statement and correct reason / *Bewering en korrekte rede***
SF	**Substitution correctly in correct formula / *Korrekte vervanging in die korrekte formule***
NPU	**No penalty for omitting units / *Geen penalisering vir eenhede uitgelaat***

QUESTION/*VRAAG* 1

1.1	$m_{AD} = \frac{6-4}{3-(-1)}$ $m_{AD} = \frac{1}{2}$ **AO: Full marks / *Volpunte***	✓ **SF** **A** ✓ $\frac{1}{2}$ **CA** (2)
1.2	$P\left(\frac{3+4}{2} ; \frac{6+1}{2}\right)$ $P\left(\frac{7}{2} ; \frac{7}{2}\right)$	✓ **SF** **A** ✓ $\left(\frac{7}{2} ; \frac{7}{2}\right)$ **CA** (2)
1.3	$m_{AB} = m_{CD} = \frac{1}{2}$ ($\parallel$ lines / *lyne*) $\therefore y = \frac{1}{2}x + c$ **OR** $\therefore y - y_1 = \frac{1}{2}(x - x_1)$ $(4 ; 1): 1 = \frac{1}{2}(4) + c$ $(4;1): y - 1 = \frac{1}{2}(x - 4)$ $\therefore 1 = 2 + c$ $\therefore y - 1 = \frac{1}{2}x - 2$ $\therefore -1 = c$ $\therefore y = \frac{1}{2}x - 2 + 1$ $\therefore y = \frac{1}{2}x - 1$ $\therefore y = \frac{1}{2}x - 1$	✓ $m = \frac{1}{2}$ **CA** ✓ sub/*verv* $(4 ; 1)$ **CA** ✓ $y = \frac{1}{2}x - 1$ **A** (3)
1.4	$0 = \frac{1}{2}x - 1$ $1 = \frac{1}{2}x$ $2 = x$ $\therefore E(2 ; 0)$	✓ $y = 0$ **A** ✓ $(2 ; 0)$ **CA** (2)

1.5	$m_{AE} = \frac{4-0}{-1-2}$	$\checkmark m_{AE} = \frac{4}{-3}$ **CA**
	$m_{AE} = \frac{4}{-3}$	
	$\tan\theta = m_{AE}$	
	$\tan\theta = \frac{4}{-3}$	$\checkmark$ **SF** **CA**
	$\therefore ref.\angle = \tan^{-1}\left(\frac{4}{3}\right) = 53{,}13°$	$\checkmark$ **ref.** $\angle$ **CA**
	$\therefore \theta = 180° - 53{,}13°$	$\checkmark \theta = 126{,}87°$**CA**
	$\therefore \theta = 126{,}87°$	(4)
1.6	$m_{CE} = \frac{1}{2}$	$\checkmark m_{CE} = \frac{1}{2}$ **A**
	$\tan\alpha = m_{CE}$	
	$\tan\alpha = \frac{1}{2}$	$\checkmark$ **SF** **CA**
	$\alpha = \tan^{-1}\left(\frac{1}{2}\right) = 26{,}57°$	$\checkmark \boldsymbol{\alpha} = 26{,}57°$ **CA**
	$\therefore A\hat{E}D = 126{,}87° - 26{,}57° = 100{,}30°$	$\checkmark A\hat{E}D = 100{,}30°$ **CA**
		(4)
		[17]

	$m_{AE} = \frac{4-0}{-1-2}$	

QUESTION/*VRAAG* 2

2.1		
2.1.1	$m_{OQ} = \frac{3-0}{2-0}$ $m_{OQ} = \frac{3}{2}$ **AO: Full marks / *Volpunte***	✓**SF** **A** ✓m_{OQ} **CA** (2)
2.1.2	$m_{tan} = -\frac{2}{3}$	✓ $m = -\frac{2}{3}$ **CA**

Details for 2.1.2:

$m_{tan} = -\frac{2}{3}$

$\therefore y = -\frac{2}{3}x + c$ $(2\,;3)$: $3 = -\frac{2}{3}(2)+c$ $\therefore 3 = -\frac{4}{3} + c$ $\therefore \frac{13}{3} = c$ $\therefore y = -\frac{2}{3}x + \frac{13}{3}$	**OR/*OF***	$\therefore y - y_1 = -\frac{2}{3}(x - x_1)$ $(2;3)$: $y - 3 = -\frac{2}{3}(x - 2)$ $\therefore y = -\frac{2}{3}x + \frac{13}{3}$

✓ $m = -\frac{2}{3}$ **CA**

✓ sub/*verv* (2 ; 3) **CA**

✓ equation/*vergelyking* **CA**

OR/OF	**OR/OF**
$xx_1 + yy_1 = r^2$ $\therefore x(2) + y(3) = 13$ $\therefore y = -\frac{2}{3}x + \frac{13}{3}$	✓ **F** **A** ✓ **SF** **A** ✓ equation/*vergelyking* **CA** (3)

2.2		✓ elliptical shape with horizontal axis as major axis / *Elliptiese vorm met groter-as as die horisontale-as* **A** ✓ x-intercepts/ *x-afsnitte* **A** ✓ y-intercepts/ *y-afsnitte* **A** (3)
		[8]

QUESTION/*VRAAG* 3

3.1.1	$\cos(128,2° + 204,7°) \approx 0,89$	✓ SF	A
		✓ **0,89**	A
	AO: Full marks / *Volpunte*		**(2)**
3.1.2	$\text{cosec}(204,7° − 128,2°)$	✓ SF	A
	$= \dfrac{1}{\sin(76,5°)}$	✓ reciprocal ratio / *resiprook verh.*	A
	$\approx 1,03$	✓ **1,03**	A
	AO: Full marks / *Volpunte*		**(3)**
3.2.1	$\therefore \sin 15° = k$	✓ **Diagram**	A
		✓ $\sqrt{1-k^2}$	A
		✓ **k**	CA
			(3)
3.2.2	$\tan 255° = \tan(180° + 75°)$	✓ Reduction/*reduksie*	A
	$\therefore \tan 75° = \dfrac{\sqrt{1-k^2}}{k}$	✓ numerator/*teller*	CA
		✓ denominator/*noemer*	CA
			(3)
3.3	$\sec\theta = −1,583$	✓ $\dfrac{1}{−1,583} = \cos\theta$	A
	$\dfrac{1}{\cos\theta} = −1,583$		
	$1 = −1,583 \times \cos\theta$		
	$\dfrac{1}{−1,583} = \cos\theta$	✓ ref. $\angle$	A
	$50,82\ldots° = \text{ref.}\angle$		
	$\therefore \text{Q2: } \theta = 180° − 50,82° = 129,18°$	✓ Quadrants/*kwadrante*	A
	AND/*EN*		
	$\therefore \text{Q3: } \theta = 180° + 50,82° = 230,82°$	✓ Both answers/*beide antwoorde*	CA
			(4)
			[15]
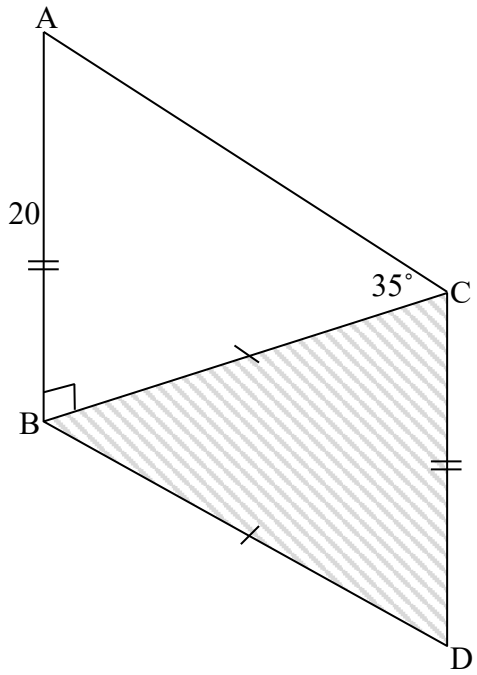
QUESTION/*VRAAG* 4

4.1	$\cosec^2(180° + \theta) + \dfrac{\sin(180°- \theta).\cot^2(180°+ \theta).\sin 270°}{\cos(360°-\theta).\tan(180°+ \theta)}$	✓ $\cosec^2\theta$	**A**
	$= \cosec^2\theta + \dfrac{(\sin\theta).(\cot^2\theta).(-1)}{(\cos\theta).(\tan\theta)}$	✓ $\sin\theta$	**A**
		✓ $\cot^2\theta$	**A**
		✓ -1	**A**
	$= \cosec^2\theta - \dfrac{\tan\theta.\cot^2\theta}{\tan\theta}$	✓ $\cos\theta$	**A**
		✓ $\tan\theta$	**A**
	$= \cosec^2\theta - \cot^2\theta$	✓ $\tan\theta$ **I**	**A**
		✓ S	**A**
	$= 1$	✓ 1 **I**	**A**
			(9)
4.2	$\dfrac{1}{(1 - \sin\theta)(1 + \sin\theta)} = \sec^2\theta$		
	$\therefore \text{LHS} = \dfrac{1}{(1-\sin\theta)(1+\sin\theta)}$		
	$\therefore \text{LHS} = \dfrac{1}{1-\sin^2\theta}$	✓ $1 - \sin^2\theta$ **A**	
	$\therefore \text{LHS} = \dfrac{1}{\cos^2\theta}$	✓ $\dfrac{1}{\cos^2\theta}$ **A**	
	$\therefore \text{LHS} = \sec^2\theta = \text{RHS}$		
			(2)
			[11]

QUESTION/*VRAAG* **5**

5.1	
	✔ *f* and *g* start points and end points / *beginpunte en eindpunte* **A**
	g: ✔ shape/*vorm* **A** ✔ turning points/*draaipunte* **A** ✔ *y*-intercept/*y-afsnit* **A**
	f: ✔ shape/*vorm* **A** ✔ *x*-intercepts/*x-afsnitte* **A** ✔ Asymptotes/*Asimptote* **A**
	(7)

5.2	$y \in [-2 \,; 0\,]$ **OR/***OF* $-2 \leq y \leq 0$	✔ Notation/*notasie* **A** ✔ start and endpoints/*begin- en eindpunte* **CA** (2)
5.3	$180°$	✔ Answer/*Antw.* **A** **(1)**
5.4	$x \in (180°; 270°)$ **OR/***OF* $180° < x < 270°$	✔ endpoints / *eindpunte* **CA** ✔ notation /*notasie* **CA** (2)
		[12]

QUESTION/VRAAG 6

6.1	$\tan 35° = \dfrac{20}{BC}$ $BC \times \tan 35° = 20$ $BC = \dfrac{20}{\tan 35°}$ $BC = 28,56$ units/*eenhede*	✓ **SF** **A** ✓ BC Subj /*onderwerp* **CA** ✓ answer/antw **CA** (3)
6.2	$a^2 = b^2 + c^2 - 2b.c.\cos A$ $CD^2 = BC^2 + BD^2 - 2BC.BD.\cos C\hat{B}D$ $(20)^2 = (28,56)^2 + (28,56)^2 - 2(28,56)(28,56)\cos C\hat{B}D$ $0,75\ldots = \cos C\hat{B}D$ $40,99\ldots° = C\hat{B}D$ $\therefore C\hat{B}D = 41°$	✓ F **A** ✓ SF **CA** ✓ S **CA** ✓ Answer/Antw. **A** (4)
6.3	Area $\Delta BCD = \dfrac{1}{2}.a.b.\sin C$ Area $\Delta BCD = \dfrac{1}{2}.BC.BD.\sin C\hat{B}D$ Area $\Delta BCD = \dfrac{1}{2}(28,56)(28,56)\sin 41°$ Area $\Delta BCD = 267,57$ square units	✓ F **A** ✓ SF **CA** ✓ Answer/*Antw*. **A** (3)
		[10]

QUESTION/*VRAAG* **7**

7.1	Equal/*gelyk*		✓	(1)

7.2	

7.2.1	$\hat{B}_3 = 80°$ (ext. ∠ of cq / *buite ∠ van kvh*) $\hat{C}_1 = 80°$ (∠'s opp = sides / *∠'e teenoor = sye*) $\hat{B}_1 = 80°$ (∠'s in same seg. / *∠'e in dieselfde seg.*)	✓ **ST** ✓**RE** ✓ **ST** ✓**RE** ✓ **ST** ✓**RE** (6)

7.2.2	(a)	$\hat{C}_3 = 70°$ (ext. ∠ of cq / *buite ∠ van kvh*)	✓ **ST** ✓**RE** (2)
	(b)	$\hat{E} = 30°$ (int. ∠ of Δ / *binne ∠ van Δ*)	✓ **ST** ✓**RE** (2)
	(c)	$\hat{D}_1 = \hat{C}_2$ (∠'s in same seg. / *∠'e in dieselfde seg.*) ∴ $\hat{C}_2 = 180° - 70° - 80°$ (∠'s on a straight line / *∠'e op 'n reguitlyn*) ∴ $\hat{D}_1 = 30°$	✓ **ST** ✓**RE** ✓ **ST**
		OR/*OF*	**OR/***OF*
		$\hat{D}_1 + \hat{A} + \hat{B}_1 = 180°$ (int. ∠ of Δ / *binne ∠ van Δ*) ∴ $\hat{D}_1 = 180° - 70° - 80° = 30°$	✓ **ST** ✓**RE** ✓ **ST** (3)

7.2.3	$\hat{D}_1 = \hat{E} = 30°$ ∴AD is a tangent / *AD is 'n raaklyn* (∠ between the line and a chord / *∠ tussen die lyn en 'n koord*)	✓ **ST** ✓**RE** (2)
		[16]

QUESTION/*VRAAG* 8

8.1	twice / *twee keer*	✓	(1)
8.2			
8.2.1	$\hat{L}_1 = 76°$ (∠'s in same seg. / *∠'e in dieselfde seg.*)	✓ **ST** ✓**RE**	(2)
8.2.2	$\hat{O}_1 = 76°$ (corresp. ∠'s =; ON∥LM / *ooreenk. ∠'e =; ON∥LM*)	✓ **ST** ✓**RE**	(2)
8.2.3	$\hat{M}_4 = 38°$ (< at centre = 2 × ∠ at circumf. / *middelpts ∠ = 2 × omtreks ∠*)	✓ **ST** ✓**RE**	(2)
8.2.4	$\hat{N}_1 + \hat{N}_2 = 104°$ (opp ∠'s of cq / *teenoorst. ∠'e van kvh*)	✓ **ST** ✓**RE**	(2)
			[9]

QUESTION/*VRAAG* 9

9.1	Divides / *verdeel*	✔ **A** (1)
9.2		
9.2.1	In ΔBDC: $\hat{D} = 90°$ (∠ in semi-circle / *∠ in semi-sirkel*) $\therefore DC^2 = 17^2 - 8^2$ (Pyth) $\therefore DC = 15$ cm	✔ **ST** ✔**RE** ✔ **SF** ✔ Answer / *Antw* **.CA** (4)
9.2.2 (a)	$\frac{CE}{BC} = \frac{CF}{CD}$ (line ∥ one side of Δ / *lyn ∥ een sy van Δ)* $\frac{1}{4} = \frac{CF}{15}$ $\therefore CF = 3{,}75$ cm	✔ **ST** ✔**RE** ✔ **SF** ✔ Answer / *Antw.* **CA** (4)

(b)	In ΔBAC and/*en* ΔFEC:
	    1. $\hat{C}$ is a common angle/*gemene hoek*
	    2. $E\hat{B}A = 90°$ (rad ⊥ tan)
	       $E\hat{F}C = 90°$
	       (corresp. ∠'s =; BD∥EF / *ooreenk. ∠'e =; BD∥EF*)
	       $\therefore E\hat{B}A = E\hat{F}C$
	    3. $C\hat{E}F = B\hat{A}C$ (int. ∠of Δ / *binne ∠ van Δ*)
	$\therefore$ ΔBAC∥∥ΔFEC
	
	              OR / *OF*
	
	In ΔBAC and/*en* ΔFEC:
	    1. $\hat{C}$ is a common angle/*gemene hoek*
	    2. $E\hat{B}A = 90°$ (rad ⊥ tan)
	
	       $E\hat{F}C = 90°$
	       (corresp. ∠'s =; BD∥EF / *ooreenk. ∠'e =; BD∥EF*)
	       $\therefore E\hat{B}A = E\hat{F}C$
	
	$\therefore$ ΔBAC∥∥ΔFEC (∠∠∠)
		✔ **ST**
		✔ **ST** ✔ **RE**
		
		✔ **ST RE**
		
		✔ **ST**
		
		    OR / *OF*
		
		✔ **ST**
		✔ **ST** ✔ **RE**
		
		✔ **ST RE**
		
		✔ **RE**
		(5)
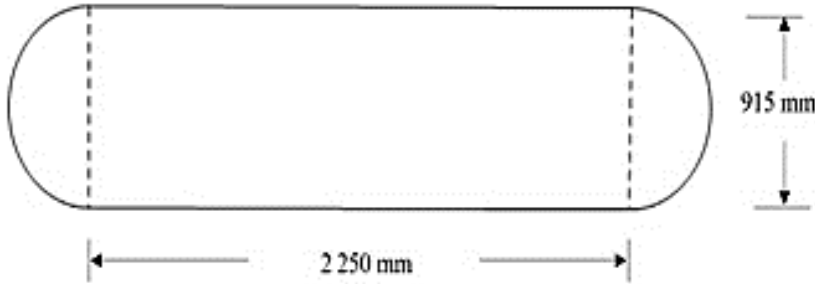
| (c) | In ΔBAD and/*en* ΔCBD:
　　1.　$B\hat{D}A = B\hat{D}C$ (∠'s on str. line/∠*'e op reguit lyn*)
　　2.　$A\hat{B}D = B\hat{C}D$ (tanchord/*raaklynkoord*)
　　3.　$B\hat{A}D = D\hat{B}C$ (int. ∠of Δ / *binne* ∠ *van* Δ)
∴ ΔBAD\|\|\|ΔCBD (∠∠∠)
∴ $\frac{BA}{CB} = \frac{AD}{BD} = \frac{BD}{CD}$ (ΔBAD\|\|\|ΔCBD)
∴ $\frac{AD}{8} = \frac{8}{15}$
∴ $AD = \frac{64}{15} \approx 4{,}27$ cm

<div align="center">**OR/*OF***</div>
$\frac{AC}{EC} = \frac{BC}{FC}$ (ΔBAC\|\|\|ΔFEC)
But, *maar* $EC = \frac{1}{4}BC = \frac{17}{4}$ and/*en*
$FC = \frac{1}{4}DC = \frac{15}{4}$ (Prop th/*ewerig st.* BD \|\| EF)
∴$\frac{AC}{\frac{17}{4}} = \frac{17}{\frac{15}{4}}$
∴ $AC \approx 19{,}27$
∴ $AD = 19{,}27 - 15 = 4{,}27$ | ✔ **ST RE**

✔ **ST RE**

✔ **ST**
✔ Answer / *Antw.***CA**

✔ **ST**

✔ **ST**

✔ **ST**
✔ **ST**

(4) |
| | | **[18]** |

QUESTION/*VRAAG* 10

10.1.1	$n = \dfrac{No.\ of\ rev.}{\text{time}} = \dfrac{1\ rev}{50\ \text{min}} = 0,02$ rev/*min*			✔conv. /*Herleid* **A** (1)
10.1.2	14 6425 km $= 14,6425 \times 1\ 000 = 14\ 642,5$ m			✔conv. /*Herleid* **A** (1)
10.1.3	$V = \pi Dn$ $V = \pi(14\ 642,5)(0,02)$ $V \approx 920,02$ m/min			✔ **F** **A** ✔ **SF** **CA** ✔**Answer/*Antw.*CA** (3)
10.2	$n = 15$ rev / sec $= 900$ rev / min $\omega = 2\pi n$ $\omega = 2\pi(900)$ $\omega \approx 5\ 654,87$ rad / min			✔**conv. /*Herleid* A** ✔ **F** **A** ✔ **SF** **CA** ✔**Answer/*Antw.*CA** (4)
10.3	10.3.1	80 mm = 8 cm $4h^2 - 4dh + x^2 = 0$ $4h^2 - 4(10)h + (8)^2 = 0$ $4h^2 - 40h + 64 = 0$ $\div\ 4: h^2 - 10h + 16 = 0$ $(h - 8)(h - 2) = 0$ $\therefore h = 8$ or $h = 2$ $\therefore h = 2$ cm	80 mm = 8 cm $4h^2 - 4dh + x^2 = 0$ $4h^2 - 4(10)h + (8)^2 = 0$ $4h^2 - 40h + 64 = 0$ $\div\ 4: h^2 - 10h + 16 = 0$ $h = \dfrac{-b \pm \sqrt{b^2 - 4ac}}{2a}$ $h = \dfrac{-(-10) \pm \sqrt{(-10)^2 - 4(1)(16)}}{2(1)}$ $\therefore h = 8$ or $h = 2$ $\therefore h = 2$ cm	✔ **F** **A** ✔ **SF** **CA** ✔Factors/*faktore* **A** ✔Answer/*Antw.*CA ✔ $h = 2$ cm **A** (5)
	10.3.2	$r = 10 \div 2 = 5$ $s = r\theta$ $5,24 = 5\theta$ $1,048$ rad $= \theta$ $\therefore \theta = 1,048 \times \dfrac{180°}{\pi}$ $\therefore \theta = 60°$		✔ **F** **A** ✔ **SF** **CA** ✔conv. /*Herleid* **A** ✔Answer/*Antw.* CA (4)
	10.3.3	$Area = \dfrac{r^2\theta}{2}$ $Area = \dfrac{(5)^2\,(1,048)}{2}$ $Area = 13,1$ cm^2	$Area = \dfrac{rs}{2}$ $Area = \dfrac{(5)(5,25)}{2}$ $Area = 13,1$ cm^2	✔ **F** **A** ✔ **SF** **CA** ✔ *Area* **CA** (3)
				[21]

QUESTION/*VRAAG* 11

11.1	

$Area = a\left(\frac{O_1+O_n}{2} + O_2 + O_3 + \ldots + O_{n-1}\right)$ $149{,}38 = \frac{x}{5}\left(\frac{12+12{,}5}{2} + 12{,}3 + 11{,}8 + 11{,}6 + 11{,}8\right)$ $149{,}38 = \frac{x}{5}(59{,}75)$ $\frac{x}{5} = \frac{149{,}38}{59{,}75} \approx 2{,}50$ $x = 12{,}5$ cm		✓ **F** **A** ✓ $\boldsymbol{a = \frac{x}{5}}$ **A** ✓ **SF** **CA** ✓ value of / *waarde van x* **CA** (4)
OR/*OF*		**OR / *OF***
$Area = a(m_1 + m_2 + m_3 + \ldots + m_{n-1})$ $149{,}38 = \frac{x}{5}\left(\frac{12+12{,}3}{2} + \frac{12{,}3+11{,}8}{2} + \frac{11{,}8+11{,}6}{2} + \frac{11{,}6+11{,}8}{2} + \frac{11{,}8+12{,}5}{2}\right)$ $149{,}38 = \frac{x}{5}(12{,}15 + 12{,}05 + 11{,}7 + 11{,}7 + 12{,}15)$ $149{,}38 = \frac{x}{5}(59{,}75)$ $\frac{x}{5} = \frac{149{,}38}{59{,}75} \approx 2{,}50$ $x = 12{,}5$ cm		✓ **F** **A** ✓ $\boldsymbol{a = \frac{x}{5}}$ **A** ✓ **SF** **CA** ✓ value of / *waarde van x* **CA** (4)

11.2		
11.2.1	915 *mm* = 91,5 *cm* and/*en* 2 250 *mm* = 225 *cm*	✓ conversion /*herleiding* **A** (1)
11.2.2	Volume of storage tank = Volume of sphere + volume of cylinder *Volume van opgaartenk* $= \frac{4}{3}\pi r^3 + \pi r^2 h$ $= \frac{4}{3}\pi \left(\frac{91,5}{2}\right)^3 + \pi \left(\frac{91,5}{2}\right)^2 \times 225$ $= 598\,615,875\pi \text{ cm}^3$ $= 598,62 \text{ litres}$	✓radius **A** ✓**F** **A** ✓**SF** **A** ✓ ans in litres / *antw in liters* **CA** (4)
11.2.3	0,5 ton = 500 kg ∴ Weight/*Gewig* of/*van* gas $= 598,62\, l \times \frac{1\,kg}{1,96\,l} = 305,42\,kg$ ∴Tare weight /*tarra gewig* = 500 – 305,42 = 194,58 kg ∴ Tare weight % / *tarra gewig %* $= \frac{194,58}{500} \times 100\% \approx 38,92\%$	✓ conversion / *herleiding* **A** ✓Weight of gas / *gewig van gas* **CA** ✓ tare weight / *tarra gewig* **CA** ✓ tare perct /*tarra perst* **CA** (4)
		[13]
		TOTAL/*TOTAAL***: 150**