Province of the
EASTERN CAPE
EDUCATION

NATIONAL SENIOR CERTIFICATE

GRADE 12

JUNE 2024

TECHNICAL MATHEMATICS P2

MARKS: **150**

TIME: **3 hours**

This question paper consists of 15 pages, including a 2-page information sheet, and an answer book of 25 pages.

INSTRUCTIONS AND INFORMATION

Read the following instructions carefully before answering the questions.

1. This question paper consists of ELEVEN questions.

2. Answer ALL the questions in the SPECIAL ANSWER BOOK provided.

3. Clearly show ALL calculations, diagrams, graphs, et cetera which you have used in determining the answers.

4. Answers only will NOT necessarily be awarded full marks.

5. You may use an approved scientific calculator (non-programmable and non-graphical) unless stated otherwise.

6. If necessary, round off your answers to TWO decimal places, unless stated otherwise.

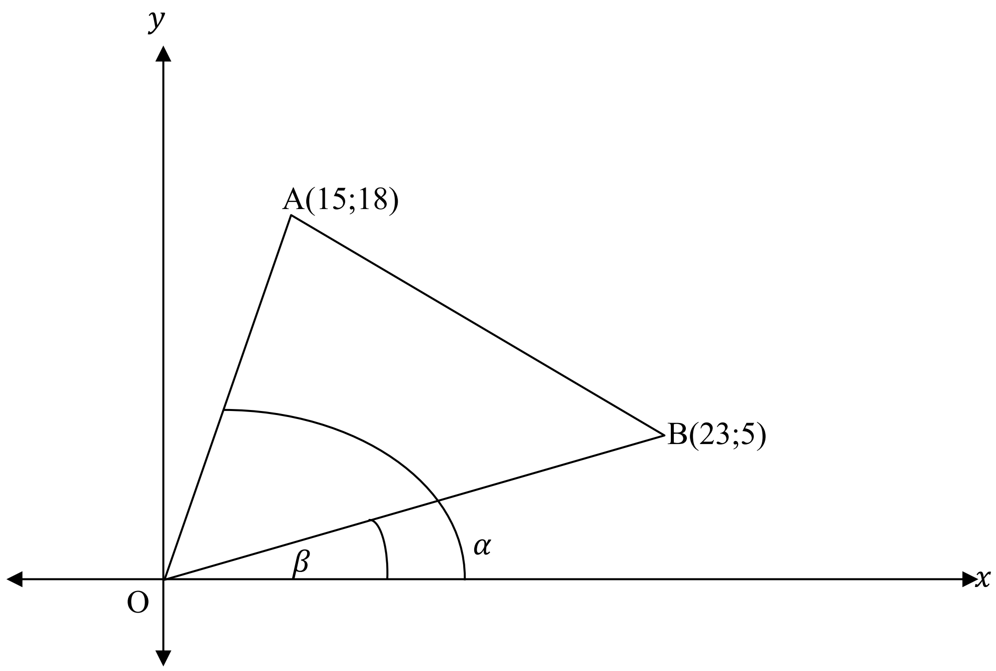
7. Diagrams are NOT necessarily drawn to scale.

8. An information sheet with formulae is included at the end of the question paper.

9. Write neatly and legibly.

QUESTION 1

In the diagram below, AOB is a triangle with vertices A$(15\ ;\ 18)$; O$(0\ ;\ 0)$ and B$(23\ ;\ 5)$.
β is the angle of inclination of line OB and α is the angle of inclination of line OA.



1.1 Determine the gradients of OA and OB. (4)

1.2 Determine the angle of inclination of line OB. (3)

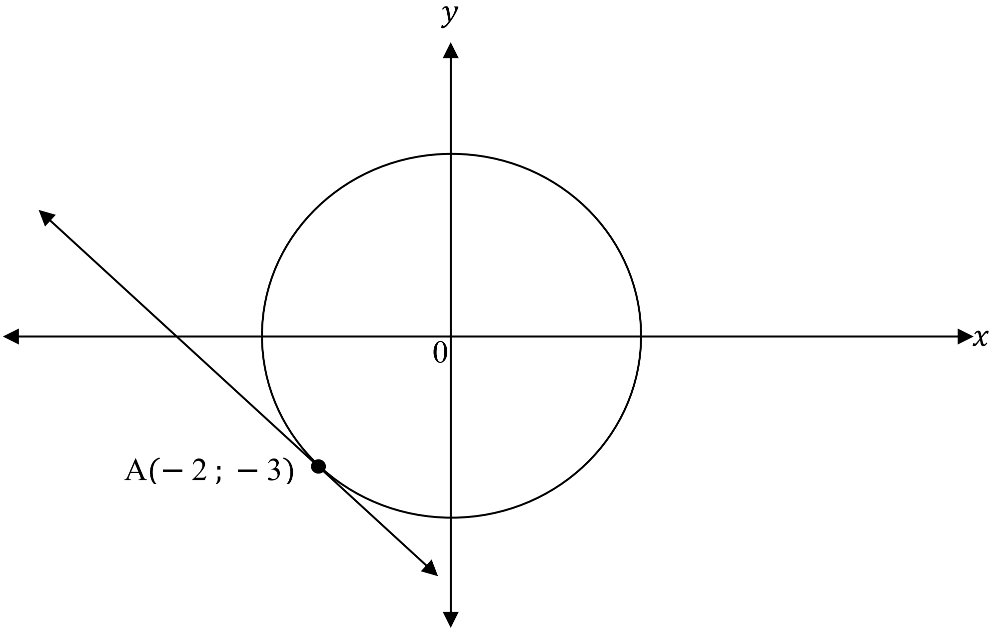
1.3 Find the size of $A\hat{O}B$, correct to the nearest whole number. (4)

1.4 AOBM is a parallelogram. Find the coordinates of M. (5)

[16]

QUESTION 2

2.1 The diagram below shows the circle with equation $x^2 + y^2 = 13$. The contact point of a tangent to the circle is at A$(-2\,;\,-3)$.



2.1.1 Write down the radius of the circle in simplified surd form. (1)

2.1.2 Determine the equation of the tangent to the circle at point A in the form $y = \ldots$ (4)

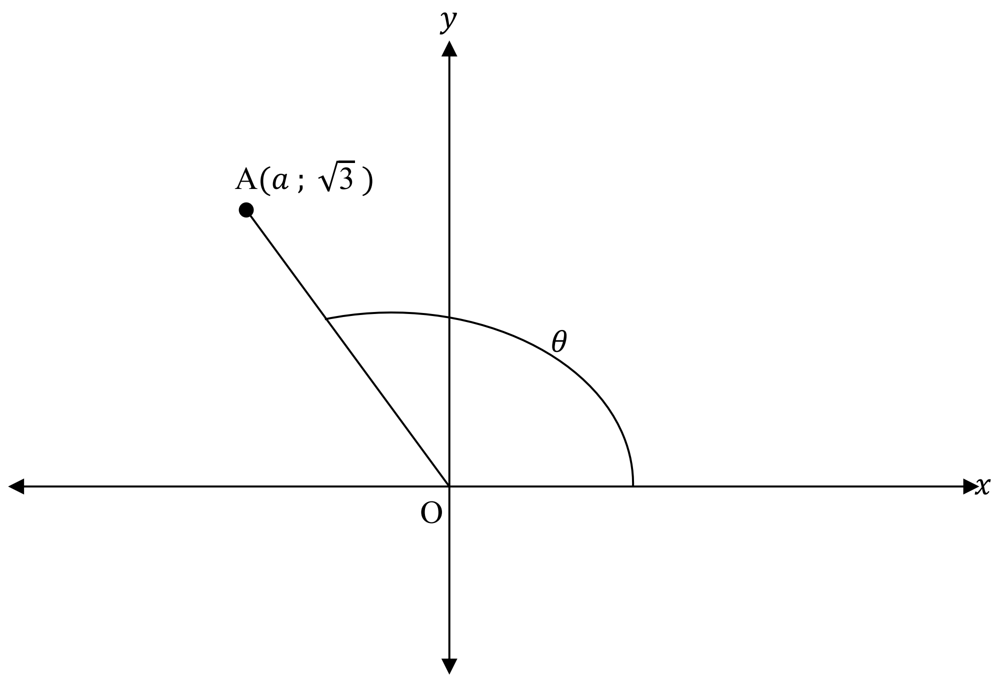
2.1.3 Write down the coordinates of another point where the line AO intersects with the circle. (2)

2.2 Sketch the graph of $\frac{x^2}{3} + \frac{y^2}{9} = 1$. Clearly indicate the intercepts. (3)

[10]

QUESTION 3

3.1 In the diagram below A(a ; $\sqrt{3}$) and OA = 3.



Determine the value of the following, without using a calculator:

3.1.1 a (3)

3.1.2 $\sec\theta$ (1)

3.1.3 $cosec\,(\theta + 360°)$ (3)

3.2 Determine the values of x, if $\tan(x - 30°) = -0{,}982$ and $0° \leq x - 30° \leq 360°$. (4)

[11]

QUESTION 4

4.1 Simplify: $\dfrac{\sin(180° - \theta)\tan(180° + \theta)\sin(270°)}{\cos(360° - \theta)\tan(180° - \theta)}$ (6)

4.2 Prove that: $(cosec\ B - \cot B)^2 = \dfrac{1 + \cos B}{1 - \cos B}$ (6)

[12]

QUESTION 5

Given the functions defined by $f(x) = \cos(x - 30)$ and $g(x) = 2\sin x$ for $x \in (0°\ ;360°)$.

5.1 Write down the period of *f*. (1)

5.2 Write down the amplitude of *g*. (1)

5.3 On the same axes, given in your SPECIAL ANSWER BOOK, draw the graphs of *f* and *g*. Clearly show the turning points, endpoints and the intercepts with the axes. (8)

5.4 Use your graphs to determine for which values of *x* is:

5.4.1 $g(x) \geq 0$ (2)

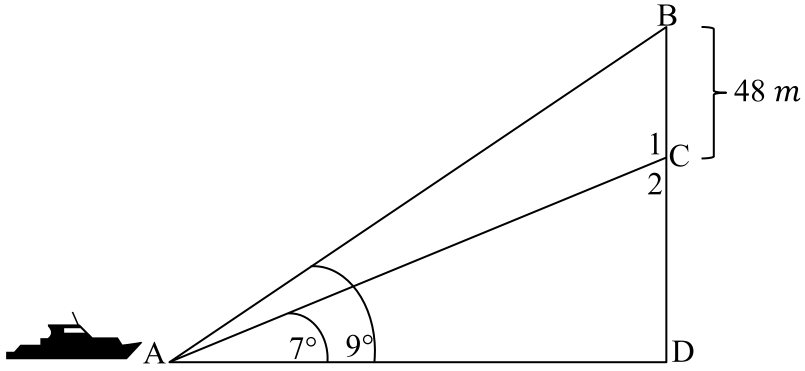
5.4.2 $f(x).g(x) < 0$ in the second quadrant (2)

[14]

QUESTION 6

6.1 Write down the sine rule for ΔABD. (1)

6.2 A ship at sea, observes that the angles of elevation to the top and bottom of a lighthouse on a cliff are 7° and 9° respectively. It is known that the height of the lighthouse is 48 m.



Determine:

6.2.1 The size of $B\hat{A}C$, stating a reason (2)

6.2.2 The size of $A\hat{B}D$, stating a reason (2)

6.2.3 The length of AC (4)

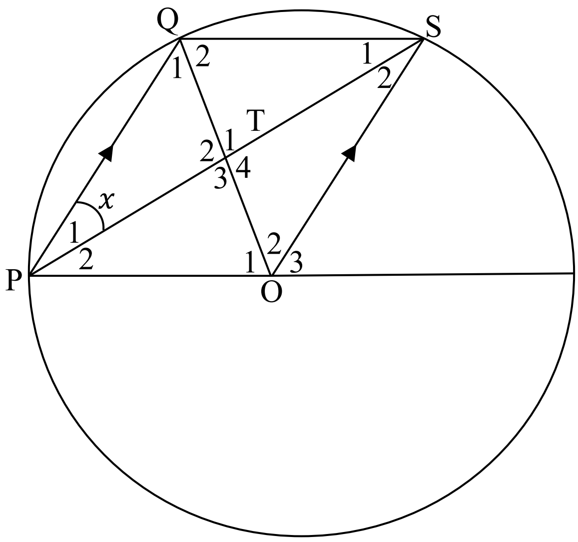
6.2.4 The distance between the ship and the bottom of the cliff (2)

6.2.5 The height of the cliff (3)

[14]

QUESTION 7

In the diagram below, O is the centre of the circle. OS ∥ PQ and PS meet OQ at T.



7.1 If $P_1 = x$, express T_1 in terms of x. Give reasons. (6)

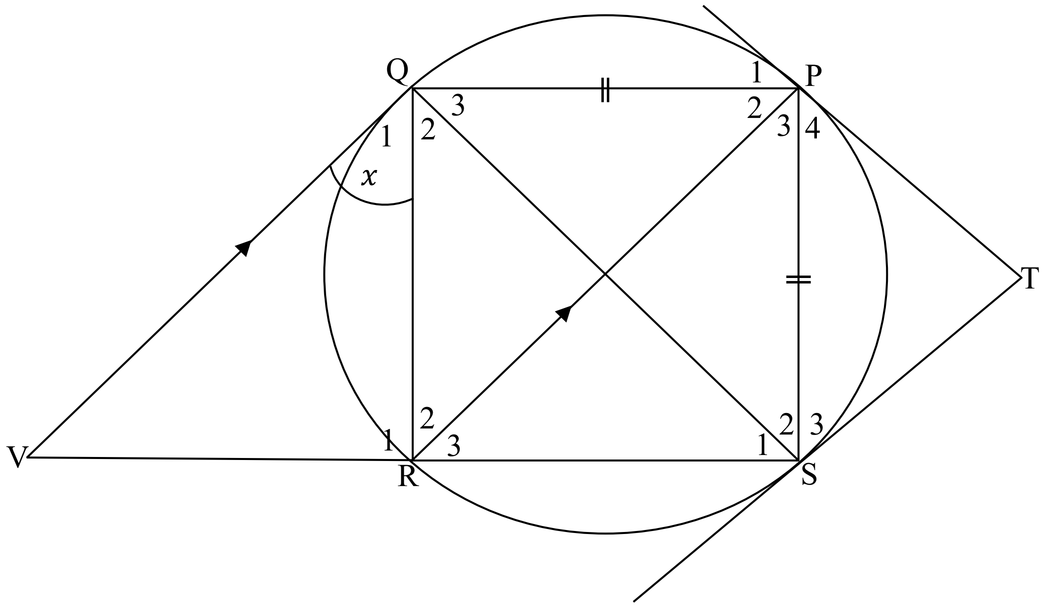
7.2 If $x = 30°$, calculate the sizes of the angles in ΔQST. Give reasons where necessary. (5)

7.3 Show that ΔPQS ≡ ΔSOP. (3)

[14]

QUESTION 8

PQRS is a cyclic quadrilateral with PS = PQ. SR is produced to meet V such that PR ∥ QV.
TP and TS are tangents to the circle. $Q_1 = x$.



8.1 Name, with reasons, four other angles equal to x. (8)

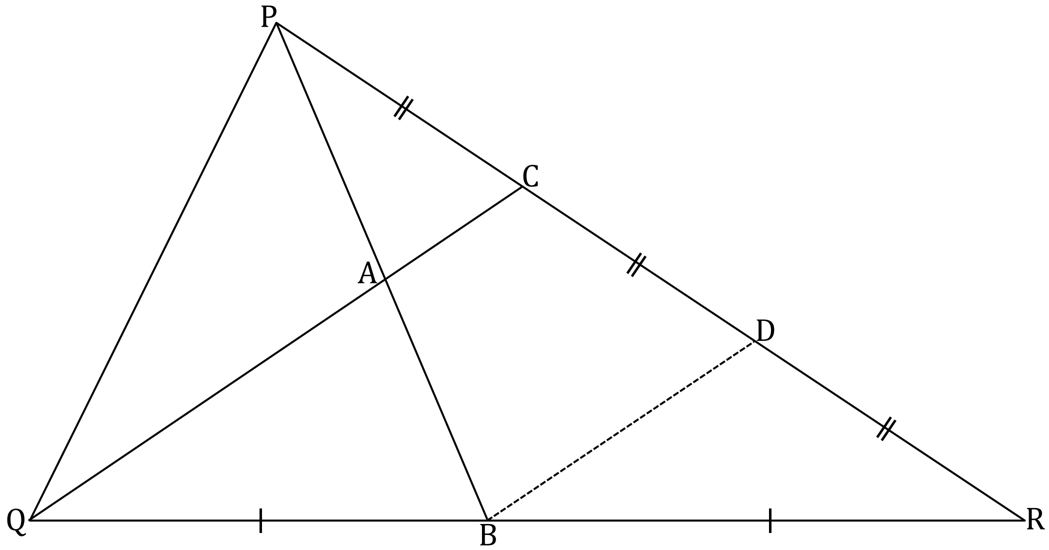
8.2 Give a reason for $P_4 = S_3$. (1)

8.3 Prove, with reasons, that $\hat{T} = Q\hat{P}S$. (5)

[14]

QUESTION 9

In the diagram below, B is the midpoint of side QR. C and D are points on PR such that PC = CD = DR. PR = 15 cm.



9.1 Show that BD ∥ QC. (3)

9.2 Prove that PA = AB. (3)

9.3 Determine the length of QR, if PD : DR = 2 : 1. (6)

[12]

QUESTION 10

A fan in a jet engine has a diameter of 340 cm and a circumferential velocity of 568 metres per second.

10.1 Convert 568 m/s to km/h. (2)

10.2 Determine the rotational frequency of the wheel in hours. (5)

10.3 Determine the angular velocity of the wheel in seconds. (3)

10.4 Determine the distance, in km, a point on the fan will cover in 15 seconds. (3)

10.5 Determine how long it will take the fan to make half a revolution. (2)

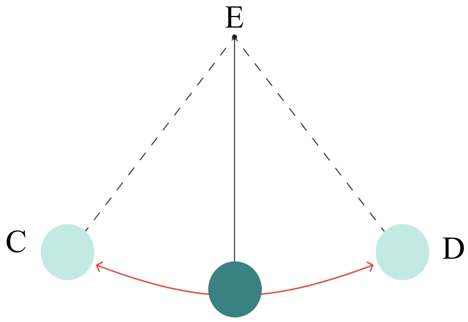
[15]

QUESTION 11

11.1 A pendulum in a clock, FIGURE A, follows the path as depicted in the diagram below, FIGURE B. There is a radius of 30 cm and the angle formed is 60°.

FIGURE A

FIGURE B



11.1.1 Determine the length of arc CD, that the pendulum follows. (3)

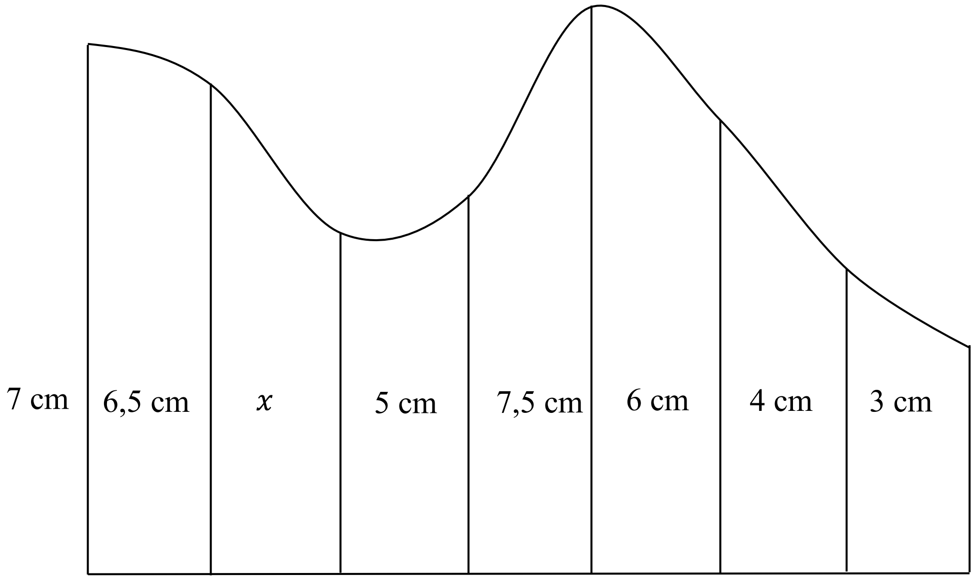
11.1.2 Determine the area of sector ECD. (3)

11.1.3 Calculate the length of the pendulum. (3)

11.2 An analogue clock has a diameter of 30 cm, and a chord length of 20 cm.

Determine the length of the hour hand. (5)

11.3 The ordinates in the irregular figure are: 7 cm, 6,5 cm, x , 5 cm; 7,5 cm, 6 cm, 4 cm and 3 cm respectively as indicated in the diagram below. The width of the irregular figure is 11,55 cm and the area is 63,525 cm^2.



Determine the length of the unknown ordinate x. (4)

[18]

TOTAL: 150

INFORMATION SHEET

$x = \frac{-b \pm \sqrt{b^2 - 4ac}}{2a}$ $\qquad$ $x = -\frac{b}{2a}$ $\qquad$ $y = \frac{4ac - b^2}{4a}$

$a^x = b \Leftrightarrow x = \log_a b, \quad a > 0,\ a \neq 1$ and $b > 0$

$A = P(1+ni)$ $\qquad$ $A = P(1-ni)$ $\qquad$ $A = P(1+i)^n$ $\qquad$ $A = P(1-i)^n$

$i_{eff} = \left(1 + \frac{i}{m}\right)^m - 1$

$f'(x) = \lim_{h \to 0} \frac{f(x+h) - f(x)}{h}$

$\int x^n dx = \frac{x^{n+1}}{n+1} + C \quad , \ n \neq -1$ $\qquad$ $\int kx^n dx = k \cdot \frac{x^{n+1}}{n+1} + C \quad , \ n \neq -1$

$\int \frac{1}{x} dx = \ln(x) + C, \quad x > 0$ $\qquad$ $\int \frac{k}{x} dx = k \cdot \ln(x) + C, \quad x > 0$

$\int a^x dx = \frac{a^x}{\ln a} + C \quad , \ a > 0$ $\qquad$ $\int ka^{nx} dx = k \cdot \frac{a^{nx}}{n \ln a} + C \quad , \ a > 0$

$d = \sqrt{(x_2 - x_1)^2 + (y_2 - y_1)^2}$ $\qquad$ $\mathrm{M}\left(\frac{x_1 + x_2}{2}; \frac{y_1 + y_2}{2}\right)$

$y = mx + c$ $\qquad$ $y - y_1 = m(x - x_1)$ $\qquad$ $m = \frac{y_2 - y_1}{x_2 - x_1}$ $\qquad$ $m = \tan\theta$

$\frac{x^2}{a^2} + \frac{y^2}{b^2} = 1$

In ΔABC:

$\frac{a}{\sin \mathrm{A}} = \frac{b}{\sin \mathrm{B}} = \frac{c}{\sin \mathrm{C}}$ $\qquad$ $a^2 = b^2 + c^2 - 2bc \cos \mathrm{A}$ $\qquad$ $\text{Area} = \frac{1}{2} ab.\sin \mathrm{C}$

$\sin^2\theta + \cos^2\theta = 1$ $\qquad$ $\tan^2\theta + 1 = \sec^2\theta$ $\qquad$ $1 + \cot^2\theta = \mathrm{cosec}^2\theta$

$\pi rad = 180°$

Angular velocity = $\omega = 2\pi n$ where n = rotation frequency

Angular velocity = $\omega = 360°n$ where n = rotation frequency

Circumferential velocity = $v = \pi Dn$ where D = diameter and n = rotation frequency

Circumferential velocity = $v = \omega r$ where ω = Angular velocity and r = radius

Arc length $s = r\theta$ where r = radius and θ = central angle in radians

Area of a sector $= \dfrac{rs}{2}$ where r = radius and s = arc length

Area of a sector $= \dfrac{r^2\theta}{2}$ where r = radius and θ = central angle in radians

$4h^2 - 4dh + x^2 = 0$ where h = height of segment, d = diameter of the circle and x = length of chord

$A_T = a\left(m_1 + m_2 + m_3 + \ldots + m_{n-1}\right)$ where a = width of equal parts, $m_1 = \dfrac{o_1 + o_2}{2}$ and n = number of ordinates

OR

$A_T = a\left(\dfrac{o_1 + o_n}{2} + o_2 + o_3 + o_4 + \ldots + o_{n-1}\right)$ where a = width of equal parts, $o_i = i^{th}$ ordinate and n = number of ordinates