Province of the
EASTERN CAPE
EDUCATION

NATIONAL SENIOR CERTIFICATE

GRADE 12

JUNE 2024

MATHEMATICS P2

MARKS: 150

TIME: 3 hours

FONT SIZE: 18 PT

This question paper consists of 20 pages, a formula sheet and an answer book of 25 pages.

INSTRUCTIONS AND INFORMATION

Read the following instructions carefully before answering the questions.

1. This question paper consists of 10 questions.

2. Answer ALL the questions in the SPECIAL ANSWER BOOK provided.

3. Clearly show ALL calculations, diagrams, graphs, etc. which you have used in determining the answers.

4. Answers only will NOT necessarily be awarded full marks.

5. You may use an approved scientific calculator (non-programmable and non-graphical), unless stated otherwise.

6. If necessary, round off answers to TWO decimal places, unless stated otherwise.

7. Diagrams are NOT necessarily drawn to scale.

8 An information sheet with formulae is included at the end of the question paper.

9. Write neatly and legibly.

QUESTION 1

The maximum daily temperatures in Bloemfontein for the first 11 days in January were recorded as indicated in the table below.

27	32	35	36	30	27	17	26	34	37	40

1.1 Calculate the mean for the maximum daily temperatures for the first 11 days in January. (2)

1.2 Calculate the standard deviation. (1)

1.3 How many days were the temperatures more than one standard deviation of the mean? (3)

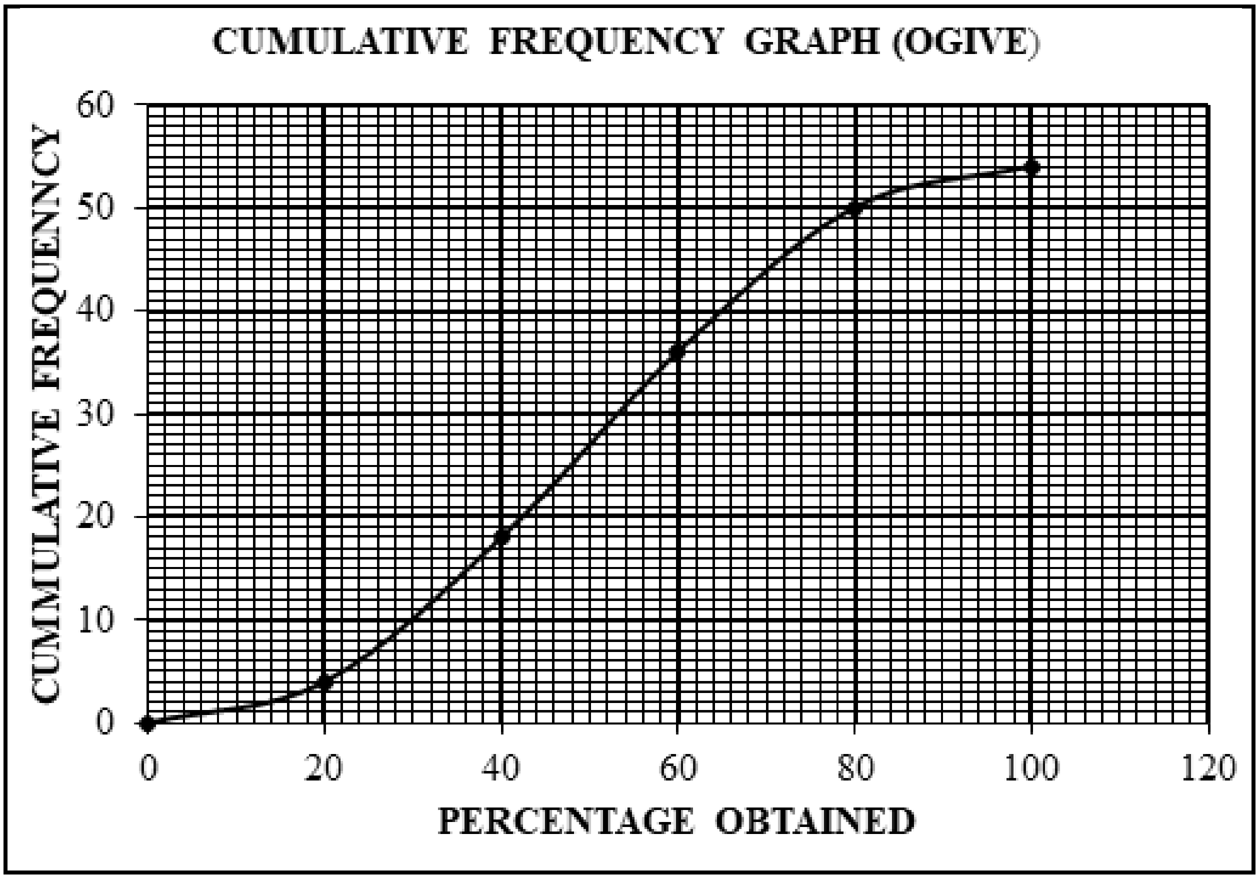
1.4 Determine the interquartile range of the data. (3)

1.5 Draw a box-and-whisker diagram in the grid provided on the ANSWER BOOK. (3)

[12]

QUESTION 2

In a certain school, the analysis of mathematics matric results in percentages were represented in the cumulative frequency graph (ogive) given below.



Use the above graph to answer the following questions.

2.1 Complete the frequency table provided in the ANSWER BOOK.

Percentage obtained	**Frequency**	**Cumulative frequency**
$0 \leq x < 20$		4
$20 \leq x < 40$		18
$40 \leq x < 60$		36
$60 \leq x < 80$		50
$80 \leq x < 100$		54

(2)

2.2 Write down the total number of matriculants who wrote mathematics in this school. (1)

2.3 Write down the modal class. (1)

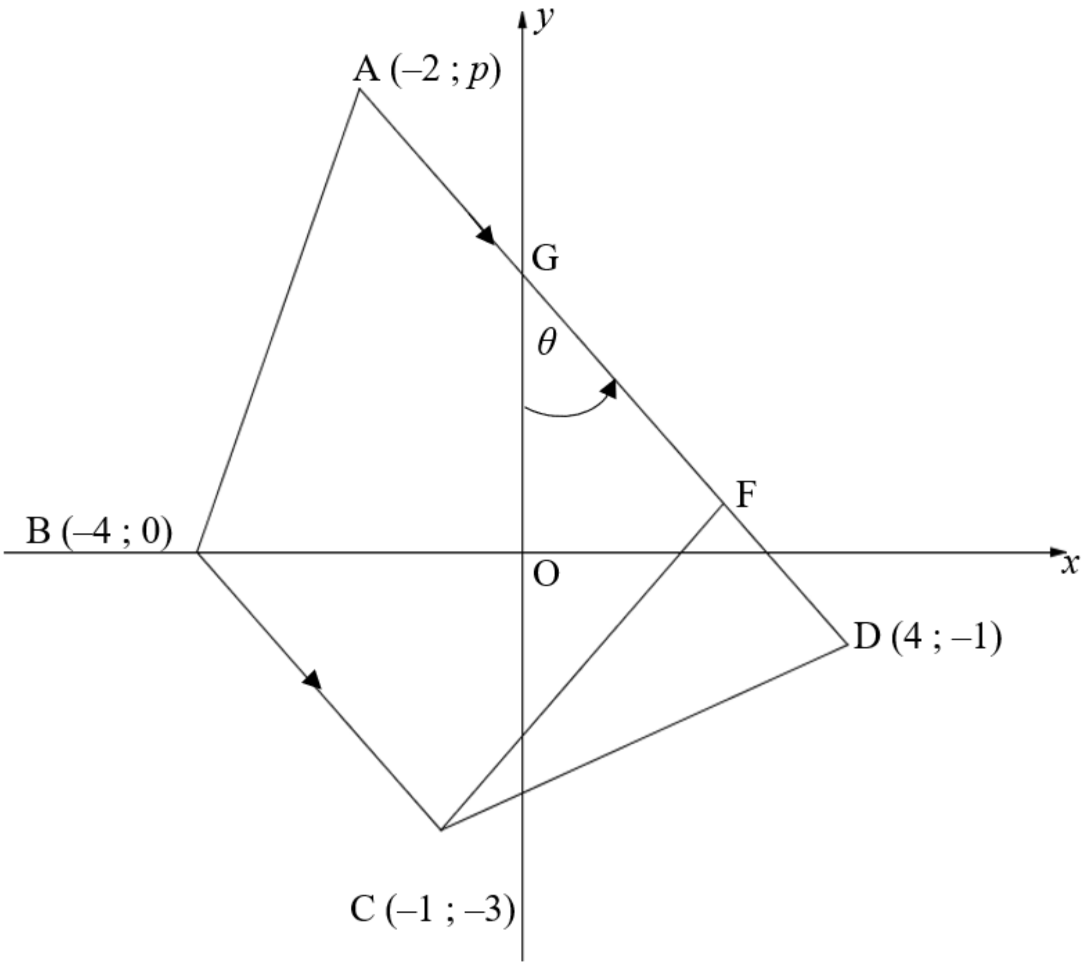
2.4 Estimate the median percentage for mathematics of this school. (2)

2.5 If the requirement for a learner to be admitted in a certain institution is 70% and more in mathematics, determine how many matriculants will qualify for admission. (2)

[8]

QUESTION 3

In the diagram below, A $(-2\ ;\ p)$, B$(-4\ ;\ 0)$, C$(-1\ ;\ -3)$ and D$(4\ ;\ -1)$ are the vertices of a trapezium. AD||BC. Point G is the y-intercept of line AD. F lies on line AD.



3.1 Determine the length of BC. (2)

3.2 Determine the gradient of BC. (2)

3.3 Determine the equation of line AD in the form $y = mx + c$. (3)

3.4 Calculate the value of p. (2)

3.5 If the coordinates of F are $\left(\frac{5}{2};\frac{1}{2}\right)$, show that CF ⊥ AD. (2)

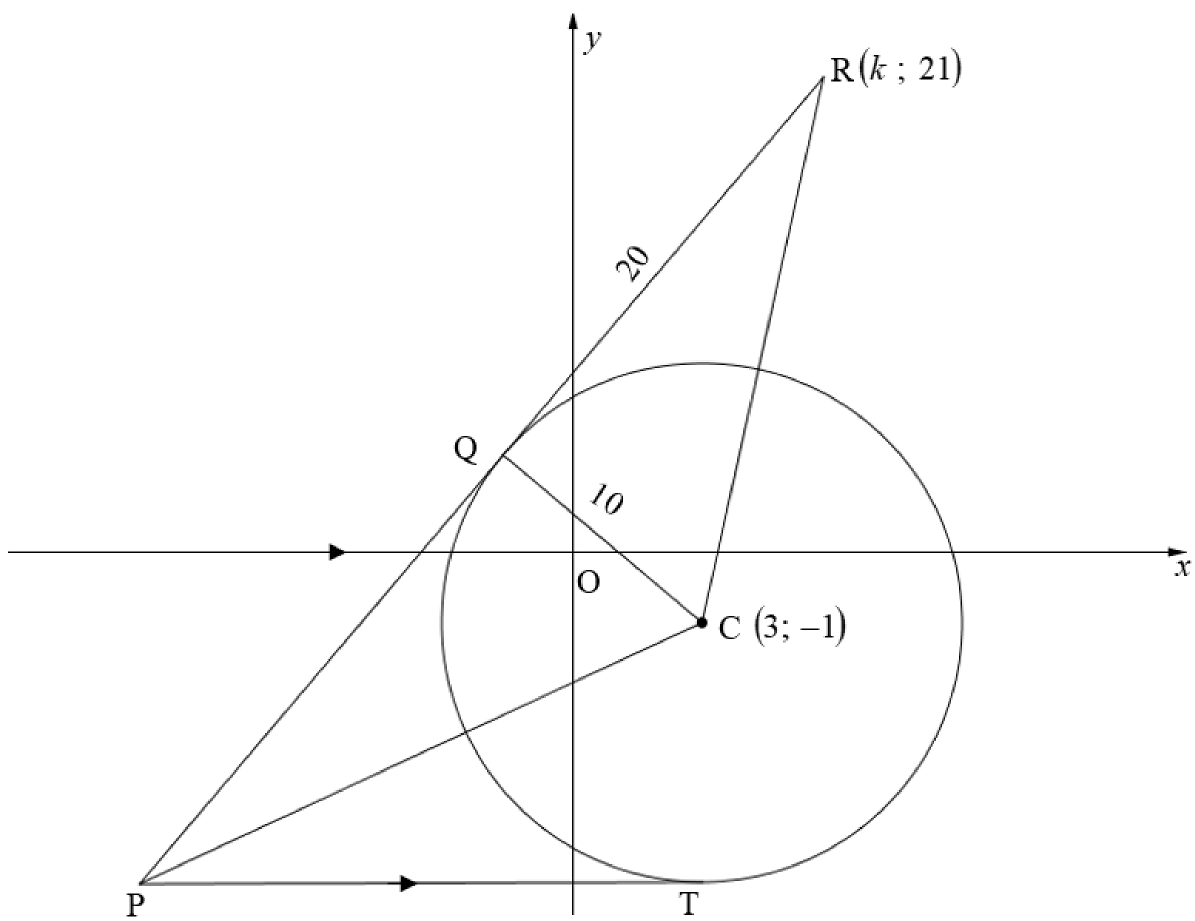
3.6 Calculate the size of θ. (3)

3.7 Calculate the area of trapezium ABCD. (4)

[18]

QUESTION 4

In the diagram below, a circle with centre C(3;–1) and a radius of 10 units is drawn. PQR and PT are tangents to the circle at Q and T respectively. PT is parallel to the x-axis.
R (k ; 21), C and P are vertices of ΔRCP. QR = 20 units.



4.1 Write down the size of $C\hat{Q}R$. (1)

4.2 Calculate the length of RC, and leave your answer in surd form. (2)

4.3 Calculate the value of k, if R lies in the first quadrant. (4)

4.4 Determine the equation of the circle with centre C, passing through T and Q. Write your answer in the form $(x-a)^2+(y-b)^2=r^2$. (2)

4.5 Determine the equation of PT. (2)

4.6 The equation of line PR is given by $3y-4x=35$.

4.6.1 Calculate the coordinates of P. (2)

4.6.2 Calculate the length of PQ with a reason. (2)

4.6.3 Is the area of ΔQRC = area of ΔQCP? Motivate your answer. (3)

4.7 Consider another circle with equation $(x-3)^2+(y+16)^2=16$ and having centre M.

4.7.1 Write down the coordinates of the centre M. (1)

4.7.2 Write down the length of the radius of the circle with centre M. (1)

4.7.3 Prove that the circle with centre C and the circle with centre M, do not touch each other (intersect). (3)

[23]

QUESTION 5

5.1 If $\sin 14^\circ = p$, **without using a calculator**, determine the values of the following in terms of p:

5.1.1 $\cos 76^\circ$ (2)

5.1.2 $\cos 44^\circ$ (4)

5.1.3 $2\sin 218^\circ . \cos 38^\circ$ (3)

5.2 Given: $1 + \dfrac{\sin(90^\circ + \theta).\cos(\theta - 360^\circ)}{\sin(\theta - 30^\circ).\cos\theta - \sin\theta.\cos(\theta - 30^\circ)}$

5.2.1 Simplify to a single trigonometric ratio of θ **without using a calculator**:

$$1 + \frac{\sin(90^\circ + \theta).\cos(\theta - 360^\circ)}{\sin(\theta - 30^\circ).\cos\theta - \sin\theta.\cos(\theta - 30^\circ)}$$ (6)

5.2.2 Write down the maximum value of

$$y = 1 + \frac{sin(90^0 + \theta) . cos(\theta - 360^0)}{sin(\theta - 30^0) . cos\,\theta - sin\,\theta . cos(\theta - 30^0)}$$ (1)

5.3 Prove that $\dfrac{sin\,3x}{sin\,x} = 3 - 4\,sin^2\,x$ (5)

5.4 Given: $\sin^2 x + \sin 2x - 3\cos^2 x = 0$.

5.4.1 Determine the general solution of the above equation. (5)

5.4.2 Hence, or otherwise, determine the values of x in the interval $x \in [-90^\circ ; 180^\circ]$. (3)

[29]

QUESTION 6

6.1 Sketch the graphs of $f(x) = 3\,sin\,x$ and $g(x) = tan\frac{1}{2}x$ in the interval of $x \in [-180^0; 180^0]$ on the grid provided in the ANSWER BOOK. Clearly show all intercepts with the axes, turning points and the asymptotes. (6)

6.2 Use your graphs to answer the following questions for $x \in [-180^{\circ}; 180^{\circ}]$

6.2.1 Write down the period of g. (1)

6.2.2 Write down the value(s) of x for which the graph of g is undefined. (2)

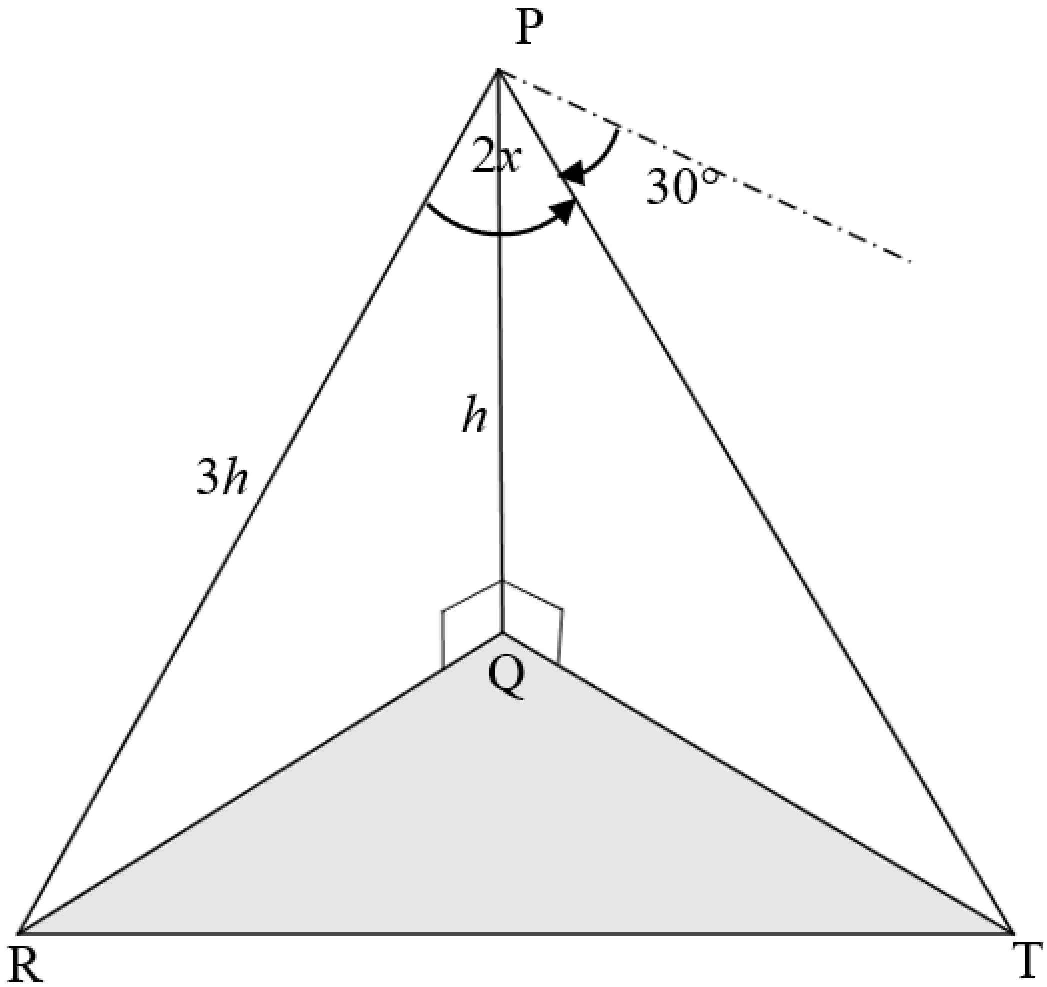
6.2.3 Write down the range of h if $h(x) = f(x) - 2$. (2)

6.2.4 How many solutions exists for $f(x) = g(x)$? (1)

[12]

QUESTION 7

In the diagram below, PQ is a vertical pole having height h metres. R, Q and T are three points on the same horizontal plane. PR and PT are cables and the angle of depression from P to T is 30°. PR = $3h$ and angle $R\hat{P}T = 2x$.



7.1 Write down the size of $P\hat{T}Q$. (1)

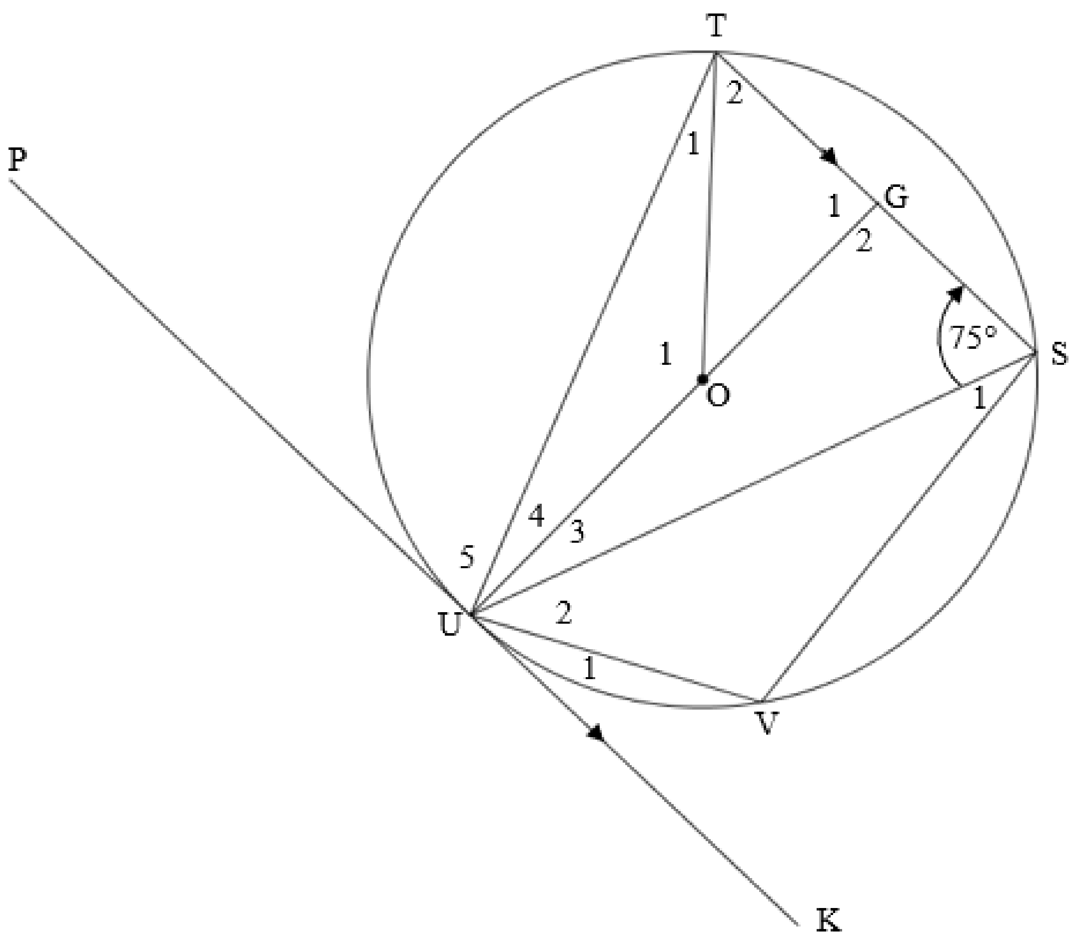
7.2 Determine the length of PT in terms of h. (3)

7.3 Calculate the size of x if $RT = \sqrt{7}h$. (5)

[9]

QUESTION 8

In the diagram below, the circle having centre O, passes through U, T, S and V. PUK is a tangent to the circle at U. TS|| PK. UOG is a straight line. $T\hat{S}U = 75^\circ$.



8.1 Calculate with reasons the size of:

8.1.1 $\hat{O}_1$ (2)

8.1.2 $\hat{U}_5$ (2)

8.1.3 $\hat{T}_1$ (3)

8.1.4 $\hat{V}$ (3)

8.1.5 $\hat{U}_3$ (2)

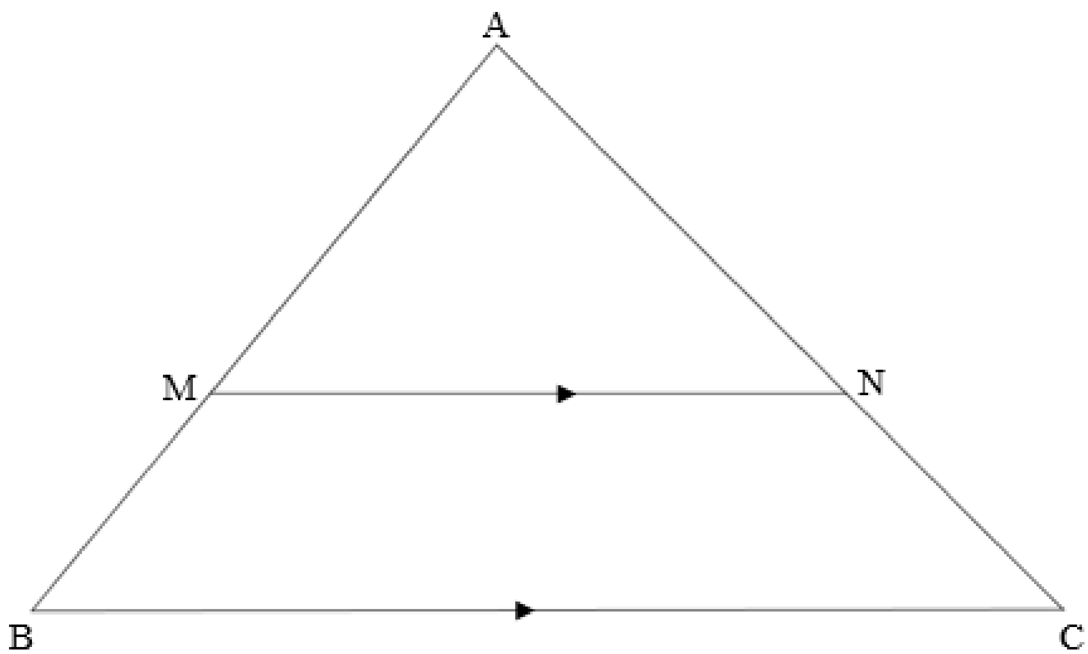
8.1.6 $\hat{G}_2$ (3)

8.2 If it is further given that $TS = \sqrt{80}$, calculate the length of TG with reasons, and leave your answer in simplest surd form. (2)

[17]

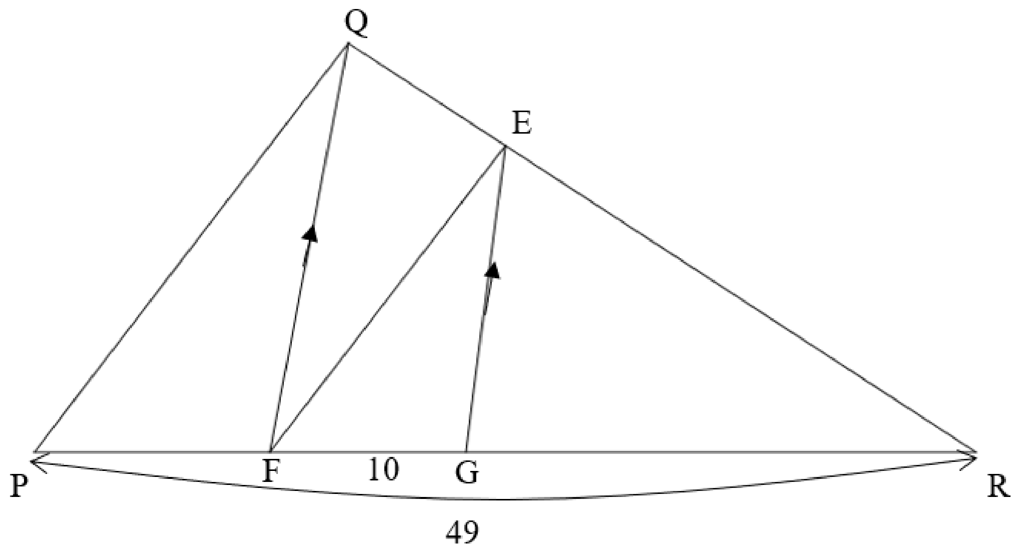
QUESTION 9

9.1 In ΔABC below, M is a point on AB and N is a point on AC, such that MN || BC.



Prove the theorem which states that $\frac{\text{AM}}{\text{MB}} = \frac{\text{AN}}{\text{NC}}$. (5)

9.2 In the diagram below, ΔPQR is drawn, EG || QF and EF is a straight line. $QE:ER = 2:5$. $PR = 49$ units and $FG = 10$ units.



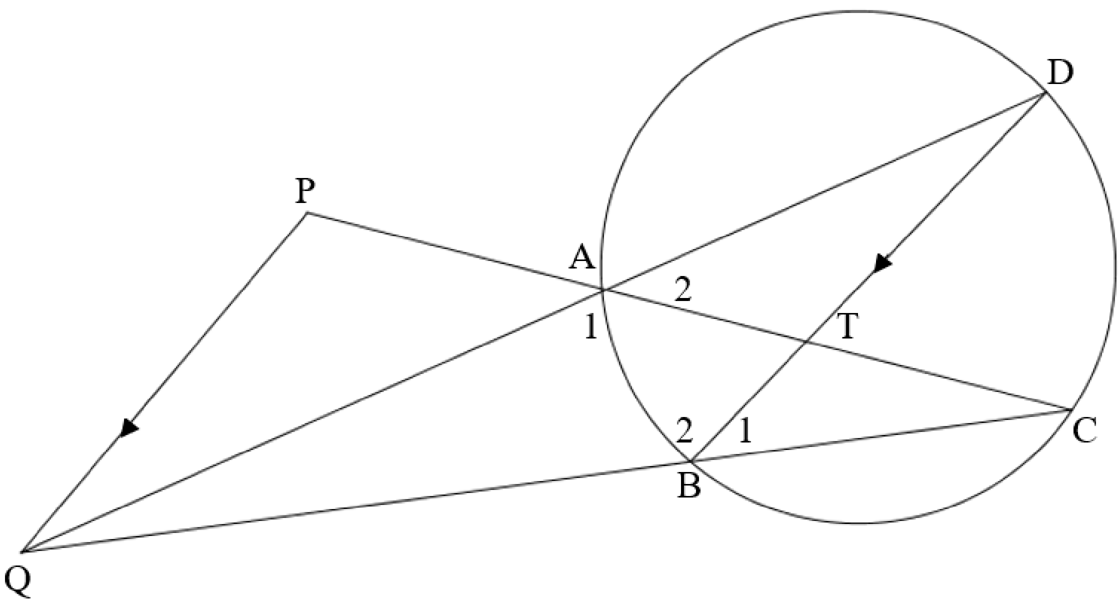
9.2.1 Calculate the length of GR with reasons. (4)

9.2.2 Prove that FE || PQ. (3)

[12]

QUESTION 10

In the diagram below, A, B, C and D are points on the circumference of the circle. PC and QC are drawn from P and Q respectively and intersect at C. QP is joined. DB||PQ.
$QB = 5BC$.



Prove that:

10.1 $\frac{CT}{PC} = \frac{1}{6}$ (3)

10.2 $\Delta QAC ||| \Delta QBD$ (4)

10.3 $QD.QA = 30BC^2$ (3)

[10]

TOTAL: 150

INFORMATION SHEET

$$x = \frac{-b \pm \sqrt{b^2 - 4ac}}{2a}$$

$$A = P(1 + ni) \quad A = P(1 - ni) \qquad A = P(1 - i)^n$$

$$A = P(1 + i)^n$$

$$T_n = a + (n - 1)d \qquad S_n = \frac{n}{2}[2a + (n - 1)d]$$

$$T_n = ar^{n-1} \qquad S_n = \frac{a(r^n - 1)}{r - 1} \; ; r \neq 1$$

$$S_\infty = \frac{a}{1 - r}; -1 < r < 1$$

$$F = \frac{x[(1+i)^n - 1]}{i} \qquad P = \frac{x[1 - (1+i)^{-n}]}{i}$$

$$f'(x) = \lim_{h \to 0} \frac{f(x+h) - f(x)}{h}$$

$$d = \sqrt{(x_2 - x_1)^2 + (y_2 - y_1)^2} \qquad \mathrm{M}\left(\frac{x_1 + x_2}{2}; \frac{y_1 + y_2}{2}\right)$$

$$y = mx + c \qquad y - y_1 = m(x - x_1) \qquad m = \frac{y_2 - y_1}{x_2 - x_1}$$

$$m = \tan\theta$$

$$(x - a)^2 + (y - b)^2 = r^2$$

In ΔABC: $\frac{a}{\sin A} = \frac{b}{\sin B} = \frac{c}{\sin C}$

$$a^2 = b^2 + c^2 - 2bc.\cos A$$

$$area\ \Delta ABC = \frac{1}{2}ab.\sin C$$

 Please turn over

$$\sin(\alpha+\beta)=\sin\alpha.\cos\beta+\cos\alpha.\sin\beta$$

$$\sin(\alpha-\beta)=\sin\alpha.\cos\beta-\cos\alpha.\sin\beta$$

$$\cos(\alpha+\beta)=\cos\alpha.\cos\beta-\sin\alpha.\sin\beta$$

$$\cos(\alpha-\beta)=\cos\alpha.\cos\beta+\sin\alpha.\sin\beta$$

$$\cos 2\alpha=\begin{cases}\cos^2\alpha-\sin^2\alpha\\ 1-2\sin^2\alpha\\ 2\cos^2\alpha-1\end{cases}$$

$$\sin 2\alpha=2\sin\alpha.\cos\alpha$$

$$\bar{x}=\frac{\sum fx}{n}$$

$$\sigma^2=\frac{\sum_{i=1}^{n}(x_i-\bar{x})^2}{n}$$