Province of the
EASTERN CAPE
EDUCATION

NATIONAL SENIOR CERTIFICATE

GRADE 12

JUNE 2024

MATHEMATICS P1

MARKS: **150**

TIME: **3 hours**

This question paper consists of 11 pages, including 1 information sheet.

INSTRUCTIONS AND INFORMATION

Read the following instructions carefully before answering the questions.

1. This question paper consists of 10 questions.
2. Answer ALL the questions.
3. Clearly show ALL calculations, diagrams, graphs, et cetera that you have used in determining your answers.
4. Answers only will NOT necessarily be awarded full marks.
5. You may use an approved scientific calculator (non-programmable and non-graphical), unless stated otherwise.
6. If necessary, round off answers to TWO decimal places, unless stated otherwise.
7. Diagrams are NOT necessarily drawn to scale.
8. An information sheet, with formulae, is included at the end of the question paper.
9. Number the answers correctly according to the numbering system used in this question paper.
10. Write neatly and legibly.

QUESTION 1

1.1 Solve for x:

1.1.1 $x^2 - 8(x-2) = 25$ (3)

1.1.2 $-3x^2 + 2x + 2 = 0$ (correct to TWO decimal places) (3)

1.1.3 $(x+3)(5-x) \leq 0$ (3)

1.1.4 Given: $\dfrac{x+3}{\sqrt{x+5}} = 1;\ x \in \mathbb{R}$

(a) For which value(s) of x will $\dfrac{x+3}{\sqrt{x+5}}$ be undefined? (2)

(b) Solve for x. (4)

1.2 Solve simultaneously for x and y:

$y + 2x = 5$

$2x^2 - xy - 4y^2 = 8$ (6)

1.3 Given that: $M = \dfrac{108}{x^2 - 4x + 8};\ x \in \mathbb{R}$, determine the maximum value of M. (4)

[25]

QUESTION 2

2.1 Given the following arithmetic sequence: $2; -3; -8; \ldots$

2.1.1 Determine the value of T_{43}. (3)

2.1.2 Calculate the sum of the first 43 terms in the row, i.e. S_{43}. (2)

2.1.3 Calculate the value of n for which $T_n = -2023$. (3)

2.2 Given: $2(3x-1)+2(3x-1)^2+\ldots$

2.2.1 For which values of x is the series above a convergent geometric series? (3)

2.2.2 Calculate $\sum_{k=1}^{\infty} 2(3x-1)^k$; if $x=\frac{1}{2}$ (3)

2.3 The first three terms of a geometric sequence has a sum of 21 and their product is 64. Determine the value of the first term, if the common ratio is an integer, i.e. $r \in \mathbb{Z}$. (4)

[18]

QUESTION 3

Consider the following quadratic number pattern: 3 ; 12 ; 33 ; . . .

3.1 Write down the next term in the quadratic number pattern. (1)

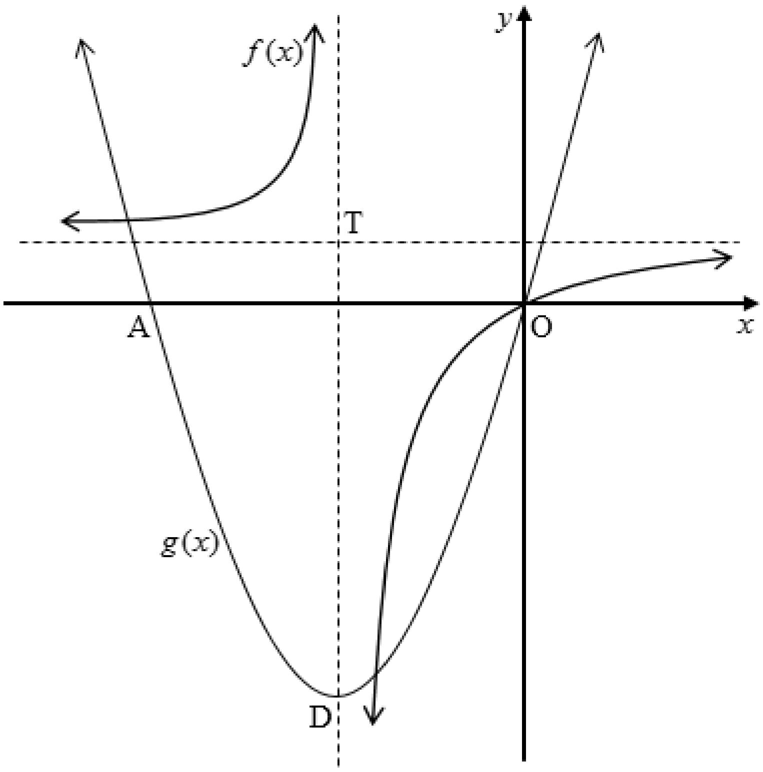
3.2 Determine the general term of the quadratic number pattern in the form $T_n = an^2 + bn + c$. (3)

3.3 Which TWO terms in the quadratic number pattern will have a difference of 345? (3)

[7]

QUESTION 4

The diagram below shows the graphs of $f(x)=\dfrac{2}{x+2}+1$ and $g(x)=a(x+2)^2-8$.
Both graphs pass through the origin, O. The vertical asymptote of f passes through D, the turning point of g. The asymptotes of f intersect at T. A is the other x-intercept of g.



4.1 Write down the coordinates of D, the turning point of g. (1)

4.2 Write down the equations of the asymptotes of f. (2)

4.3 Determine:

4.3.1 The value of a (2)

4.3.2 The length OA (3)

4.3.3 The range of f (1)

4.3.4 The equation of the axis of symmetry of f with a negative gradient (2)

4.4 For which values of x will:

4.4.1 $g(x)<0$? (2)

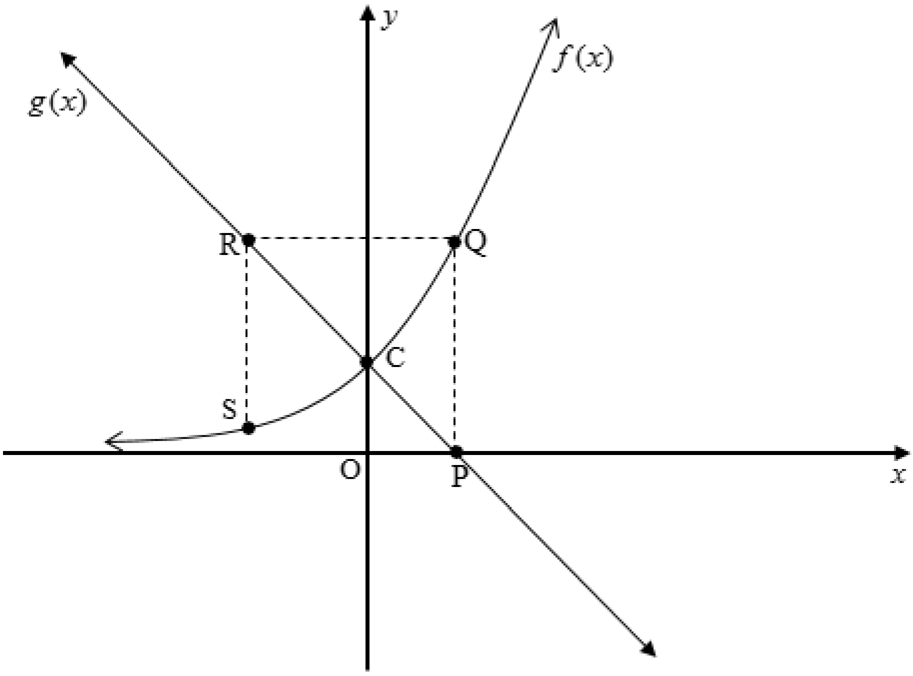
4.4.2 $g(x).f(x)\geq 0$? (2)

4.5 Determine the value(s) of k, for which $h(x)=-g(x)+k$ will have two distinct roots with the same sign. (3)

[18]

QUESTION 5

In the diagram below, the graphs of $f(x) = 3^x$ and $g(x) = -x + 1$ are given.



5.1	Write down the coordinates of C.	(1)
5.2	Write down the range of $f(x)$.	(1)
5.3	Determine the equation of $f^{-1}(x)$, in the form $y = ...$	(2)
5.4	For which values of x is $f^{-1}(x) < -1$?	(2)
5.5	If PQ || SR || *y-axis* and QR ‖ *x-axis*, determine the coordinates of S.	(4)
5.6	Describe the translation(s) of $f(x)$ to $p(x) = 3(3^x) - 2$	(2)
		[12]

QUESTION 6

6.1 The purchase price of machinery bought by a company 5 years ago was R80 000. Using the reducing-balance method, calculate the annual rate of depreciation if the current book value of the machinery is R20 000. (3)

6.2 Calculate the effective interest rate per annum of an investment earning interest at 8,5% p.a. compounded quarterly. (3)

6.3 A parent made an initial deposit of R x into a study investment account. Three years later a further amount of R15 000 is deposited into the account. Five years after the initial deposit was made, R7 000 was withdrawn from the account. The interest rate for the first five years was 11% p.a. compounded monthly. Thereafter the interest rate changed to 12% p.a. compounded half-yearly.

6.3.1 Calculate, in terms of x, how much money was in the account 3 years after the initial deposit was made. (This answer should not include the second deposit.) (2)

6.3.2 If the investment was worth R90 132,56 after 8 years, calculate the initial amount that was deposited, i.e. the value of x. (5)

[13]

QUESTION 7

7.1 Determine $f'(x)$, from first principles, if $f(x)=\frac{1}{2}x^2$. (4)

7.2 Determine:

7.2.1 $f'(x)$, if $f(x)=\frac{1}{5}x^5-6x^{-2}$ (2)

7.2.2 $\frac{d}{dx}\left(x+\sqrt{x}\right)^2$ (4)

[10]

QUESTION 8

8.1 Given: $f(x) = -x^3 + 12x - 16$

8.1.1 Show that $(x-2)$ is a factor of $f(x)$. (2)

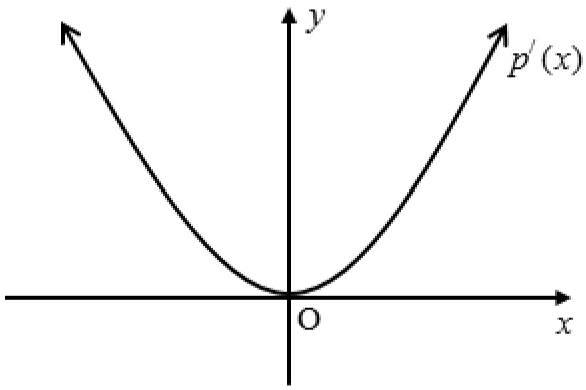
8.1.2 Determine the x-intercepts of f. (3)

8.1.3 Determine the coordinates of the turning points of f. (4)

8.1.4 Sketch the graph of f, clearly indicating turning points and intercepts with the axes. (3)

8.1.5 Determine the equation of the tangent at the point of inflection. (4)

8.2 A sketch graph of $p'(x)$ is given below.



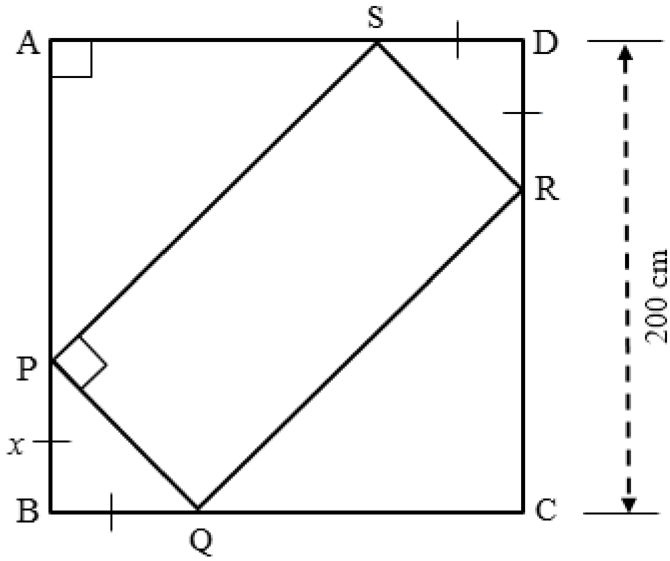
8.2.1 For which values of x is the graph of $p(x)$ increasing? (2)

8.2.2 For which values of x is the graph of $p(x)$ concave up? (2)

[20]

QUESTION 9

In the diagram below, ABCD is a square with side length CD = 200 cm. PQRS is a rectangle with vertices on the sides of the square. PB = BQ = SD = DR = x cm.



9.1 Show that the area of the rectangle is given by, $\mathrm{A} = 2\left(200x - x^2\right)$. (3)

9.2 Determine the value of x for which the area of the rectangle will be a maximum. (3)

9.3 What is the ratio of the maximum area of PQRS : area of ABCD? (3)

[9]

QUESTION 10

10.1 Events A and B are independent events. It is further given that:

- P(A) = 0,6
- P(B) = 0,5

10.1.1 Are the events mutually exclusive? Motivate your answer. (2)

10.1.2 Represent the information on a Venn-diagram. (3)

10.1.3 Calculate:

(a) P(only A) (1)

(b) P(not A or not B) (2)

10.2 The contingency table below represents 100 learners' responses regarding camping.

	Boys	**Girls**	**Total**
Like Camping	24	30	54
Dislike Camping	14	32	46
Total	38	62	100

10.2.1 If a learner from this group is chosen randomly, what is the probability that it is a girl? (1)

10.2.2 Is the event, "like camping" independent of the gender? (4)

10.3 There are only red balls and green balls in a bag. A ball is taken at random from the bag. The probability that the ball is green is $\frac{3}{7}$. The ball is replaced in the bag. 2 more red balls and 3 more green balls are put in the bag. Thereafter, a ball is taken at random from the bag and the probability that this ball is green is $\frac{6}{13}$.

Determine how many of each colour ball was originally in the bag. (5)

[18]

TOTAL: 150

INFORMATION SHEET: MATHEMATICS

$$x = \frac{-b \pm \sqrt{b^2 - 4ac}}{2a}$$

$A = P(1+ni)$ $\quad A = P(1-ni)$ $\quad A = P(1-i)^n$ $\quad A = P(1+i)^n$

$T_n = a + (n-1)d$ $\quad S_n = \frac{n}{2}(2a + (n-1)d)$

$T_n = ar^{n-1}$ $\quad S_n = \frac{a(r^n - 1)}{r - 1}$; $r \neq 1$ $\quad S_\infty = \frac{a}{1-r}$; $-1 < r < 1$

$F = \frac{x\left[(1+i)^n - 1\right]}{i}$ $\quad P = \frac{x\left[1 - (1+i)^{-n}\right]}{i}$

$$f'(x) = \lim_{h \to 0} \frac{f(x+h) - f(x)}{h}$$

$d = \sqrt{(x_2 - x_1)^2 + (y_2 - y_1)^2}$ $\quad \mathrm{M}\left(\frac{x_1 + x_2}{2} ; \frac{y_1 + y_2}{2}\right)$

$y = mx + c$ $\quad y - y_1 = m(x - x_1)$ $\quad m = \frac{y_2 - y_1}{x_2 - x_1}$ $\quad m = \tan\theta$

$(x-a)^2 + (y-b)^2 = r^2$

In ΔABC:

$\frac{a}{\sin A} = \frac{b}{\sin B} = \frac{c}{\sin C}$ $\quad a^2 = b^2 + c^2 - 2bc.\cos A$ $\quad area\,\Delta ABC = \frac{1}{2}ab.\sin C$

$\sin(\alpha + \beta) = \sin\alpha.\cos\beta + \cos\alpha.\sin\beta$ $\quad \sin(\alpha - \beta) = \sin\alpha.\cos\beta - \cos\alpha.\sin\beta$

$\cos(\alpha + \beta) = \cos\alpha.\cos\beta - \sin\alpha.\sin\beta$ $\quad \cos(\alpha - \beta) = \cos\alpha.\cos\beta + \sin\alpha.\sin\beta$

$\cos 2\alpha = \begin{cases} \cos^2\alpha - \sin^2\alpha \\ 1 - 2\sin^2\alpha \\ 2\cos^2\alpha - 1 \end{cases}$ $\quad \sin 2\alpha = 2\sin\alpha.\cos\alpha$

$\bar{x} = \frac{\sum x}{n}$ $\quad \partial^2 = \frac{\sum_{i=1}^{n}(x_i - \bar{x})^2}{n}$ $\quad P(A) = \frac{n(A)}{n(S)}$

P (A or B) = P (A) + P (B) – P (A and B)

$\hat{y} = a + bx$ $\quad b = \frac{\sum(x - \bar{x})(y - \bar{y})}{\sum(x - \bar{x})^2}$